Contents Sliding Response Spectra

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SUMMARY:
Building content damage can cause significant economic loss in major earthquakes. This paper quantifies the contents damage to structures with similar backbone hysteresis curves, but with different unloading and reloading characteristics under both impulse and earthquake excitations scaled to the design level for Wellington. These sliding demands are expressed in terms of sliding spectra and they show that the amount of sliding depends on the shape of the hysteresis curve used. Two simple explanations are proposed and evaluated to determine which hysteresis loops would result in the greatest sliding damage. It is shown that a sudden increase in stiffness when the structure is moving at a high velocity is not the key factor. Instead, the velocity at the time of sliding initiation seems to correlate better with the calculated sliding distance.

Keywords: contents, sliding spectra, response spectrum, friction force

1. INTRODUCTION

The recent Canterbury earthquakes resulted in significant contents damage in buildings which had little structural damage. In some cases, building contents such as microwaves moved several metres across rooms, HVAC units sitting on top of roofs of multi-storey buildings moved, material came off shelves, and contents expected to behave as rocking systems slid on the floors. As vehemently moving heavy objects like photocopiers, and furniture can be hazardous and dangerous, it is important to assess the sliding demands of contents.

The key engineering demand parameters commonly used to determine damage within a building during earthquakes are generally assumed to be (i) the *total acceleration*, and (ii) the *interstorey drift* (e.g. Bradley et al. 2010). Since the floor acceleration dictates whether or not a contents item moves, it is often assumed that the acceleration is the key parameter controlling damage. However, there are a number of other parameters that are also important. For example, in the E-defence shake-table tests, photocopiers moved not because of acceleration, because the copier was on wheels and somewhat isolated from the structural accelerations. As the structure moved, because of the isolation the copier wanted to remain in the same place. Because the structure was moving, it collided with the copier, producing an impact force that imparted momentum to the copier. The copier then moved across the room at high speed. Perhaps the key factor causing damage from the photocopier is *total* (ground plus relative building) *displacement*, because it is the magnitude of this displacement that determines whether or not the structure is likely to impact items such as these copiers. Furthermore, while acceleration may relate to the initiation of movement of some contents items, the duration of acceleration, velocity of excitation, or other factors may influence the amount of movement and hence the likely damage.

Furthermore, different structural systems may cause contents to move different amounts because they cause different responses. For example, a number of newer structural systems have different characteristics from the bilinear or Takeda systems commonly assumed for regular structures. These include the many post-tensioned building systems such as those in moment-frames, or rocking structures (e.g. Priestley and MacRae 1996, Priestley 1996, NZ South Rangitikei Rail Bridge (Tilby 1981), Gledhill et al. 2008, Roke et al. 2009, Deierlein et al. 2010). These have a non-linear elastic, or flag-shaped, hysteresis loop. It is possible that these structures will have more severe effects on than those with the more traditional hysteretic loops. Two reasons for this possibility are explained below.

Explanation 1. High increase in stiffness at high velocity. Pinched loop and flag-shaped structures have a hysteretic characteristic which, during unloading, has a rapid increase in stiffness near the position of zero displacement when compared with elastoplastic structures, as shown in Fig.1.1. The increased stiffness (force) is then turned into high velocity, since the momentum needs to be conserved. Such systems have been referred to as "clickety-clack" systems (Buchanan et al. 2012). Other systems of this type with a high increase in stiffness at high velocity include structures with traditional (buckling) concentric braces with medium to high slenderness ratios, pure steel plate shear walls, concrete walls and others (MacRae 2010). The behaviour of "clickety-clack" systems may not be desirable as shown by the example below.

