Calibration of a Combined Site Parameter of $V_{S30}$ and Bedrock Depth for Ground-Motion Prediction Equations Using Strong-Motion Records from Japan

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SUMMARY:
In modern ground-motion prediction equations (GMPEs), site period or average shear-wave velocity of the top 30m ($V_{S30}$) has been used to model site effect. Some studies show that site period is a better site parameter than $V_{S30}$ for long-period sites. The drawback for using site period is the high cost of obtaining the soil shear-wave velocity profile for a deep soil site. In many engineering applications, it may be relatively easy to estimate the depth of bedrock, without high cost. We propose to use $T_{VS30H} = 4H_{rock}/V_{S30}$ where $H_{rock}$ is the depth of engineering bedrock to replace the site period in a GMPE. For KiK-Net stations in Japan, the correlation between site period and $T_{VS30H}$ is surprisingly good and this means that $T_{VS30H}$ can be used to replace site period as the site parameter. We will use the surface and borehole records from the KiK-Net stations to validate the proposed site parameter.

Keywords: Ground-motion prediction equations, site effect, $V_{S30}$, site period, site amplification ratios

1. INTRODUCTION

$V_{S30}$, the average shear-wave velocity of the top 30m of soil, has been used in many ground-motion prediction equations (GMPEs), including four of the next generation attenuation (NGA) models, Abrahamson and Silva (2008), Boore and Atkinson (2008), Campbell and Bozorgnia (2008) and Chiou and Youngs (2008). However, many studies have found that $V_{S30}$ alone cannot represent the site effect because the shear-wave velocity of soil layers below 30m, the depth to bedrock and the impedance ratio between the last soil layer and the bedrock all contribute significantly to site response (Castellaro et al. 2008). Realizing the limitations of $V_{S30}$, Zhao et al. (2006a), Zhao et al. (2006b), Zhao (2010) and Zhao and Xu (2012a) used site classes based on site period (4 times the shear-wave travel time from engineering bedrock to the ground surface). McVerry (2011) shows that using site period can improve the model prediction significantly over using site classes.

Recently, Zhao (2011) and Zhao and Xu (2012b) evaluated site period and $V_{S30}$ using two strong motion datasets. The first dataset consisted of surface and borehole records from KiK-Net in Japan. Zhao (2011) and Zhao Xu (2012b) used over 3000 records from 95 earthquakes, and all of the events had reliable source parameters such as moment magnitude, focal mechanism and tectonic location (shallow crustal, subduction interface and subduction slab). Zhao (2011) and Zhao and Xu (2012b) found that using site period as a site parameter could lead to improved estimates for surface/borehole amplification ratios compared with those obtained by using $V_{S30}$.

Zhao (2011) and Zhao and Xu (2012b) also used a sub-dataset from Zhao (2010) and Zhao and Xu (2012a) studies for GMPEs. The records used in the Zhao (2011) and Zhao and Xu (2012b) studies included the surface records from KiK-Net stations and records from K-net stations and stations from Port Airport Research Institute for which site periods can be estimated from geotechnical information and spectral ratios of horizontal and vertical components with confidence (Zhao 2006a). Zhao (2011) and Zhao and Xu (2012b) found that the use of site periods in the GMPE dataset did not lead to statistically significant reduction in the inter-site variability, which is not consistent with the findings from the surface/borehole amplification ratios. Zhao and Xu (2012b) attributed the discrepancies to
the larger standard deviations of the inter-site residuals in the GMPE dataset. Zhao and Xu (2012b) suggested that inter-event error and error associated with path effect propagated into inter-site residuals, because of imperfect data distribution with respect to the number of records for each earthquake and for each site. Though the use of site period did not lead to statistically significant reduction in model variability for long period sites in the GMPE dataset, the amplification ratios computed using site periods were much more reasonable than those obtained using $V_{S30}$. For example, the peak amplification ratios for long period sites derived from using $V_{S30}$ did not increase with increasing site period. The poor performance of $V_{S30}$ was attributed to the large variability in the correlation between $V_{S30}$ and site period for soft soil sites. These results suggested that site period is a better site parameter than $T_{VS30}$ for both datasets.

