A Simplified Approach for the Seismic Vulnerability Assessment of R.C. Bridges with Simply Supported Deck

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SUMMARY
The paper proposes a simplified procedure for the seismic vulnerability analysis of girder bridges with simply supported decks and single-circular piers. The proposed method can be applied in all the cases for which the seismic response of the whole bridge depends on the most critical pier. For an assigned limit state, the procedure determines the capacity curve of the critical pier as a function of 3 parameters (elastic stiffness, displacement at yielding, displacement at collapse), taking into account the different possible collapse modes (shear failure; lap splice debonding of the longitudinal bars; buckling) and the geometric non linearity. A N2-like procedure is then applied in order to identify the “Capacity Return Period”, which represents the performance level, to be compared with the design spectrum. A significant numerical example is presented in which the traditional FEM solution is compared with the proposed simplified procedure.

Keywords: Bridges, reinforced concrete, seismic vulnerability

1. INTRODUCTION
In seismic prone areas, the seismic risk assessment of strategic buildings and infrastructures is a crucial question. A particularly relevant category is represented by bridges, to which a great attention has been devoted in the last few years, in order to perform the vulnerability inventory at the regional scale and possibly obtain priority lists and preliminary indications for the retrofitting interventions. To this aim, many research studies have been devoted to the definition of “damage indexes” or “indexes of criticality” (Wen-I Liao and Ching, 2004; Mezzina and Raffaele, 2007). The safety assessment of existing bridges is based on the appraisal of the dissipative capacity of the structure and its actual available ductility by means of suitable non linear methods of analysis. A very effective method, if applicable, is the pushover analysis (PO). The theoretical foundation of this procedure, which uses static structural analysis for appraising the non linear behaviour under seismic loads, is dated back to more than thirty years ago (Freeman et al., 1975; Fajfar and Fischinger, 1988). Since then, the method has been extensively developed, so that today many variants exist, which are characterized by a greater accuracy, but also by greater complexity (Antoniou and Pinho, 1996; Chopra and Goel, 2002; Fajfar and Gaspersic, 2004). Anyway, it should be noticed than most of these studies are aimed at the analysis of buildings, while specific procedures for the assessment of existing bridges are relatively few (Isakovic and Fischinger, 2006; Kappos et al., 2006). In this field, research has been mostly oriented at the development of the multi-modal and adaptive extensions of PO methods, which are very interesting, but unfortunately are computationally very expensive. In the last few years, an increasing attention has been devoted to simplified PO procedures in which the capacity curve of RC structures is calculated only by defining the position of some characteristic points, and without performing any analysis of thrust (Borzi et al. 2007; Crowley et al. 2004). In these approaches, the non-linear properties of the structure are modelled by introducing equivalent systems - usually a single degree of freedom system – which corresponds to the original structure in terms of vibration period, energy dissipation and displacement capacity (Pinto et al., 2007). The fundamental objective is to assess the seismic vulnerability of building by relying only on poor data that can be easily retrieved even by a limited preliminary investigation, without the need to perform a detailed protocol of investigation aimed at the complete geometric and mechanical characterization of the structure.
The research study presented in the paper is referred to this theoretical framework, and proposes a simplified procedure to define the capacity curve of existing bridges, expressed by Moment-displacement relationship $M-\delta$, taking into account the different failure modes that can possibly affect the structural ductility. The capacity curve is defined starting from the Moment-Curvature response ($M-\chi$) of the base section for an equivalent SDoF system. Strength and ductility (in terms of displacement) are then properly modified in order to account for a possible early collapse triggered by one of the following collapse mechanisms: i. inadequate overlapping length of the longitudinal bars; ii. buckling of the longitudinal bars, iii. shear. The ultimate shape of the capacity curve is finally determined by evaluating the further loss of strength related to the II order effects induced by geometric nonlinearity. The research study has been developed for the analysis of vulnerability of RC girder bridges with simply supported deck and single-column piers with solid circular section, which were very popular in Italy in the 70's. Anyway, the methodology can be easily extended to the case of piers having different types of section. In the paper, after outlining the sequential steps of the procedure, a numerical validation is presented on a representative example, comparing the proposed simplified response under combined axial stress and bending (which is the basis of the entire procedure) with a more refined non linear FEM pushover analysis.