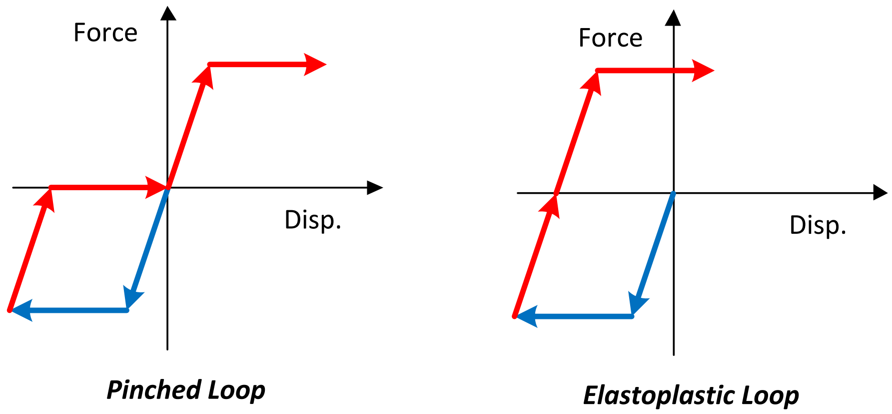
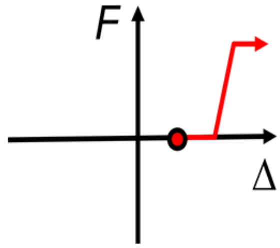


Figure 1.1 Different hysteretic loops

Consider a motorbike travelling at a constant velocity as shown in Fig. 1.2. Because it is at constant velocity, the horizontal forces and accelerations on the motorcyclist are zero. However, when the motorbike suddenly hits a wall, the forces on the bike suddenly increase, until the wall is pushed over. The hysteresis loop for the motorcycle in Fig.1.2b is similar to that for many "clickety-clack" structures as there is a sudden increase in stiffness at high velocity. As a result of this the following provocative question was asked: "Is the difference between the motorcyclist, and a person in a "clickety-clack" building during an earthquake, only the amount of protection they are wearing?" (MacRae 2010). Anecdotal evidence (e.g. Bull 2011, Clifton 2011) indicates that in buildings of this type, including concrete shear wall buildings, due to the high increase in stiffness at high velocity, many items and people were thrown across rooms during the 22nd February 2011 Christchurch earthquake. This is similar to the way the motorcyclist may be expected to be thrown of their motorcycle. The response of systems with pinched hysteresis loops (the flag-shaped bilinear hysteresis characteristic of rocking system, for example) is different from the response of some more traditional systems which have hysteretic loops which are more bilinear, or Ramberg-Osgood shaped where the increase in stiffness generally occurs at the position of maximum displacement where the velocity is zero.

It may be that the "low-damage" systems, like rocking walls and some post-tensioned beam-column systems, while seeking to protect the structure, actually impose greater demands (higher velocity profiles) on their contents (including physical items, and the people within the buildings) than some more traditional systems, and that the actual risk of severe injury, or even life loss, is significantly increased.

(a) Motorbike and Wall



(b) Hysteresis Curve

Figure 1.2 Hysteresis for sudden stiffness change at high velocity (WWW, 2010)

Explanation 2. High velocity at negative sliding acceleration, a_s. A related, but different argument, simply considers the likely velocity when the structure develops the sliding acceleration in the negative direction after being released from the position of maximum displacement. The maximum displacement is assumed to be roughly similar for all structures of the same stiffness as described by the equal displacement assumption. This may be understood for the linear-elastic, nonlinear elastic and bilinear hysteresis loops in Fig. 1.3 where the unloading energy of the structure is shown as the yellow shaded area multiplied by the structural mass. If there is no additional damping in the system, then upon releasing the oscillator from the peak displacement, the potential energy represented by the yellow area times the mass is transformed into kinetic energy when the oscillator reaches the zero acceleration line. This kinetic energy then wants to cause the oscillator to move in the negative acceleration direction to a displacement which can be found when the kinetic energy is transformed into potential energy in that direction. As it approaches the peak displacement in the negative acceleration direction it gradually slows down. The velocity at the point of sliding in the negative force direction can be found from the kinetic energy at that point which is the energy released (equal to the shaded area times the mass) minus the elastic energy used to move the structure to the point of sliding (as indicated with a_s). The difference in relative velocity at the point of sliding in the negative force direction between these three types of hysteresis characteristic is roughly estimated from the relative shaded areas under the loop in the positive loading direction. It may be seen that as the elastic response contains more energy, it is likely to have a greater velocity at the point of sliding than the other loops, so the sliding displacement is likely to be bigger. The bilinear loop is likely to have the lowest contents movement upon unloading from the same displacement.

