Seismic risk assessment in Bridge Management Systems

Y. C. Yue
School of Human Settlement and Civil Engineering, Xi'an Jiaotong University, China

M. Pozzi, D. Zonta, F. Bortot & R. Zandonini
DIMS, University of Trento, Italy

SUMMARY:
This paper is dedicated to the development of a comprehensive framework for seismic vulnerability assessment for bridge management system. This framework can provide network level seismic risk assessment after the earthquake. In this paper, the system level seismic risk analysis is performed by way of the connectivity analysis. The seismic risk correlation between the components is incorporated into the analysis based on the Bayesian Network with continuous variables. The results are very helpful for bridge managers and government officials in understanding the network status and can assist them in making rapid decisions in near-real time, under post earthquake conditions.

Keywords: seismic vulnerability assessment, bridge management system, connectivity analysis, correlation

1. INTRODUCTION

We have already described, in Yue et al. (2010), how the Department of Transportation of Autonomous Province of Trento (APT) is addressing the problem of the seismic vulnerability of its bridge stock. APT manages more than 1000 bridges and approximately 2400 kilometers of roads, through a comprehensive Bridge Management System (BMS). The APT’s BMS includes evaluation of seismic vulnerability of each bridge, based on the fragility curve approach, consistent with Hazus guidelines (FEMA 2003). In Yue et al. (2010), we learned that the seismic risk in the APT stock is moderate. However, the system operation at network level is of concern in a post earthquake situation. Approximately 15% of the bridges in the APT stock have a relatively high risk of suffering operational problems. It is therefore necessary to understand the network operation after the earthquake. In this paper, the connectivity between any two given places within the network is calculated. The connectivity reliability of a network states the probability that the traffic can reach the destination from the origin. It is very helpful for bridge managers and government officials in understanding the network status and can assist them to make rapid decisions in near-real time, under post earthquake conditions. However, in the network level calculation, we find that it is important to consider the correlations between different bridges. In order to understand the correlation between bridges, in the second part of this paper, we proposed a probabilistic framework to estimate the condition state of a bridge stock in a post-earthquake situation based on the knowledge of the state of other bridge.

2. CONNECTIVITY ANALYSIS IN APT-BMS

2.1. Definition of network connectivity

![Figure 2.1 Simple network with two nodes and three bridges](image)

Figure 2.1 Simple network with two nodes and three bridges
The connectivity reliability of a network states the probability that the traffic can reach the destination from the origin (Liu and Frangopol, 2006). In this paper, it is assumed that the bridge elements are the only vulnerable parts of the network, and that the roads between any two bridges will never fail. Fig. 2.1 is a simple network with two nodes and three bridges. Bridge i, for i = 1, 2, 3, is in the operational mode with probability $p_i$, and in the failed mode with probability $q_i = 1 - p_i$. The values of $p_i$ (i = 1, 2, 3) are 0.7, 0.8, and 0.9 respectively. In this example, we assume that there is no correlation between these bridges; they are all independent of each other. The links between the nodes and the bridges are assumed to be safe. A bridge mode vector $V$ is used to denote the state mode of the bridges: $V_i = 1$ if bridge $i$ is in failed mode, and 0 if in operational mode.

### Table 2.1. all the network states in Fig. 2.1

<table>
<thead>
<tr>
<th>Network state</th>
<th>Vector V</th>
<th>Probability of vector V</th>
<th>Connectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110</td>
<td>0.054</td>
<td>Disconnected</td>
</tr>
<tr>
<td>2</td>
<td>101</td>
<td>0.024</td>
<td>Disconnected</td>
</tr>
<tr>
<td>3</td>
<td>011</td>
<td>0.014</td>
<td>Connected</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0.216</td>
<td>Connected</td>
</tr>
<tr>
<td>5</td>
<td>001</td>
<td>0.056</td>
<td>Connected</td>
</tr>
<tr>
<td>6</td>
<td>010</td>
<td>0.126</td>
<td>Connected</td>
</tr>
<tr>
<td>7</td>
<td>000</td>
<td>0.504</td>
<td>Connected</td>
</tr>
<tr>
<td>8</td>
<td>111</td>
<td>0.006</td>
<td>Disconnected</td>
</tr>
</tbody>
</table>