2. REFERENCE FRAMEWORK OF THE PROCEDURE

For bridges with simply supported decks and single-columns piers it is possible to define simplified approaches for the structural vulnerability assessment as an alternative to more accurate methods (Pinto et al., 2007). In this type of bridges, in fact, the deck is not significantly involved in the seismic response and thence the global behaviour of the structure under seismic loads is determined by the supporting structures (piers). In particular, the vulnerability of the deck is strongly dependent on the behaviour of the “critical” pier, which is defined as the pier characterized by the lowest value of the Seismic Vulnerability Coefficient $CVS=\delta_{SL}^{c}/\delta_{SL}^{d}$ (where $\delta_{SL}^{c}$, $\delta_{SL}^{d}$ respectively are the displacement “Capacity” and “Demand” of the structure for the Limit State assumed for the assessment), both in the trasversal and in the longitudinal direction (Mezzina et al., 2010). It should be remarked that if the retrofitting interventions programmed provide a substantial modification of the static scheme (for example a solidarization of the deck, or its complete replacement with a continuous deck), the aforementioned assumptions for the simplified vulnerability analysis cannot be applied.

In order to evaluate the deformation capacity of the structure beyond the elastic limit, it is fundamental to determine the capacity curve of the structure, which is usually obtained by the application of non-linear static procedures (PO). In view of vulnerability analyses at a regional scale, involving a large number of structures, the availability of simplified and rapid approaches to be used as an alternative to the full non linear procedures, is particularly useful. In a simplified approach, the capacity curve expressed in terms of the Force-displacement relationship $F-\delta$ (or equivalently, the Moment-displacement relationship $M-\delta$), can be represented by a bi-linear function (Fig. 2.1.), that is completely known once the two points A and B are identified.

![Figure 2.1](image)

**Figure 2.1.** The simplified bi-linear representation of the F-\delta relationship

The pair of coordinates $A(F_y - \delta_y)$ and $B(F_u - \delta_u)$ characterize the structure, respectively, at the elastic limit state and at the ultimate limit state (Pinto et al., 2009[2]). The reference model is an equivalent
Single degree of Freedom system (SDoF, Fig. 2.2, right). In the transversal direction to the axis of the bridge (“T”), each pier behaves as a simple oscillator. In the longitudinal direction (“L”), in the presence of seismic bearings (the so called “fixed” pier), the pier can also be model by a SDoF scheme, in which the mass is given by the reactions of the supported decks.

![Fig. 2.2. The equivalent SDoF schematization.](image)

Thence, there are two distinct simplified models: one in the transversal direction, having a mass \( M_T \) and height \( H_T \), another in the longitudinal direction, having a mass \( M_L \) and height \( H_L \) (Fig. 2.2). Depending on the direction of the analysis, the centers of application of the masses have different heights. The masses of the two models can be calculated by means of the following expressions:

\[
M_T = \frac{W_T}{g} - \frac{n_T W_f}{g} + \frac{W_p}{g} + \frac{W_f}{g} \quad ; \quad M_L = \frac{W_L}{g} - \frac{n_L W_f}{g} + \frac{W_p}{g} + \frac{W_f}{g}
\]

where \( W_f, W_p, W_f \) respectively are the weight of the deck, of the pulvino and of the pier; \( g \) is the gravitational acceleration; \( n_T \) and \( n_L \) are the number of spans loading the pier in the transversal and longitudinal direction, as a function of the constraint type. The height of the centers of the masses \( M_T \) and \( M_L \), measured from the base section of the pier, are given by:

\[
H_T = \frac{0.3 W_f 0.85 H_p + W_p H_p + W_f H_f}{W_T} \quad ; \quad H_L = \frac{0.3 W_f 0.85 H_p + W_p H_p + W_f (H_f + \Delta_{up})}{W_T}
\]

where: \( H_T, H_P, H_f \) are the heights of the pier, of the pulvino and of the deck (see Fig.2.2); \( \Delta_{up} \) is the overall height of the pulvino-bearing device (measured from the top section of the pier). In order to simplify the procedure and reduce the parameters involved, in the proposed procedure the interaction between the super-structure and the foundation structures is neglected, and the piers are supposed to be fully clamped at the base.