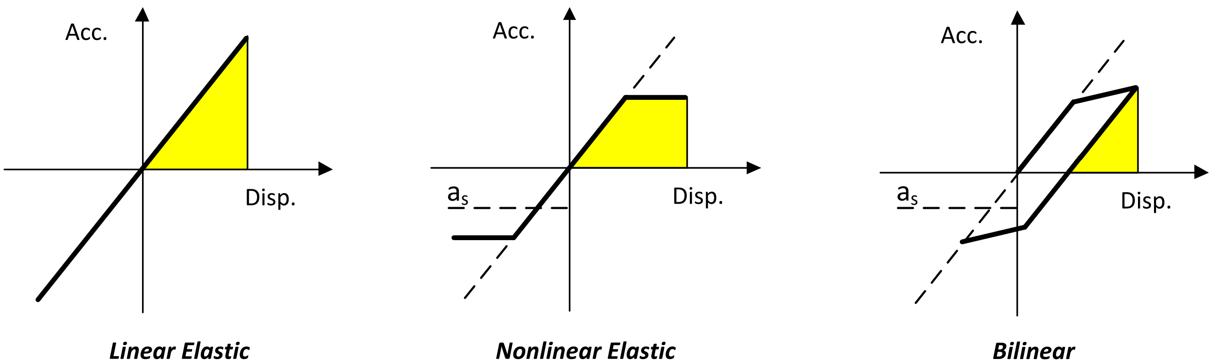


Figure 1.3 Unloading energy for different hysteresis loops

It should be noted that neither of these explanations represent the full reality of the problem (English al. 2011, Lin et al. 2012), but simply point to the issue that contents sliding may be different for structures with different types of hysteresis loop. For example, Explanation 2 is unlikely to be robust because during the earthquake shaking, contents sliding may occur at the peak displacement in the

positive direction.
The second explanation is more consistent with a recent study by English (2011) that indicates that the peak sliding seems to generally be affected more by the energy dissipated by the hysteresis loop than by than by the likelihood of a high stiffness increase at high velocity.

This paper, which describes contents movement in terms of sliding, develops sliding response spectra for both impulse and earthquake records in order to evaluate which of the simplified explanations above is most likely to reflect the behavior of the contents of structures during significant earthquake shaking.

2. BACKGROUND LITERATURE

Many studies have been carried out on sliding, including the effects of buildings sliding on sand, sliding base isolation dissipaters such as the friction pendulum system, and in field of soil mechanics where sliding blocks are analyzed. Also, there are many observations of contents sliding and falling off shelves during earthquakes (e.g. NZSEE 2010). Whittaker and Soong (2003) state that building contents damage, together with non-structural damage contents, may be 4 times greater than the structural damage. This high rate of contents to structural damage is generally higher in moderate levels of ground shaking such as the 2010 Darfield earthquake where there was little structural damage to well-designed structures, but other losses were significant (MacRae et al. 2011).

Relatively little work has been done to quantify the likely magnitude and understand the mechanisms of sliding of building contents. Generally, the sliding behaviour is simulated as a rigid block supported on a frictional contact surface (Coulomb friction) on a horizontal plane (e.g. Shenton et al. 1991). Hutchinson et al. (2006) conducted fragility analysis of sliding dominated equipment and contents, and a simple numerical example was illustrated for a building structure design. Gazetas et al. (2009) showed that the slippage of a soil block is not only dependent on the peak acceleration, but also on other ground motion characteristics, such as the peak velocity, frequency, presence of strong pulses. They found that the effect of vertical components of ground motions is not generally significant, and this was reconfirmed in Gazetas et al. (2012) using the 22nd February Canterbury earthquake records. Sliding spectra of a block for an individual earthquake motion were generated, and the influence of soil profiles was discussed (Gazetas et al. 2012). Gazetas et al. (2012) also provide a good summary of past work on this topic. No studies are known to have been conducted to address the questions raised above.