The calculation of site periods requires values for the shear-wave velocities of all soil layers above the engineering bedrock and the shear-wave velocity that defines the engineering bedrock. Many values have been used for bedrock shear-wave velocity, and Zhao (2011) and Zhao and Xu (2012b) used $V_s=700m/s$ so as to be consistent with the GMPE by Zhao et al. (2006a).

However, there is a practical problem with the use of site period. For many recording stations in many parts of the world, shear-wave velocity below 30m depth may not be available. To obtain shear-wave velocity at greater depth for many engineering applications may bring high cost. The depth to engineering bedrock in many cases can be easily obtained from nearby borehole logs that do not provide either shear-wave velocity or standard penetration test (SPT) results that could be used to estimate soil shear-wave velocity. Abrahamson and Silva (2008) and Chiou and Youngs (2008) used the depth to bedrock with a $V_s=1000m/s$ as a site term combined with $V_{S30}$ based on residual analyses on the NGA dataset. In this study we attempt to evaluate such a combination as an alternative site parameter to site period.

Figure 1. Correlation between site period and $T_{VS30}$ in (a) and between site period and $T_{VS30H}$ in (b)

In a GMPE or any empirical relationships, if two parameters have a good correlation, either parameter can be used to replace the other in a GMPE. Figure 1 shows the correlations among site period, $T_{VS30}$ and $T_{VS30H}$ which are defined by

$$ T_{VS30} = \frac{120m}{V_{S30}} \quad T_{VS30H} = \frac{4H_{rock}}{V_{S30}} $$

where $H_{rock}$ is the depth to engineering bedrock. When $H_{rock} < 2.0$, $H_{rock}=2.0$ is selected and when $T_{VS30H} < 0.01$, $T_{VS30H} = 0.01$ is selected, for the convenience of using a logarithmic scale for plotting $T_{VS30H}$. The minimum value for site period is also taken as 0.01s, e.g. when site period is less than 0.01 site period is set as 0.01. All data are from the KiK-Net stations in Japan. Fig. 1(a) shows the correlation between site period calculated for a bedrock shear-wave velocity of 700m/s and $T_{VS30}$, and Fig. 1(b) shows the correlation between site period and $T_{VS30H}$. The standard deviation of the residuals for the fitted solid curve between $T_{VS30}$ and site period is 0.21 in the natural logarithmic scale for all sites, 0.16 for sites with a site period less than or equal to 0.45s, and 0.28 for sites with a site period over 0.45s. The good correlations between site period and $T_{VS30}$ for short period sites (between 0.02s and 0.3s) means that site period and $T_{VS30}$ lead to similar site amplification ratios as shown by Zhao (2011) and Zhao and Xu (2012b). The large scatter for moderate and long period sites and the slow
increase in $T_{VS30}$ with increasing site period lead to underestimates of amplification ratios (Zhao 2011, and Zhao and Xu 2012b) compared with those based on site period. The correlation between $T_{VS30H}$ and site period is excellent at site periods over 0.45s with a standard deviation of 0.15 only, considerably smaller than that for the correlation curve between $T_{VS30}$ and site period in the same site period range. The correlation for short period sites is still reasonable, with a standard deviation of 0.31 for sites with a site period less than or equal to 0.45s, slightly larger than that for the correlation curve between site period and $T_{VS30}$ in the same site period range. The good correlation between $T_{VS30H}$ and site period means that $T_{VS30H}$ is an excellent alternative to site period as a site parameter for most sites.

![Figure 2](image)

**Figure 2.** Distribution of earthquakes with respect to focal depth and magnitude for KiK-Net data

![Figure 3](image)

**Figure 3.** Distribution of strong motion records from KiK-Net stations used in the present study for PGA. A magnitude-dependent cut-off distance is used to avoid the effect of un-triggered instruments.