Given a specific mode vector, if there is at least one path connecting node 1 and node 2, then we say that node 1 and node 2 are connected; otherwise they are disconnected. Table 2.1 gives all the network states and the corresponding probability for each network state. Table 2.1 shows that there are five network states that are connected. The sum of the probabilities for these five states is 0.916. In this case, we say that the connectivity for the network is 0.916. From this example, the connectivity can be defined as the sum of the probabilities of the network states that are connected. In this simple network, there are only 3 bridges and 2 nodes; therefore it is very easy to check the connectivity between two nodes. For a complex network with a large number of nodes, it is extremely difficult to check the connectivity between any two nodes; the following gives the procedure to solve this problem. Assume that there is a graph with $n$ nodes and $m$ links. To check the connectivity between any two nodes:

1. Rank all the nodes from 1 to $n$ randomly, and assuming a need to check the connectivity between node 1 and node $n$, set node 1 as the start node;
2. If there is a direct link between node $i$ and the start node, then node $i$ is called the linked node; find all the linked nodes, and mark the other nodes as unlinked nodes;
3. For each of the linked nodes, repeat step 2 until the linked nodes set and the unlinked nodes set become unchanged. Finally, if node $n$ is in the linked nodes set, then node 1 and node $n$ are connected, otherwise they are unconnected.

Using this method, the connectivity for a specific network state can be obtained. However, due to the exponential effect, it is difficult to enumerate the state space for a network with more than a few nodes. For a network with 50 bridges, the number of network states is $2^{50} = 1.13 \times 10^{15}$, which is a huge number. On the other hand, in many cases, it is not necessary to enumerate all the possible states. In some cases, for all the connected states, there are only a finite number of states that account for the majority of the probability of being connected, while the probabilities of others states are very low.

Take the network in Fig. 2.1 for example; network states 3, 4, 5, 6, and 7 are connected. The sum of probabilities of states 4, 6, and 7 is 0.846, which accounts for 92.4% of all the probabilities being in connected states. Therefore, in the following we can restrict our attention to the most likely states and give bounds on the network performance. In order to enumerate the most likely states, algorithm ORDER-II (Lam 1986) is used in the following. Algorithm ORDER-II can generate states in the appropriate order, and does not require fixing the number of states beforehand. The algorithm can be run until a desired degree of accuracy is obtained, thus optimizing the use of computational resources. In the following section, the algorithm ORDER will be implemented in the network of APT-BMS to calculate the connectivity between any two nodes.
2.2. Network simulation

There are 983 bridges in the APT stock, located along SP (province owned) roads and SS (state owned) roads. The whole APT road network, including all bridges and roads, is simulated as a graph. The key phase of network simulation is identifying all the nodes of the graph. The following points are defined as nodes: the intersections or endpoints of SP and SS roads. Each node has 3 variables: ID number, longitude, latitude. Fig. 2.2 is the simulated graph from Trento to Ala. Trento is the capital city of the APT region, while Ala is an important town in the south of the APT, near the high risk seismic zones in Northern Italy. There are 40 bridges that have different probabilities of being in operational limit state, as represented by the colored dots. SS12 and SP90 are two main roads connecting Trento and Ala. The Adige River and A22 highway are between SS12 and SP90. Only the intersection and the endpoints of SS and SP roads can be identified as nodes. Based on this definition, there are 16 nodes in this graph.

![Figure 2.2 The simplified graph between Trento-Ala](image)

After identifying all the nodes, the next step is to identify all the links. Not all connections between two nodes can be regarded as links; these must be along the SP or SS roads. There are 22 links in Fig. 2.2. Every link has 6 variables: ID number, start node ID, end node ID, ID of the road forming the link, the relative position of the start node on the road, and the relative position of the end node on the road. When all the nodes and links are identified, the whole APT network is simulated as a graph in Google Earth as shown in Fig. 2.3. The small red points represent the nodes, and the red lines represent the links. In total, there are 558 nodes and 740 links in the APT stock. All the bridges are located on the links. Now the algorithms can be performed on the APT network.