English et al. (2011, 2012) carried out a study to mechanism of sliding and to evaluate, what properties of the structure and ground motions cause the sliding effects. The analytical model of the contents sitting on a single-structure was modelled as a two degree-of-freedom system where the mass of the contents was much less than that of the structure, and the two were connected with a near-rigid plastic spring. The strength of the spring was set as μmg, where μ is the coefficient of friction (COF), m is the mass of the contents, and g is the acceleration due to gravity. Some findings from the response of the structure after impulse analysis include:

1) Sliding initiates when the contents acceleration is μg
2) The amount of sliding tended to increase with a greater initial velocity of sliding.
3) The sliding stops when the velocity of the sliding and contents becomes the same. However, if this occurs at an acceleration which is greater than μg, then the structure will immediately slide again. The time when sliding stops depends on the characteristics of the structure. This consideration is not included in either of the simplified explanations in Section 1.
4) Bilinear yielding structures tend to have lower sliding displacements than elastic structures, because the relative velocities between the structure and contents is greater for the elastic structure than for the inelastic structure, even if the time of sliding is similar.
5) Non-linear elastic hysteresis loops tend to have greater sliding displacements than bilinear loops, because they allow sliding for a greater length of time before the velocities of the structure and contents becomes similar.

6) The effects found in (4) and (5) were independent of period, and the elastic oscillator generally had the greatest sliding demand in the study undertaken.
7) As the structural yield strength decreased, the sliding displacement also decreased.
8) Some very high frequency forces occurred between the contents and structure when there were sharp changes in force at high velocity in the structural hysteresis loop, especially for short period structures.
9) Peak sliding seemed to generally be affected more by the energy dissipated by the hysteresis loop than by than by the likelihood of a high stiffness increase at high velocity.

Also, the oscillators were subject to the unscaled suite of SAC LA10in50 records. The peak sliding was generally greatest for oscillators with linear elastic or elastic hysteresis loops, than for oscillators with slackness, bilinear, Ramberg-Osgood, or flag-shaped hysteresis loops. Also, the demand decreased with lower structural yield strength. All other hysteresis loops had similar backbone curves. Amongst these, the structures with non-linear elastic loops, flag-shaped loops, and loops with slackness, bilinear, and Ramberg-Osgood showed decreasing amounts of sliding.

It should also be noted that while idealized hysteresis loops are often used in design, actual loops are often significantly more complex. For example, experimental studies have shown that (i) the Takeda loop does not describe the behaviour of all models well (MacRae et al. 1993), (ii) in post-tensioned concrete beam-column tests without dissipaters, the behaviour may not be non-linear elastic due to other systems also dissipating energy (Priestley and MacRae, 1996), and (iii) during rocking response, high frequency impact effects may be observed (Yu et al. 2011). These effects were not considered in this study.

3. STRUCTURE-CONTENTS SLIDING MODEL DEVELOPMENT & IMPULSE ANALYSIS

A simple two-degree-of-freedom structure-contents sliding system, similar to that used in previous studies (e.g. English et al. 2011) was used to model the building and contents as shown in the top left corner of Fig. 3.1. The hysteresis behaviour of structure is described via spring component *S*, while the behaviour between contents and floors follows Coulomb's law of friction. The frictional behaviour of the contents-floor interaction was modelled using a bilinear hysteresis rule to approximate a rigid-plastic relationship. The sliding force, $F_{resisting}$, was set to μmg, where μ is the coefficient of friction (COF) that is assumed to be constant, and m is the contents mass, and g is the acceleration of gravity. Sliding will initiate only when the total structural acceleration, $a_{structure}$, is greater than μg. A ratio between m and the structural mass, M, of 1:1000 was used. Only hysteresis damping was considered in this initial study. The contents movement was defined as the relative displacement between the structure floor and the contents. The computer software RUAUMOKO-2D (Carr 2008) was used to perform the numerical analyses.