2. CALIBRATION FOR $T_{VS30H}$ USING SURFACE AND BOREHOLE AMPLIFICATION RATIOS FROM KIK-NET STRONG MOTION STATIONS

We used the first dataset in the Zhao (2011) study to calibrate the proposed site parameters in Eqn. 1.1b. Fig. 2 shows the distribution of earthquakes with respect to focal depth and moment magnitude used by Zhao (2011). More than half of the earthquakes are subduction slab earthquakes with a
maximum depth of 130km. Fig. 3 shows the record distribution with respect to source distance and magnitude for four site classes as defined by Zhao (2011). Engineering bedrock is defined as having a shear-wave velocity equal to or larger than 700m/s. The depth used to calculate the site period depends on the shear-wave velocity of the engineering bedrock; the site period is set as 0.01s when the engineering bedrock is at the ground surface for the convenience of using a logarithmic scale.

Amplification ratios between surface and borehole records differ from those between surface soil and surface rock site records that are used for engineering designs. However, Zhao (2011) and Zhao and Xu (2012b) suggested that, if a site parameter can be used to model surface/borehole amplification ratios well, it can also model the amplification ratios between a surface soil and a nearby rock site very well. We first calculate the amplification ratios for each pair of records and then calculate the average ratio for each site so as to eliminate the effect of magnitude and source distance (Zhao 2011, and Zhao and Xu 2012b). The following equation is fitted to the average amplification ratios for all sites,

$$\ln[A_{SB}(T, T_{SP})] = a_{SB}(T_{SP})T + b_{SB}(T_{SP})\ln(T) + c_{SB}(T_{SP})[\ln(T)]^2 + d_{SB}(T_{SP})$$  \hspace{1cm} (2.1)

where $T$ is either the site period or $T_{VS30H}$, $T_{SP}$ is spectral period, and $a_{SB}(T_{SP})$, $b_{SB}(T_{SP})$, $c_{SB}(T_{SP})$ and $d_{SB}(T_{SP})$ are regression coefficients for a given value of $T_{SP}$. Only the terms with estimates that differ from zero at a significance level of 5% are retained. For all periods, only three terms in Eqn. (2.1) are sufficient to model the average amplification ratios. The standard deviation of the residuals from Eqn. 2.1 is referred to as the inter-site standard deviation (Zhao 2011, and Zhao and Xu 2012b). We then compare the standard deviations of Eqn. 2.1 for the cases where $T$ is site period or $T_{VS30H}$. When the standard deviation for one site parameter (site period or $T_{VS30H}$) is statistically smaller than that for the other site parameter, we take the site parameter for this particular case as a statistically better site parameter.

![Figure 4](image)

Figure 4. Amplification ratio as a function of site period in the top row, and as a function of $T_{VS30H}$ using bedrock $V_s=700m/s$ in the bottom row. The left panel is for PGA and the right panel is for 0.5s spectral period.

Fig. 4 shows the amplification ratios for peak ground acceleration (PGA) and 0.5s spectral period together with the inter-site standard deviation labeled as $\tau_s$. The standard deviation is on natural logarithm scale and the 84-percentile amplification ratio equals the median value multiplied by a factor of $\exp(\tau_s)$. The solid line is the fitted function of Eqn. 2.1 with only statistically significant coefficients being used. The top row uses site period based on $V_s=700m/s$ for bedrock. The second row uses $T_{VS30H}$. For PGA and 0.5s spectral period the inter-site standard deviations for two cases are
nearly identical. The inter-site standard deviations for PGA are 0.49, and the inter-site standard deviations for 0.5s spectral period from site period and $T_{VS30H}$ are very similar (0.44 and 0.45). For PGA in Fig. 4(a), coefficients $a_{SB}$ and $c_{SB}$ are not statistically significant. For 0.5s spectral period, only terms $b_{SB}$ and $d_{SB}$ are statistically significant.

Figure 5. Amplification ratio as a function of site period in the top row, and as a function of $T_{VS30H}$ using bedrock $V_S=700\text{m/s}$ in the bottom row. The left panel is for 1.0s spectral period and the right panel is for 2.0s spectral period.