![Figure 2.3 Google Earth map of the APT-BMS network](image)

2.3. Network simulation

After simulating the network, the algorithms ORDER and ORDER-II are used to find $m$, the most probable states of the network. Let’s start with the simple network from Trento to Ala in Fig. 2.2. Since there are 40 bridges in this network, there will be $2^{40} = 1.1 \times 10^{12}$ states for this network. For the whole network in APT-BMS, there are 984 bridges, and so the number of network states will be 2984, which is an enormous number. In order to simplify the computation, all the bridges within one link are combined into one bridge, and the probability of this new bridge having operational problems is the
sum of probabilities of all the bridges having operational problem. It must be noted that here we make an approximation. Take an example with two bridges (A and B) on the link. As we know, the probability of the link being disconnected is \( P_{\text{fail}}(\text{link}) = P_{\text{fail}}(A) + P_{\text{fail}}(B) - P_{\text{fail}}(AB) \). If we neglect the correlation between bridges A and B, we have \( P_{\text{fail}}(AB) = P_{\text{fail}}(A) \cdot P_{\text{fail}}(B) \), so \( P_{\text{fail}}(\text{link}) = P_{\text{fail}}(A) + P_{\text{fail}}(B) - P_{\text{fail}}(A) \cdot P_{\text{fail}}(B) \). Since the values of \( P_{\text{fail}}(A) \) and \( P_{\text{fail}}(B) \) are very small, their product is negligible. Therefore \( P_{\text{fail}}(\text{link}) = P_{\text{fail}}(A) + P_{\text{fail}}(B) \). The components have been reduced to 17, so the total number of network states becomes \( 2^{17} = 1.3 \times 10^5 \).

In this example, the number of component is \( n = 17 \), and we want to consider the \( m \) most probable states for this network. For each network state, we calculate the connectivity of the network. If there is at least one path to connect node 1 and node 16, then the connectivity for this state is 1, otherwise 0. After considering all the \( m \) most probable states, the approximate total connectivity for this network is:

\[
C = \sum_{i=0}^{m} C_i \cdot P_i
\]  

(2.2)

where \( P_i \) is the probability of the \( i \)-th network state, and \( C_i \) is the connectivity for the \( i \)-th network state. \( C_1 = 1 \), if there is at least one path from node 1 to node 16; \( C_i = 0 \), if not. Obviously,

\[
C_{2^n} \leq C_i \leq C_1 \quad i = 1, 2, \ldots, 2^n
\]  

(2.3)

where \( C_1 \) is the connectivity when all the components are in operational mode, and \( C_{2^n} \) is the connectivity when all the components are in failure mode. So \( C_1 = 1 \), and \( C_{2^n} = 0 \). If we consider the \( m \) most probable states, we have:

\[
C = \sum_{i=1}^{m} C_i \cdot P_i + \sum_{i=m+1}^{2^n} C_i \cdot P_i
\]  

(2.4)

From Eqn. 2.3, we have:

\[
\sum_{i=m+1}^{2^n} C_i \cdot P_i \leq \sum_{i=m+1}^{2^n} C_1 \cdot P_i \leq \sum_{i=m+1}^{2^n} C_1 \cdot P_i
\]  

(2.5)

Since \( C_1 = 1 \), and \( C_{2^n} = 0 \), Eqn. 2.5 becomes

\[
0 \leq \sum_{i=m+1}^{2^n} C_i \cdot P_i \leq \sum_{i=m+1}^{2^n} P_i = 1 - \sum_{i=1}^{m} P_i
\]  

(2.6)

Substituting Eqn. 2.6 into Eqn. 2.4, we get:

\[
\sum_{i=0}^{m} C_i \cdot P_i \leq C \leq \sum_{i=0}^{m} C_i \cdot P_i + 1 - \sum_{i=0}^{m} P_i
\]  

(2.7)

Table 2.2 gives the connectivity of the network when considering different \( m \) values. When the 10 most probable states are considered, the range becomes [0.99448, 0.99869]. As the value of \( m \) increases, the upper and lower bounds of \( C \) converge quickly. From this example, we can say that the connectivity for this network is 0.99448. After performing the algorithm on the network from Trento to Ala, we want to consider the connectivity of the whole network in APT-BMS. Lavazè Pass and Riccomassimo are two remote places in Trentino Province located at the north and south path of the
APT region as shown in Fig. 2.3, respectively. Given the start node as Riccomassimo and the end node as Lavazè Pass, the connectivity, using the algorithm ORDER-II, is analysed below.