Impulse analysis was first conducted, because this has been shown to be a powerful tool to understand seismic response (e.g. MacRae et al. 2008). This involves using a rectangular impulse acceleration (0.125*g*) during the first 0.01 second of the record followed by a period of no further acceleration following the approach of English et al. (2011) enabling the free vibration response to be understood. The structural behaviour (*S*) was initially assumed to be linear elastic. Figure 3.1 shows the total acceleration, the structure floor and contents velocity and displacement history relative to the ground. As can be seen, a full sliding behaviour can be divided into the following stages:

- O-A: Structure and contents respond together
- A-B: Sliding has initiated at A, and continues to the peak displacement of the structure.
- B-C: Continued sliding, past the peak displacement position, until the velocity in the contents becomes the same as that in the structure. The total acceleration of the structure at C is greater than μg required to initiate sliding. Because of this, sliding immediately initiates in the opposite direction to that previously.
- After C: Continued sliding, occurs, but it tends to reduce the total sliding displacement between the contents and structure.

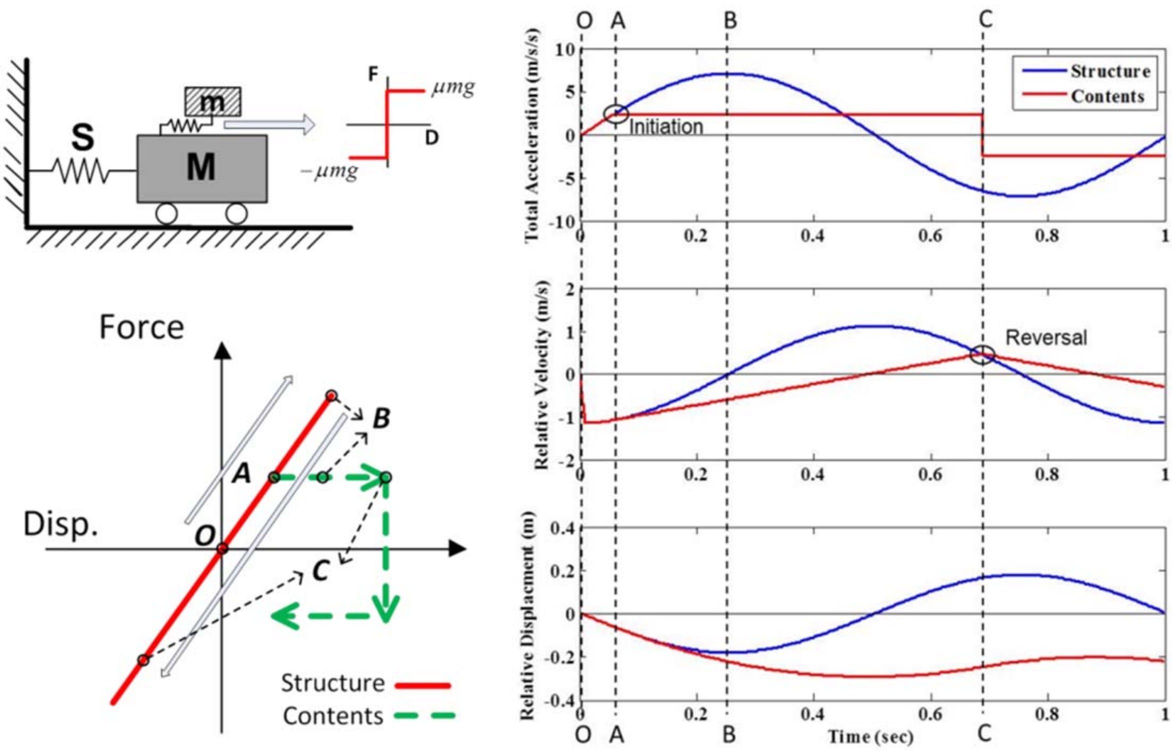
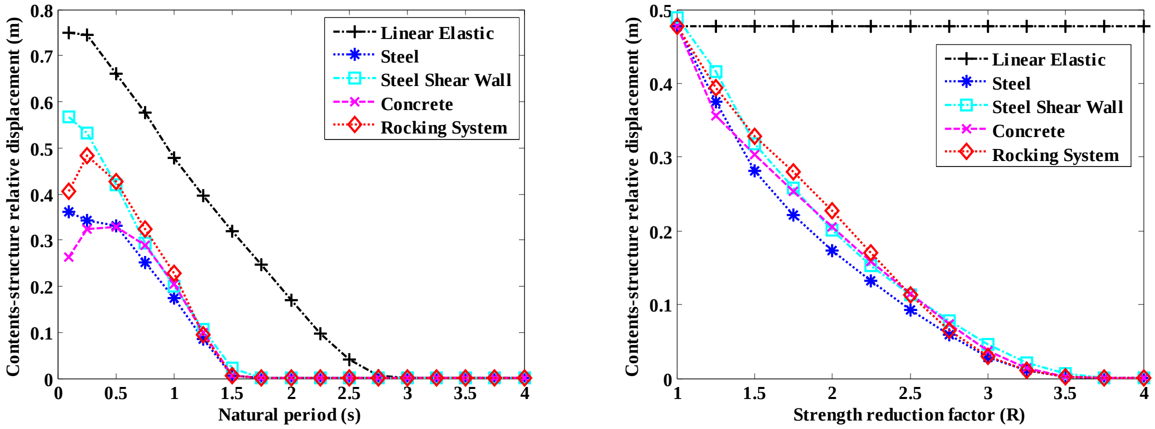