Fig. 5 shows the amplification ratios and the fitted function to the data for 1s spectral period in the left panel and 2s spectral period in the right panel. Again, the inter-site standard deviations computed using site period or $T_{VS30H}$ are nearly the same for each spectral period. The inter-site standard deviations for 1s spectral period are larger than those for 2s spectral periods.

Figure 6. Amplification ratios as a function of site period in the top row and a function of $T_{VS30H}$ using bedrock $V_S=700\text{m/s}$ in the bottom row. The left panel is for 3.0s spectral period and the right panel is for 5.0s spectral period.
Fig. 6 shows the amplification ratios 3s spectral periods in the left panel and 5s spectral periods in the right panel. Again, for each spectral period, the inter-site standard deviations for the two site parameters are nearly identical and these results suggest that both parameters can be used equally well to represent site effect in a GMPE.

Fig. 7 shows the comparison of inter-site standard deviations between using site period and $T_{VS30H}$ and probabilities of $F$-test for all sites as one group in (a); and probabilities of $F$-test for each site class in (b).

Fig. 7(a) shows the standard deviations derived from Eqn. 2.1 and the $F$-test probability of accepting equal standard deviations from the use of site period and $T_{VS30H}$ for all sites as a single group for the selected bedrock shear-wave velocity of 700m/s. Fig. 7(a) shows that the inter-site standard deviations calculated by using site period and $T_{VS30H}$ for a bedrock shear-wave velocity of 700m/s are nearly identical for all spectral periods and the lowest probability for accepting a hypothesis that the residuals from two site parameters have similar standard deviations is over 10%. Fig. 7(b) shows the standard deviations and $F$-test probability for each site class (SC). Apart from the results for SC IV sites in Figure 7(b), the inter-site standard deviations using site period or $T_{VS30H}$ are nearly identical and the $F$-test probability is over 60%. The $F$-test probability for SC IV sites in a period range of 2.0-4.5s is less than 5% and the inter-site standard deviations from using site periods are less than those from using $T_{VS30H}$, suggesting that site period is a better site parameter than $T_{VS30H}$. The largest difference in standard deviation is 0.063, leading to an 84-percenter factor difference less than 7%, which is practically negligible.

The absolute values of surface/borehole ratios are not relevant to a GMPE, it is the differences between the $A_{SB}$ for a soil site and the $A_{SB}$ for a rock site that are important. It is also important that two appropriate site terms can lead to consistent or similar amplification ratios. We will use the following response spectral ratios to confirm that the two site parameters can lead to similar site amplification ratios. The spectral ratio is computed from

$$A_{pseudo-site}(T_{site}, T_{SP}) = \frac{A_{SB}(T_{site}, T_{SP})}{A_{SB}(T_{rock}, T_{SP})}$$

where $A_{pseudo-site}$ is defined as pseudo-amplification ratio for a soil site over a rock site, in order to differentiate from the amplification ratio in a GMPE. $T_{site}$ is a site parameter, either site period or $T_{VS30H}$, and $T_{rock}$ is the site parameter for a rock site, either site period or $T_{VS30H}$ for the rock site. The pseudo-amplification ratio does not equal the amplification ratios between surface soil / surface rock sites that can be derived from the site terms in a GMPE. However, the shape of the pseudo-amplification ratios, e.g., the variation of ratios with increasing spectral periods, are very similar to those of the surface soil / surface rock amplification ratios derived from the site terms of a GMPE as shown later in this manuscript.

Fig. 8 compares the pseudo-amplification ratios with respect to an engineering rock site that has a site period of 0.1s. The pseudo-amplification ratios were calculated from Eqn. 2.2 for a given site period using site period or $T_{VS30H}$. For PGA, the pseudo-amplification ratios from the two site parameters differ by about 10% for short period sites as shown in Figs 8(a) and 8(b), and are nearly identical for long-period sites as shown in Figures 8(e) and 8(f). The pseudo-amplification ratios from the two site
parameters for a site with a site period of 0.8s in Fig. 8(c) and 1.0s in Figure 8(d) are also nearly identical at all spectral periods. These results confirm that $T_{VS30H}$ is an excellent alternative for site period.