3. COLLAPSE MODES

In existing bridges, which are mostly characterized by an unsatisfactory level of the technical design (mainly because the reference building codes are very obsolete), piers often reveal a number of critical states such as: poor confinement at the base; inadequate amount of longitudinal reinforcements (sometimes a brittle behaviour of the section is engaged). A number of experimental studies carried out in this field (Calvi et al. 2005), and in particular concerning single-column piers with circular section (Albanesi et al., 2008), have shown that the crisis can be triggered by different mechanisms, according to the value of the significant structural parameters (height of the pier, diameter of the pier’s section, amount of longitudinal and transversal reinforcements, ...). The variation of even one of these parameters can change the actual type of failure mechanism, and it is nearly impossible to predict which of these occurs at the limit state, and thence it is difficult to evaluate the capacity curve (points
A and B in Fig. 2.1). Thus, it becomes important to quantify with a good approximation the effects induced by each possible collapse mechanism, in order to determine the variations on the response of the pier in terms of strength (force or moment) and deformation capacity (displacement or ductility). Different failure modes are taken into account and for each of them, a procedure for properly correcting the capacity curve, as obtained by the basic analysis of the critical section (base section) under bending and normal stress, is defined. The analysis is performed with reference to the base section, because the non linear zone is assumed to be located here (fully clamped constraint), in the tract of length $L_P$ (usually referred to as "length of the plastic hinge"). The collapse modes included in the procedure are:

a) Failure under **combined axial stress and bending**, in the presence of effective confinement, $\chi=\chi_U$ (§ 3.1);

b) **Lap-splice failure of longitudinal bars** (§ 3.2);

c) **Buckling of longitudinal bars** (§ 3.3);

d) **Shear** (§ 3.4).

Whenever the failure under combined axial stress and bending is anticipated by one of the mechanisms listed at the points b), c) e d), and the final capacity curve is correspondingly modified.

### 3.1. Failure under combined axial stress and bending

As previously remarked, in the simplified model of Fig. 2.2, the capacity curve $M-\delta$ is defined starting from the moment-curvature relationship ($M-\chi$) of the critical section, which is located at the base of the pier, within the tract of length $L_P$. The first simplification assumed in the proposed procedure consists in the transformation of the $M-\chi$ relationship calculated at the level of the section (red line in Fig 3.1a), in an elastic-perfectly plastic law ($M_y=M_u$) (black dotted line). In order to identify the characteristic points of the moment-curvature curve of the critical section, that is to say the curvatures $\chi_y$ and $\chi_u$ and the ultimate moment $M_u$, the following geometrical and mechanical parameter should be first determined: i. longitudinal reinforcement area ($A_s$); ii. confinement level, expressed as the geometric volumetric percentage of the transverse reinforcement ($\rho$); iii. compressive strength of concrete ($f_c$); iv. yield strength of the steel $f_y$; v. compressive normal stress acting on the $N$-th pier. The values $f_c$ and $f_y$ are to intended as average values of the results provided by experimental in-situ tests, or derived by existing information, divided by the "Partial Safety Factor" (PSF) corresponding to the "Knowledge level" (KL) attained (CEN, 2005 [1]).

![Fig. 3.1a. M-\chi relationship (of the critical section. 3.1b. Capacity curve of the SDoF oscillator.](image-url)

For circular sections, when the $M-\chi$ law can be expressed by means of a bilinear relationship, Priestley (Priestley MJN, 2003) proposes a numerical expression in which the curvature at yielding ($\chi_y$) is independent from the above mentioned parameters, and is only a function of the strain at yielding of the steel ($\epsilon_{sy}$) and of the diameter D of the pier section. Consequently, the capacity at yielding in terms of chord rotation ($\theta_y$) can be obtained by a purely elastic analysis under bending that, in the specific case, corresponds to a perfectly elastic behaviour of the pier up until the yielding moment $M_y$ is attained:
where \( \delta_i \) is the displacement of the top section and \( L_V \) is the shear length (= \( M/F \)).