Figure 3.1 Response history of structure-contents sliding system

To understand the influence on contents sliding behaviour due to different structural systems, four typical structural systems were investigated in addition to the linear elastic one. The Takeda, bilinear, bilinear with slackness, and flag-shaped bilinear with the force associated with energy dissipation being 50% of the total force were used to simulate the behaviour of reinforced concrete, steel yielding without buckling, steel shear wall, and rocking systems, respectively. To maintain the comparability among different hysteresis behaviour, the same backbone characteristic is utilized for the non-linear curves.

As the structure's fundamental period increases (regardless of the individual values of its mass and stiffness) for structures with a constant strength reduction factor (R = 2) and a contents friction coefficient of 0.25, the sliding displacements generally decrease as shown in Fig. 3.2a. For long period structures the acceleration demands are insufficient to cause contents sliding. The elastically responding structure has significantly greater sliding displacements than the yielding structures over the entire period range. For yielding structures, sliding is not dependent on the type of the hysteresis loop for structures with periods greater than about 0.7s. Since the demands are similar, it implies that the unloading and reloading portions of the hysteresis curve are relatively unimportant in determining the sliding behavior in these cases. It may be that these long period structures have demands causing sliding only once in the major direction of sliding. For shorter period structures, the rocking systems (non-linear elastic loop) and steel shear wall (slackness loop) have greater sliding displacements than the loops with bilinear (steel) or Takeda (concrete) characteristics.

Figure 3.2b shows the variation in sliding for structures with a period, T, of 1s and different lateral force reduction factor, R, values. As R approaches unity, the response is similar to the elastic response as would be expected. It then drops as R increases and the lateral strength decreases. When R = 3.5, the structural accelerations are no longer sufficient to cause sliding. The variation in sliding response for the different structural types does not seem to vary significantly with different R. Figure 3.2 is similar to that obtained by English (2011). These results are consistent with the idea that the sudden increase in stiffness at high velocity is not the main parameter describing contents movement as expressed by English (2011), but that the velocity at the initiation of sliding is more important.



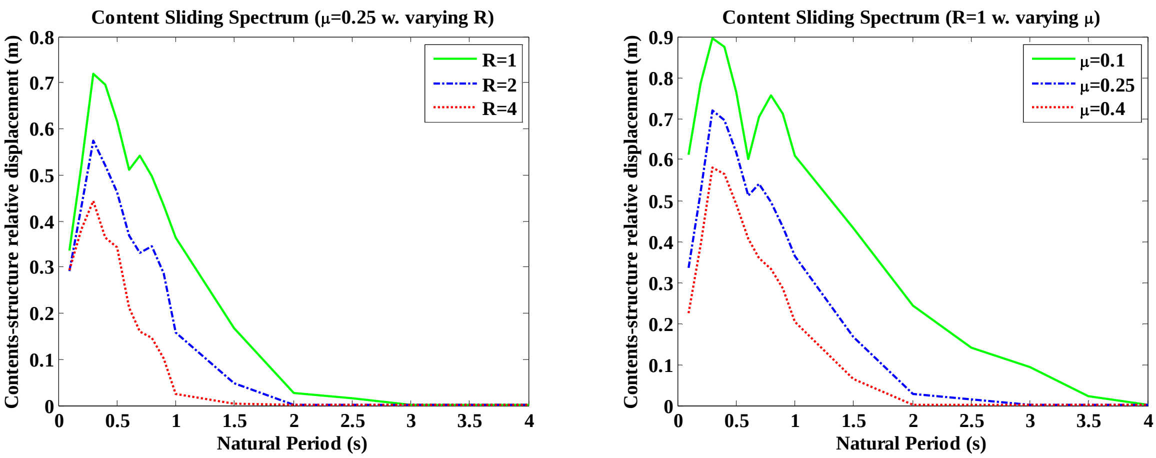
(a) Effect of Structural Period T (R = 2) (b) Effect of strength reduction factor (R) (T = 1s)