![Figure 8](image)

**Figure 8.** Comparison of pseudo-amplification ratios with respect to a rock site for two site parameters. The amplification ratios were calculated by using soil layers down to the bedrock with a shear-wave velocity of 700m/s for site sites with a site period of, (a) 0.3s, (b) 0.5s, (c) 0.8s, (d) 1.0s, (e) 1.5s and (f) 2.0s. The period for the rock site is 0.1s.

3. **CALIBRATION FOR $T_{VS30H}$ USING AMPLIFICATION RATIOS FROM A GMPE DATASET**

Zhao (2011) showed that, for long-period sites, the use of site period leads to statistically and practically significant reduction in the inter-site standard deviation for the response spectral ratios between KiK-Net surface and borehole records at long spectral periods. Zhao (2011) also showed that using site period as a site parameter did not lead to any significant reduction in the amplification ratios from a sub-dataset used for developing GMPE by Zhao et al. (2006a), Zhao (2010) and Zhao and Xu (2012a). However, site period leads to better median amplification ratios than $T_{VS30}$. We will calibrate $T_{VS30H}$ using the second dataset by Zhao (2011).

The second dataset consists of records from 39 shallow crustal earthquakes, 64 subduction-interface earthquakes and 37 subduction slab earthquakes. 2014 records, many from the first dataset, comprise 669 records from SC I sites, 467 from SC II sites, 200 from SC III sites and 678 from SC IV site. All
records are from ground surface stations and they have previously been used by Zhao (2010) and Zhao and Xu (2011) for deriving ground-motion prediction equations. The records from a small number of stations in the strong-motion network of the Port and Airport Research Institute (formerly Port and Harbour Research Institute, PHRI) and K-net stations were also used. These stations have either a measured shear-wave velocity profile down to engineering bedrock for SC I, II and III sites, or their site periods (all from the SC IV site class) can be estimated with reasonable confidence from H/V ratios (Zhao et al. 2006a). The rest of the records used by Zhao (2010) and Zhao and Xu (2012a) are not used because site periods and $T_{VS30H}$ of the recording stations are not available. The strong-motion records from the $M_W=9$ Tohoku 2011 event were included in the dataset.

In order to describe the site effects by a continuous site parameter, such as site period or $T_{VS30H}$ from the models based on site classes, we need to identify the portions of site effect that can be described by a continuous site parameter but which have been forced into residuals by the use of site classes. The site term plus the intra-event residuals contain random intra-event errors, random errors associated with site effect and the underlying portion of the site effect that can be modeled by a function of a continuous site parameter. Theoretically the inter-event residuals are associated with earthquake source parameters only, and will not be used in the present study.

Figure 9. Comparison of response spectral amplification ratios with respect to a rock site. The amplification ratios were computed by using site period and $T_{VS30H}$ for a bedrock shear-wave velocity of 700m/s, for site periods of (a) 0.3s, (b) 0.5s, (c) 0.8s, (d) 1.0s, (e) 1.5s and (f) 2.0s. The period for the rock site is 0.1s.

Again, the average value of the site class terms plus intra-event residuals for each site is used so as to minimize the error associated with path effect. The exponential of the site class term plus intra-event residuals (residuals representing the variation within a given earthquake) is referred to as the site effect.
factor (referred to as $B_{\text{site}}$). The following simple function of either site period or $T_{VS30}$ was fitted to the average values of the site effect factor

$$\ln[B_{\text{site}}(T, T_{SP})] = a_{\text{site}}(T_{SP})T + b_{\text{site}}(T_{SP})\ln(T) + c_{\text{site}}(T_{SP})[\ln(T)]^2 + d_{\text{site}}(T_{SP})$$