The capacity at collapse can be expressed instead by the relationship proposed by Panagiotakos (CEN, 2005[1], Panagiotakos & Fardis, 2001), in which a quote of plastic rotation at the critical base section is added to the yielding rotation:

\[
\theta_u = \frac{\delta_u}{L_V} = \chi_y \frac{L_V}{5} + (\chi_y - \chi_u) L_p \left( 1 - \frac{0.5L_p}{L_V} \right)
\]  
(3.2)

The only unknown term in eqn. 3.2 is the ultimate curvature \( \chi_u \), which can be derived by assuming a strain distribution associated to the collapse of the section, involving the failure of the concrete in compression and the yielding of the steel under tension. Although this approach, based on the evaluation of chord rotations, is quite simple and easy, there is the problem of defining the length of the plastic hinge \( (L_p) \), which is not so immediate, since it strongly depends on the constitutive laws of the materials, the load type, the geometry of the transversal section, the shear stress. Several formulations can be found in the literature, (Priestley & Park, 1987; Fardis, 2007) that can be all equivalently used within the proposed methodology. In the numerical application presented in Section 5 the relationship contained in Eurocode 2 has been adopted (CEN, 2005[2]):

\[
L_p = 0.1L_V + 0.17h + 0.24 \frac{d_{sl} f_y}{\sqrt{f_c}}
\]  
(3.3)

where \( d_{sl} \) is the diameter of the longitudinal bars.

The displacement capacities \( \delta_{SLD}, \delta_{SLV} \) and \( \delta_{SLC} \) corresponding, respectively, to the Damage Limit State (SLD) and the Ultimate Limit State (which are respectively represented by the Limit State of Life Safety - SLV and Near Collapse - SLC), according to the indications of the Italian Seismic Code (D.M. 14/01/2008), can be evaluated as \( \delta_{SLD} = \delta_y, \delta_{SLV} = 3/4\delta_{SLC} \) and \( \delta_{SLC} = \delta_y/\gamma_{el} \), with \( \gamma_{el} = 1.5 \) for primary structural elements and \( \gamma_{el} = 1.0 \) for secondary structural elements (Fig. 3.1b). For the calculation of the ultimate moment, a numerical formulation in a closed form is proposed (Eqn. 3.4), depending on the following parameters: i. \( v = \) axial force normalised to \( A_e f_y; \) ii. \( \omega = \) mechanical ratio of tension longitudinal reinforcement; iii. \( \rho_i; \) iv. \( f_y; \) v. \( f_c. \) The range of variation considered for the above mentioned quantities are: 0.1<\( v <0.6; \) 0.05<\( \omega <0.6; \) 0.0000475<\( \rho_i <0.000285; \) 15MPa<\( f_y <45MPa; \) 300MPa<\( f_c <405MPa. \)

\[
M_u = (aw^2 + bw + c)(aw^2 + aw + f)(gw + h)(if + l)(mf + n)
\]  
(3.4)

where \( a, b, c, d, e, \) and \( f \) are the numerical constants of the II order regression, whereas \( g, h, i, l, m, \) and \( n \) are the numerical constants of the linear regression.

3.2 Lap-splice failure of longitudinal bars

An effective model for appraising the flexural strength of the pier in the presence of longitudinal bars joined by overlapping, which is also supported by several experimental results, has been proposed by Priestley et al. (Priestley et al., 1996). In the presence of confinement, when lap-splice failure occurs (\( \rho_l < \rho_h \)), the model introduces a degradation of the flexural strength \( (\Delta M) \) given by the difference of the ultimate Moment \( M_u \) corresponding to an optimum level of confinement \( \rho_{ho} \) (see Eqn. 3.5) and a reduced moment \( M^* \) (this is obtained by interpolation between a minimum value \( M_{ho} \) corresponding to \( \rho_l = 0 \) and \( M_u \)): 
\[
\rho_h = 1.4 \frac{f_{y}^2}{4P_L f_{y}^2}; \quad \rho_s = 0 \quad \Rightarrow M^* = M_u + \frac{\rho_s}{\rho_h} (M_u - M_d)
\]  
(3.5)

\(L_s\) is the length of overlapping of the bars, \(f_{y}\) is the characteristic resistance of the steel, \(f_{y}^2\) is the maximum tension applied by the transversal reinforcements (which is assumed to be equal to 200 MPa). By applying the above described model, within the proposed procedure, in the presence of lap-splice failure, the capacity curve derived in § 3.1 is reduced by a quantity equal to \(\Delta M\) (Fig. 10).