Figure 3.2 Contents sliding displacements from impulse loading (μ = 0.25)

4. CONTENTS SLIDING RESPONSE SPECTRA FOR EARTHQUAKE RECORDS

The 20 SAC suite LA 10in50 ground motion records (Somerville et al. 1997) were selected for the non-linear time history analyses. They were scaled to match the elastic design spectrum for Wellington at each period of interest. Since the spectrum considered structures with 17 different periods, this resulted in 340 different records for each hysteresis loop shape. For a structure of a given period, the average contents sliding displacement (i.e. the contents-structure relative displacement) from the 20 records was used to obtain the average sliding. Thus, contents sliding response spectrum curves were obtained for strength reduction factors, R, of 1, 2 and 4, and coefficients of friction, μ, of 0.1, 0.25 and 0.4.

Figure 4.1 presents 2 out of 6 contents sliding response spectra plots generated for a typical steel structure (bilinear hysteresis loop). It can be observed that the peak sliding (contents-structure relative displacement) decreased, especially for buildings with periods between 0.3 and 1.5s. Generally, the higher strength reduction factors showed higher diminution in spectra between different coefficient of friction and vice versa.



(a) Varying R for μ=0.25 (b) Varying μ for R =1.0

Figure 4.1 Contents sliding response spectra under different μ and R values

Since there will be no sliding if the friction strength is more than $\mu ma_{structure}$, it is possible to prevent sliding by simply providing a stiff restraint with this force resistance. Any friction present will also provide additional safety. Another method is to provide a sufficient ledge distance for sliding before falling occurs. This can be achieved by using the generated contents sliding spectra in the following way. For example, an item of contents with a coefficient of friction of 0.25, in a single storey steel building in Wellington with a period of 0.5s which is on stiff soil, which may be significantly stronger than its minimum design strength (due to drift, wind and other limit states controlling the demand) so that the effective $R = 1$, the average amount of sliding expected (required ledge distance) is about 650 mm according to Fig. 4.1. It is noted that the above contents sliding response spectra were based on a SDOF system. Therefore, the variation of demand across the height of multi-storey buildings is not included.

5. CONCLUSIONS

In this paper a simple method was used to develop contents sliding spectra for both impulse and earthquake records. It was shown that:

- The trends in the impulse analysis were the same as that from the earthquake records showing that impulse analysis is a good way to understand the dynamics of structures and their contents.
- The maximum content sliding occurred when the structure was elastic, and the least generally occurred with bilinear hysteresis loops.
- The difference in contents sliding for structures with different hysteresis loops seemed to be best related to the velocity at the time of sliding initiation, rather than the increase in stiffness at high velocity. However, further research is needed and is underway to understand the actual behaviour.
- Procedures for obtaining sliding response spectra of contents were developed. An application of the derived contents sliding response spectra for design was also demonstrated.

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REFERENCES

Carr, A.J. (2008). Ruaumoko Manual. University of Canterbury, Christchurch, New Zealand.

Buchanan, A.H., Bull, D., Dhakal, R.P., MacRae, G.A., Palermo, P. and Pampanin, S. (2011). Base Isolation and Damage-Resistant Technologies for Improved Seismic Performance of Buildings, Report to the Royal Commission for the Canterbury Earthquakes, New Zealand, August 2011. http://canterbury.royalcommission.govt.nz/.