**Table 2.2.** Expected connectivity of between Trento to Ala given different m values

<table>
<thead>
<tr>
<th>Number of states (m)</th>
<th>$\sum_{i=0}^{m} P_i$</th>
<th>$\sum_{i=0}^{m} C_i \cdot P_i$</th>
<th>$\sum_{i=0}^{m} C_i \cdot P_i + 1 - \sum_{i=0}^{m} P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.9958</td>
<td>0.9945</td>
<td>0.9987</td>
</tr>
<tr>
<td>100</td>
<td>0.9987</td>
<td>0.9945</td>
<td>0.9958</td>
</tr>
<tr>
<td>1000</td>
<td>0.9998</td>
<td>0.9945</td>
<td>0.9946</td>
</tr>
<tr>
<td>5000</td>
<td>0.9999</td>
<td>0.9945</td>
<td>0.9946</td>
</tr>
<tr>
<td>10000</td>
<td>0.9999</td>
<td>0.9945</td>
<td>0.9945</td>
</tr>
</tbody>
</table>

**Table 2.3 Connectivity between Passo Lavaze and Riccomassimo for different m for return period of 475 years**

<table>
<thead>
<tr>
<th>Number of states (m)</th>
<th>$\sum_{i=0}^{m} P_i$</th>
<th>$\sum_{i=0}^{m} C_i \cdot P_i$</th>
<th>$\sum_{i=0}^{m} C_i \cdot P_i + 1 - \sum_{i=0}^{m} P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.5666</td>
<td>0.5666</td>
<td>1</td>
</tr>
<tr>
<td>$10^7$</td>
<td>0.7394</td>
<td>0.7313</td>
<td>0.9919</td>
</tr>
<tr>
<td>$10^8$</td>
<td>0.7899</td>
<td>0.7313</td>
<td>0.9414</td>
</tr>
<tr>
<td>$10^9$</td>
<td>0.7952</td>
<td>0.7313</td>
<td>0.9361</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>0.7978</td>
<td>0.7313</td>
<td>0.9334</td>
</tr>
</tbody>
</table>

From Table 2.3, we can see that the 100 most probable states account for 73.9% of the whole states. After that, the sum of the probabilities for the m most probable states increases very slowly. However, the approximate connectivity remains the same after m = 100. Therefore, it can be concluded that after the most 100 probable states, the network is disconnected between the Lavazè Pass node and the Riccomassimo node. From Table 2.3, we can conclude that the connectivity between the Lavazè Pass node and the Riccomassimo node is between 0.73128 and 0.93163.

In all the previous calculation, we ignore the correlation between different bridges; we know that this is not reasonable in reality. In order to address this problem, here we adopt a post earthquake assessment system, based on the framework proposed by Bensi et al. (2011) which in general allows an update of the seismic failure probability of bridges after observing some evidence. In the following, we apply a similar framework to the so called ‘twin bridges’ problem based on Bayesian Network with continuous variables. A Bayesian Network (BN) is a directed acyclic graph (traditionally abbreviated DAG) that consists of a set of nodes and a set of directed edges (Jensen and Nielsen 2007). The nodes represent variables and the edges represent condition relationships between the variables. The BN originates from the field of artificial intelligence and combines graph and probability theories. It is a useful tool that helps perform uncertainty analysis in complex systems. Due to their generality, BNs have been widely used in many areas in the last two decades: for an extensive explanation of BN, see Jensen and Nielsen (2007). Here, by ‘twin bridges’ we mean two bridges with similar characteristics as to type, material and construction year, that therefore are expected to respond similarly to an earthquake. The basic idea is that when an earthquake occurs, the limit state of one bridge is detected or the information on earthquake magnitude is obtained, the distribution of other unobserved variables such as the probability of another bridge being in the same damage state can be updated.