![Fig. 3.2. Modification of the flexural capacity curve according to the possible lap-splice failure.](image)

### 3.3 Failure for buckling of longitudinal rebars

The effects due to the possible buckling of longitudinal bars are evaluated by using the empirical model proposed by Berry & Eberhard (Berry and Eberhard, 2005). The model is based on the assumption that the instability involves only a reduction of the hysteresis loops and a decrease of the displacement capacity of the structural element. Starting by this assumption, the authors proposed a numerical relationship for the assessment of the ultimate displacement in the presence of instability, \(\delta_{bb}\), which was calibrated on the basis of a hundreds of tests.

\[
\delta_{bb} = 0.0325 L_y \left(1 + 150 \frac{\rho_s}{\rho_c} \frac{f_{y}}{f_c} d_{bh} \right) \left(1 - \frac{N}{A_c f_c} \right) \left(1 + 0.1 \frac{L_y}{D} \right)
\]  
(3.6)

In the proposed procedure, the effects of the buckling are taken into account by reducing the displacement capacity of the SDoF system, after checking the value of \(\delta_{bb}\) against the available displacement corresponding to the different limit states assumed in the verification, i.e. LS and NC (see Fig. 3.3).

![Fig. 3.3. Modification of the capacity curve in order to account for the buckling of longitudinal bars.](image)
3.4 Shear collapse

An effective model of the shear mechanism for circular sections, that provides a good feedback with experimental results, was proposed by Priestley et al. (Priestley et al., 1998). In the model, the shear strength $V_U$ of the RC elements under combined axial stress and bending is defined as the sum of three contributions: i. $V_H = \text{horizontal component of the axial force} \ N$ acting on the concrete strut; ii. $V_{ST} = \text{contribution of the transversal reinforcement}$; iii. $V_C = \text{contribution related to the aggregate interlocking effect}$. The shear collapse determines a significant reduction of the deformation capacity, and therefore of the displacement ductility of the structural element. On the basis of these observations, it can be concluded that there are four possible configurations that can be represented in the $M-\delta$ plane, as summarized in Fig. 3.4, showing the reduction of the displacements at the different limit states. The blue line corresponds to the reduced capacity curve, whereas the curve of the shear resistance is plotted in red.

![Fig. 3.4. The modification of the flexural capacity curve in the presence of a shear failure of the pier.](image)

4. APPRAISAL OF II ORDER EFFECTS

By referring to the SDoF system, II order effects can be considered as equivalent to a reduction of the resistance of the critical section. Once the capacity curve of the pier has been derived by following the procedure outlined § 3 (by applying the necessary modifications), it is finally possible to evaluate the reduction of the strength capacity $M_U$ induced by II order effects as a function of the displacement $\delta$ (Fig. 4.1): $\Delta M_{II} = N*\delta_S$ ($\delta_S$ is the displacement associated to the specific limit state).

The ductility $\mu_S = \delta_U/\delta_V$ varies from a minimum value of 1 (for the damage limit state SLD) to a maximum value corresponding to the limit state of near collapse (SLC).

![Fig. 4.1. Modification of the capacity curve under II order effects, for the different limit states.](image)
5. SYNOPSIS OF THE PROCEDURE

The whole procedure is summarized in Tab. 4.1, starting from the definition of the initial capacity curve and indicating the sequential steps in which the capacity of the section is “corrected” by considering the possible additional collapse modes.