(3.1)

where $B_{\text{site}}$ is the site effect factor, and $a_{\text{site}}$, $b_{\text{site}}$, $c_{\text{site}}$ and $d_{\text{site}}$ are regression coefficients for a given spectral period $T_{SP}$. Each term in Eqn. 3.1 is subjected to $t$-test and only those terms that are statistically significant (i.e., the absolute value is larger than zero) at a 5% significance level will be retained. The variability associated with the fitted empirical site model is again referred to as the inter-site variability. Amplification ratios $A_{\text{site}}$ between a soil site and a rock site can be calculated from

$$A_{\text{site}}(T_{\text{site}}, T_{SP}) = \frac{B_{\text{site}}(T_{\text{site}}, T_{SP})}{B_{\text{site}}(T_{\text{rock}}, T_{SP})}$$

(3.2)

where $B_{\text{site}}$ is computed from Eqn. 3.2 and $T_{\text{rock}} = 0.1s$ was selected. Figure 9 compares the amplification ratios. For all sites, the amplification ratios using $T_{VS30H}$ are very similar to those obtained by using site period, confirming that $T_{VS30H}$ is an excellent alternative site parameter to site period in a GMPE.

4. CONCLUSIONS

We have found that a site parameter $T_{VS30H}$ calculated from $V_{S30}$ (the average shear-wave velocity to a depth of 30m) and the depth to bedrock $H_{\text{rock}}$ is an excellent alternative to site period as a site parameter for ground-motion prediction equations (GMPE). The site parameter $T_{VS30H}$ is defined as $4H_{\text{rock}}/V_{S30}$. This finding may be potentially useful for sites where $V_{S30}$ and depth to bedrock are available but shear-wave velocity for soil below 30m is not. The correlation between $T_{VS30H}$ and site period for recording stations in Japan is excellent for all sites and this is the key for $T_{VS30H}$ to be an excellent alternative to site period in a GMPE. However, the correlation between $T_{VS30H}$ and site period in the other parts of the world may not be as good as for the strong-motion recording stations in Japan, and therefore the results presented in the present study may need to be verified before they are applied elsewhere.

We used 2968 pairs of ground surface and borehole strong-motion records from KiK-Net in Japan to calibrate the proposed site parameter $T_{VS30H}$. We computed the amplification ratios between each pair of records and then calculated the average amplification ratios for each site to minimize the uncertainty from earthquake source and wave-propagation path effects. We then fitted two functions to the average amplification ratios:

1) a simple function of site period; and
2) a simple function of $T_{VS30H}$ for the amplification ratios for each spectral period.

The inter-site standard deviations associated with two sets of fitted functions were used as a performance indicator for each site parameter.

We have found that, when all KiK-Net stations are considered as one group, the inter-site standard deviations from using site period or $T_{VS30H}$ are nearly identical for all spectral periods suggesting that these two parameters, site period and $T_{VS30H}$, can be used to model site effects equally well. To examine the effect of site parameter on soil sites with different site period, we divided the KiK-Net stations into four site classes according to their site periods (Zhao et al. 2006b). The standard deviations calculated by using site period are nearly identical for site class (SC) I, II and III sites at all spectral periods and the $F$-test probability of having similar standard deviations from the two site terms are over 60%. For SC IV sites, the standard deviations from $T_{VS30H}$ are statistically larger than, but practically similar to, those from site period in a spectral period range of 3-4.5s, and they are very similar at other spectral periods.
To examine if $T_{VS30}$ can be an alternative to site period for a GMPE, we have analyzed a sub-dataset of surface records used by Zhao et al. (2010) for developing geometric attenuation functions and by Zhao and Xu (2012a) for examining magnitude scaling rates for large subduction interface earthquakes in Japan. These records are from sites that have site periods, $V_{S30}$ and bedrock depth derived from measured shear-wave velocity profiles or inferred by H/V ratios. We have also found that the two site parameters lead to very similar median amplification ratios between a surface soil site and surface rock site. These results suggested that a combination of $V_{S30}$ and depth to engineering bedrock can be an excellent alternative to site period as a site parameter for a GMPE. The effect of average values and the detailed variation of shear-wave velocity below 30m depth are statistically not significant. This conclusion may be potentially useful for those sites of engineering applications where $V_{S30}$ is available and the depth to the engineering bedrock can be inferred from nearby borehole logs that do not provide shear-wave velocity or information that can be used to estimate shear-wave velocities.

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