### 3. CORRELATION ANALYSIS

The proposed framework includes three main parts: the demand model, the capacity model, and the fragility function which correlates the demand and the capacity. In seismic risk analysis, it is common to assume a lognormal distribution for component capacities ($C$) and component demands ($D$). If we assume that: $C \sim \ln N(\mu_C, \sigma_C^2)$; $D \sim \ln N(\mu_D, \sigma_D^2)$, then the probability of failure is:
\[
F = \Pr[D - C > 0] = \Pr[\ln C - \ln D < 0] = \Phi\left(\frac{\mu_g}{\sigma_g}\right) = \Phi\left(\frac{\mu_D - \mu_C}{\sqrt{\sigma_D^2 + \sigma_C^2}}\right)
\]  
(3.1)

where \(\Phi\) is the standard normal cumulative distribution function.

---

**Figure. 3.1** The conceptual BN that contains the three main parts in the framework

Fig. 3.1 is the conceptual BN representing the relationship between demand and capacity. \(g\) is the intermediate variable that is related to the probability of failure, \(g = \ln(D/C)\). In order to facilitate the calculation, all the variables follow a normal distribution or lognormal distribution. In this simple BN, obviously we have \(g \sim N(\mu_g, \sigma_g^2)\). The demand is calculated using an attenuation function depending on the local site conditions. The capacity is calculated based on some empirical or analytical models, depending on the characteristics of the bridges. Normally, the capacity model is defined with respect to several damage states. In this paper, we only consider the collapse limit state.

### 3.1. Demand model

As suggested by many researchers (Abrahamson and Silva 1997; Park et al. 2007; Sokolovet et al. 2010), the ground motion parameter \(Y_{i,j}\) is represented by

\[
\ln Y_{i,j} = f(e_i, p_i, s_i) + \eta_i + \varepsilon_{i,j}
\]  
(3.2)

where \(Y_{i,j}\) is the ground motion parameter at site \(j\) during earthquake \(i\), Peak Ground Acceleration (PGA), response Spectral Acceleration (SA), Peak Ground Velocity (PGV), or Peak Ground Displacement (PGD). In this research, we only consider the parameter PGA; \(f\) is the logarithm of the mean value of ground motion parameter that is calculated through the attenuation equation. It is a function of earthquake source \(\langle e_i \rangle\), propagation path \(\langle p_i \rangle\), and local site condition \(\langle s_i \rangle\). The random variable \(\eta_i\) is the inter-event variability that follows normal distribution. It is common to all sites during a same earthquake \(i\). The random variable \(\varepsilon_{i,j}\) is the intra-event variability that also follows normal distribution. For the same earthquake \(i\), the intra-event variables at two sites are correlated. Both inter-event variability \(\eta_i\) and intra-event variability \(\varepsilon_{i,j}\) are aleatory uncertainties that describe the variability. The inter-event error describes the variability between the different earthquakes, and the intra-event error captures the variability between different sites given the same earthquake event.

### 3.2. Hazus model

The Hazus model is a rapid approach seeking to establish dependable fragility curves (Mander 1999). In contrast to other methods that have been used in the past, such as empirical fragility curves or analytical fragility curves that require much previous damage data or extensive computation, only limited information is needed for this model. The probability of being in or exceeding a damage state in Hazus is modeled as:

\[
[P_i(PGA)]_j = \Phi\left(\frac{1}{\beta} \ln\left(\frac{PGA}{a_{2j}}\right)\right) \quad i = 1, 2, 3, 4
\]  
(3.3)