<table>
<thead>
<tr>
<th>Step / §</th>
<th>Collapse Mode/Effects</th>
<th>Modification</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - §2-3.1</td>
<td>Combined normal stress and bending</td>
<td>Linearization of the M- curve</td>
</tr>
<tr>
<td>1 - §3.2</td>
<td>Lap-splice failure of long. bars.</td>
<td>Reduction of the resistance by ΔM=M_u-M*</td>
</tr>
<tr>
<td>2 - §3.3</td>
<td>Buckling of long. bars</td>
<td>Comparison between δSL and δb and correspondent reduction of the available displacement capacity</td>
</tr>
<tr>
<td>3 - §3.4</td>
<td>Shear</td>
<td>Comparison between δSL and δv and correspondent reduction of the available displacement capacity</td>
</tr>
<tr>
<td>4 - §4</td>
<td>Geometric non linearity</td>
<td>Reduction of the resistance by ΔM_SL II = N*δSL</td>
</tr>
</tbody>
</table>

6. STEP “0”: NUMERICAL APPLICATION

As already remarked in § 2 and § 3.1, the entire proposed procedure is based on a simplified sectional analysis consisting in the linearization of the M-δ curve (determination of the two characteristic points corresponding to the yielding and collapse - step "0" in Tab. 4.1). Considering the importance of the first step for the subsequent development of the procedure, a preliminary numerical validation of the sectional analysis has been performed. The response of a circular section subjected to bending has been determined according to the procedure described in § 3.1 and then compared with the results of a FEM pushover analysis (based on fiber model) performed by using software SAP2000 V14.2.2 (Computer & Structures, 2010). The pier has been modelled by means of 12 beam elements, discretizing the cross section into 52 fibers. With regard to the constitutive aspects, law of for the concrete, the relationship shown in Fig. 6.1, proposed by Mander, Priestley e Park (Mander et al., 1988) has been adopted, whereas for the steel an elastic-plastic law with hardening has been used. In Tab. 6.1 the geometric and mechanical parameters of the pier analyzed are summarized (symbols are the same used in previous sections).

<table>
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<td>Reduction of the resistance by ΔM_SL II = N*δSL</td>
</tr>
</tbody>
</table>

The pushover analysis under a load distribution proportional to the first vibration mode has been performed, and in Fig. 6.1 the resulting capacity curves, representing the structural response of the pier under combined normal stress and bending are reported. In particular, the curve obtained with the FEM analysis by the solver SAP (dashed blue line) has been transformed into the equivalent bilinear curve (solid blue line), by assuming that the elastic branch ended up with the yielding of the first longitudinal bar and that the peak resistance was 85% of the maximum moment. The criterion adopted is coherent with the principle of the energy equivalence between the areas under the two curves.
Fig. 6.1. Capacity curves and constitutive law of concrete

From a simple graphic comparison, it can be clearly seen that the approximate procedure, that will be thereafter referred to as "VulPil_CC", is very close to the numerical solution provided by the FE model. A more detailed comparison is presented in Tab. 6.2, in terms of ultimate moment $M_U$, ultimate displacement $\delta_U$, and PGA (Peak Ground Acceleration) corresponding to the Near Collapse limit state. It can be seen that the differences are always below 7% and can therefore be considered fully satisfactory for a validation of the foundation of the proposed simplified procedure.

<table>
<thead>
<tr>
<th></th>
<th>$M_U$ [t*m]</th>
<th>$\delta_U$ [cm]</th>
<th>PGA [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VulPil_CC</td>
<td>1308.15</td>
<td>-6.95</td>
<td>0.454</td>
</tr>
<tr>
<td>SAP V14_BL</td>
<td>1405.82</td>
<td>6.87</td>
<td>0.470</td>
</tr>
</tbody>
</table>

7. FINAL REMARKS

In the last few years, the need to develop effective and manageable procedures for performing the vulnerability inventory of strategic infrastructures at a regional scale, and obtaining priority lists for the intervention, has become increasingly urgent. In this paper, a simplified procedure (which resorts to poor data that can be easily obtained) aimed at the seismic vulnerability assessment of girder bridges with simply supported decks and single-circular RC piers is proposed. The main approximation of the procedure consists in the evaluation of the $M$-$\delta$ relationship for the critical pier under combined normal stress and bending. Thence, after describing the theoretical formulation of the whole procedure, which includes the modification of the capacity curves in order to account for possible additional collapse modes, a comparative numerical analysis between the main approximation proposed and a non-linear FEM analysis (based on a traditional fiber model) has been carried out. The differences found are negligible, especially considering that the input data of the simplified procedure are very few.

AKNOWLEDGEMENT

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