where PGA is the Peak Ground Acceleration, related to the demand on the bridge; \(\Phi\) is the standard
normal cumulative distribution function; \((a_g)\) is the median spectral acceleration that causes the \(i^{th}\) limit state, related with the capacity of the bridge. There are four damage states which are (Operational limit state) OLS, (Damage control limit state) DLS, (Life safety limit state) LLS, and (Collapse limit state) CLS. The four limit states are defined based on Hazus suggestions. In this paper, we only consider the CLS; \(\beta\) is the normalized composite log-normal standard deviation which takes account of uncertainty and randomness for both capacity and demand. In Eqn. 3.3, the only unknown parameter is \((a_g)\), which is calculated using a capacity-spectrum approach. For the detail of calculating \((a_g)\), see Yue et al. (2010):

\[
(a_g)_s = \frac{C_c}{S \cdot \delta \cdot F_0} \quad \text{(3.4a)}
\]

\[
(a_g)_L = \frac{2\pi}{S \cdot \delta \cdot F_0} \cdot \sqrt{\frac{C_c \cdot \Delta}{g}} \cdot \frac{K_{3D}}{T_c} = k \cdot \sqrt{C_c} \quad \text{(3.4b)}
\]

where \(C_c\) is the capacity; \(\Delta\) is the maximum displacement response; \(S\) is a coefficient depending on the soil type; \(\delta\) is the damping correction factor with a reference value of \(\delta=1\) for 5% viscous damping; \(F_0\) is the spectral amplification factor; \(T_c\) is the upper limit of the period of the constant spectral acceleration branch; \(K_{3D}\) is a factor accounting for the 3D arching action when displacements are sufficiently large, but omitted in Eqn 3.4a because the seismic displacements are small (Basoz and Mander 1999). For the CLS, the value in Eqn. 3.4b is normally larger than that in Eqn. 3.4a, so in this paper we only consider the required spectral acceleration obtained through Eqn. 3.4b. In Eqn. 3.4b, the only parameter to be calculated is the normalized capacity \(C_c\). Based on Dutta and Mander (1998), as for single span bridges or bridges seated on weak bearings with strong piers, the capacity is assumed to arise from sliding only (Basoz and Mander 1999). In this case, the capacity is given as: \(C_c = \mu_4\)

\[
\text{where } \mu_4 \text{ is the coefficient of sliding friction of the bearings in the transverse direction. This is assumed to follow lognormal distribution. The normalized base shear capacity of a standard bridge can be expressed as:}
\]

\[
C_c = \lambda_Q \cdot k_p \cdot \frac{D}{H} \quad \text{(3.5)}
\]

where \(\lambda_Q\) is defined as a strength reduction factor that occurs due to cyclic loading; \(D, H\) are column diameter and column height; \(k_p\) is a factor related to the reinforced concrete strength of the column \(k_p = \zeta \cdot j \cdot (1+0.64 \cdot \rho_c \cdot f_y \cdot j / f_c \cdot \psi)\), where \(\zeta\) is a fixity factor taken as 1 for multi-column bends and 0.5 for single column cantilever action; \(j\) is an internal lever arm coefficient; \(\rho_c\) is the volumetric ratio of longitudinal reinforcement; \(\psi\) is the average dead load axial stress ratio in the column; \(f_y\) is yield stress of the longitudinal reinforcement and \(f_c\) is the strength of the concrete. In order to facilitate computation, \(k_p\) is better defined as the product of several parameters. Here we assume that

\[
C_c = \lambda_Q \cdot k_p \cdot \frac{D}{H} = \alpha \cdot f_y \cdot f_c \quad \text{(3.6)}
\]

where \(\alpha\) is a factor related with \(\zeta, \rho_c, \psi, D, f_y, f_c\) and \(f_c\).

3.3 BN framework for twin bridges

After introducing the three basic components in the framework, we consider the graphs of the BN framework. As we have two kinds of capacity model, we will provide two BN frameworks here, see Fig. 3.2. When the capacity of the bridge is assumed to arise from bearings, we call this kind of bridge Type 1. When the capacity of the bridge is assumed to arise from piers, we call this bridge Type 2. It should be noted that here we fix the failure type of the bridge for computation simplicity. In reality, we do not know the failure type before calculating the capacity. There are three parts in this framework:
the demand model, the capacity model and the intersection between the demand and the capacity:

1. In the demand model, $S = \log D = \log (PGA)$. $M$ is the earthquake magnitude; $\eta$ is the inter-error term for the demand and is the same value for all the sites in one earthquake.

2. From the capacity model, we have: $\ln C = \ln (e \cdot e_{\eta}) = \ln (e_{\eta}) + k + \ln C_{0}$ where $e_{\eta} = \ln e_{\eta}, e_{\eta}$ is the uncertainty term as defined and $C_{0}$ is the resistant strength. When the bridge belongs to Type 1: $\ln C_{0} = \ln \mu = R$. $\mu$ is the coefficient of the sliding friction of the bearings in the transverse direction. When the bridge belongs to Type 2: $\ln C_{0} = \ln \alpha + \ln f_{c} - \ln f_{e}$. where $F_{c} = \ln f_{c}$, $F_{e} = \ln f_{e}, \alpha, f_{c}$ and $f_{e}$ are defined as before.

3. $g$ is the intermediate parameter related to the reliability of the bridge. From Equation 11 and 21, we have $C = e_{\cdot} \cdot k \cdot C_{0}^{0.5}$. When the bridge is type 1: $g = \ln (D/C) = 2.3 S - E_{c} - \ln k - 0.5R$. When the bridge is type 2: $g = \ln (D/C) = 2.3 S - E_{c} - \ln k - 0.5 \ln \alpha - 0.5F_{y} + 0.5F_{c}$. Given the distribution parameters ($\mu_{g}, \sigma_{g}^{2}$) of $g$, the probability of failure for the bridge is $\Phi (\mu_{g} / \sigma_{g})$.

The two bridges are correlated through the global variables: $M$, $\eta$, and $C$. Since there are two bridges in Fig. 3.2, the intra-error terms should be considered with regard to the two sites where the two bridges are located. Both $Z_{a}$ and $Z_{b}$ follow normal distribution. In order to consider this correlation in BN, two parents, $U_{1}$ and $U_{2}$, are used as the sources of $Z_{a}$ and $Z_{b}$. This idea is adopted from Bensi et al. (2011).

![Bayesian Network for two bridges](image)

**Figure 3.2** Bayesian Network for two bridges (a) type 1, (b) type 2.

### 3.4 Case study

The SP135 Bridge on the River Fersina-Canezza (A) and the SP31 Bridge on the River Avisio (B) are ‘twin’ bridges in APT-BMS. Both are 3 span pre-stressed concrete bridges with wall piers, non-monolithic abutments, and both were built in 1967. The lengths of the two bridges are 58.3m and 57.5m respectively. In Fig. 3.3 and Fig. 3.4 we can see overviews and cross sections of these structures. We assume that an earthquake with a magnitude of 7 has happened. The sources to site distances for the two bridges are 15km and 10 km, and the distance between the two sites is 10km. The capacity variable $\mu_{g}$ is assumed to follow: $R = \ln \mu_{g} \sim N (\ln 0.85, 0.1^{2})$. Since the two bridges both have wall piers, they belong to the bridge Type 1 category. According to Basöz and Mander (1999), the capacities are assumed to arise from the sliding of bearings only. Table 3.1 gives the other parameters.
Having defined the relationship between these variables, we can use the computation scheme in Lauritzen and Jensen (2001) to calculate the prior distribution for $S_a$, $S_b$, $g_a$ and $g_b$. Table 3.2 gives the results. Given the distributions of $S_a$ and $S_b$, the median value of $PGA$ at bridges A and B can be calculated as 0.6118g and 0.739g; The probabilities of bridges A and B collapsing are 19.57% and 24.99%.

After the earthquake, the on-site sensor observes that bridge B has collapsed. We can enter this evidence into the BN based on the computation scheme in Lauritzen and Jensen (2001) and get the posterior of other variables given in Table 3. From Table 2 and Table 3, we can see that the expected median PGA values on the two bridges are also increased from 0.6118g to 0.8509g for bridge A, and from 0.7390g to 2.7284g for bridge B. The increase in PGA value at site B is much larger than the increase at site A; this can be explained by the large uncertainties in the attenuation equation. In the meantime, the expectation for the sliding coefficient $\mu$, is reduced from 0.85 to 0.84. The failure of bridge B shows that the capacity of the bridge is less than expected. The probability of bridge A collapsing is increased from 19.57% to 23.64%. 

Table 3.1. The parameters for calculating median spectral acceleration

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$\eta$</th>
<th>$F_0$</th>
<th>$\Delta$ (m)</th>
<th>$K_{3D}$</th>
<th>$T_C$ (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.6325</td>
<td>2.6848</td>
<td>0.3</td>
<td>1.21</td>
<td>0.3335</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.6325</td>
<td>2.6157</td>
<td>0.3</td>
<td>1.21</td>
<td>0.3629</td>
</tr>
</tbody>
</table>

Table 3.2. Results after initialization

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean value</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_a = \log(PGA_a)$</td>
<td>-0.491</td>
<td>1.067</td>
</tr>
<tr>
<td>$S_b = \log(PGA_b)$</td>
<td>-0.302</td>
<td>1.067</td>
</tr>
<tr>
<td>$g_a$</td>
<td>-2.040</td>
<td>5.662</td>
</tr>
<tr>
<td>$g_b$</td>
<td>-1.605</td>
<td>5.662</td>
</tr>
</tbody>
</table>

Table 3.3. Results given the evidence

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean value</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_a = \log(PGA_a)$</td>
<td>-0.162</td>
<td>0.596</td>
</tr>
<tr>
<td>$S_b = \log(PGA_b)$</td>
<td>1.004</td>
<td>0.003</td>
</tr>
<tr>
<td>$g_a$</td>
<td>-1.278</td>
<td>3.169</td>
</tr>
</tbody>
</table>

Having defined the relationship between these variables, we can use the computation scheme in Lauritzen and Jensen (2001) to calculate the prior distribution for $S_a$, $S_b$, $g_a$ and $g_b$. Table 3.2 gives the results. Given the distributions of $S_a$ and $S_b$, the median value of $PGA$ at bridges A and B can be calculated as 0.6118g and 0.739g; The probabilities of bridges A and B collapsing are 19.57% and 24.99%.

After the earthquake, the on-site sensor observes that bridge B has collapsed. We can enter this evidence into the BN based on the computation scheme in Lauritzen and Jensen (2001) and get the posterior of other variables given in Table 3. From Table 2 and Table 3, we can see that the expected median PGA values on the two bridges are also increased from 0.6118g to 0.8509g for bridge A, and from 0.7390g to 2.7284g for bridge B. The increase in PGA value at site B is much larger than the increase at site A; this can be explained by the large uncertainties in the attenuation equation. In the meantime, the expectation for the sliding coefficient $\mu$, is reduced from 0.85 to 0.84. The failure of bridge B shows that the capacity of the bridge is less than expected. The probability of bridge A collapsing is increased from 19.57% to 23.64%.
4. CONCLUSIONS

This paper proposes a framework for seismic vulnerability assessment for bridge management system. At the first part, the connectivity between any two places is calculated using network state enumerate algorithm ORDER-II. The results are very helpful for bridge managers and government officials in understanding the network status, and can assist them to make rapid decisions in near-real time, under post earthquake conditions. At the second part, the seismic risk correlation between the components is analysed based on the Bayesian Network with continuous various. The framework can predict the seismic risk for an individual bridge before the earthquake and update the risk after the earthquake when given some evidences on other bridges. This can be incorporated into the Decision-Support System (DSS) for near-real time emergency response, which is also the future direction of this paper.

ACKNOWLEDGEMENTS

This research was made possible thanks to the financial support of the Italian Ministry of Education (contract # PRIN_20077HK33Y_002) and by the Autonomous Province of Trento. The authors wish to thank the APT Department of Transportation, and specifically Raffaele De Col, Luciano Martorano, Stefano De Vigili, Guido Benedetti, Paolo Nicolussi and Matteo Pravda. The authors also acknowledge in particular the contributions by Francesca Bortot, David Capraro, Giovanni Cortese, Alessandro Lanaro, Devis Sonda and Paolo Zanon.

REFERENCES