Bradley, B.A., Dhakal, R.P., MacRae, G.A. and Cubrinovski, M. (2010). Prediction of spatially distributed seismic demands in specific structues: Ground motiona nd stuctural response. *Earthquake Engineering and Structural Dynamics* **39**:501-520.

Bull, D. (2011). Personal communication.

Clifton, G.C. (2011). Personal communication.

Deierlein, G.G., Ma, X., Hajjar, J.F., Eatherton, M., Krawinkler, H., Takeuchi, T., Midorikawa, M., Hikino, T. and Kasai, K. (2010). Seismic Resilience of Self-Centering Steel Braced Frames with Replaceable Energy-Dissipating Fuses ; Part 2: E-Defense Shake Table Test. *7th International Conference on Urban Earthquake Engineering & 5th International Conference on Earthquake Engineering Joint Conference.* March 3-5, 2010,Tokyo, April 2010.

English, R., MacRae, G.A., Dhakal, R. and Yeow, T. (2012). Content Sliding Mechanics in Earthquakes. *Proceedings of 2012 NZSEE Conference*. 13-15 April. Christchurch, New Zealand.

English, R. (2011). Hysteretic Influence on Earthquake Induced Sliding Damage of Contents, Final Year Undergraduate Project, Dept. of Civil and Natural Resources Engineering, University of Canterbury.

Gazetas, G., Garini, E., Berrill, J.B. and Apostolou, M. (2012). Sliding and overturning potential of Chirstchurch 2011 earthquake records, *Earthquake Engineering and Structural Dynamic*, DOI: 10.1002/eqe.2165.

Gazetas, G, Garini, E, Anastasopoulos, I. and Georgarakos, T. (2009). Effects of near-fault ground shaking on sliding systems. *Journal of Geotechnical and Geoenvironmental Engineering* **135(12):**1906–1921.

Gledhill, S.M., Sidwell, G.K., Bell, D.K. (2008). The Damage Avoidance Design of Tall Steel Frame Buildings - Fairlie Terrace Student Accommodation Project, Victoria University of Wellington, *New Zealand Society of Earthquake Engineering Annual Conference*, Wairakei.

Hutchinson, T.C. and Chaudhuri, S.R. (2006). Simplified Expression for Seismic Fragility Estimation of Sliding-Dominated Equipment and Contents. *Earthquake Spectra* **22(3)**:709–732.

Lin, S.L., MacRae, G.A., Dhakal, R., Yeow, T. and English, R. (2012). Content Sliding Mechanics in Earthquakes. *Proceedings of 2012 NZSEE Conference*. 13-15 April. Christchurch, New Zealand.

MacRae, G.A. (2010). Some Steel Seismic Research Issues. *Proceedings of the Steel Structures Workshop 2010*, Research Directions for Steel Structures, compiled by MacRae G.A. and Clifton G.C., University of Canterbury, 13-14 April.

MacRae, G. A., Au, E., Pettinga, D., Deam, B. and Sadashiva, V. (2008). A Simple Design Method for Yielding Structures Subject to Torsion. *14th World Conference on Earthquake Engineering*, Beijing, China, 12-17 October, Paper ID: 05-01-0514.

MacRae, G.A., Hair, J. and Dhakal, R. (2011). Ceiling Damage in the 2010 Canterbury Earthquake. *Joint 8th Center for Urban Earthquake Engineering (CUEE) and 8th International Conference on Earthquake Engineering (ICEE) Conference Proceedings.* March 7-8. Tokyo Institute of Technology, Tokyo, Japan.

MacRae, G.A., Priestley, M.J.N. and Tao, J. (1993). P-Δ in Seismic Regions. Structural Systems Research Project, Report No. SSRP 93/05, Department of Applied Mechanics and Engineering Sciences, University of California, San Diego, May 1993, 115pp.

NZSEE. (2010). Special Issue: Preliminary observations of the 2010 Darfield (Canterbury) Earthquakes. *Bulletin of the New Zealand Society for Earthquake Engineering*; **43(4)**: 215-439.

Priestley, M.J.N. and MacRae, G.A. (1996). Seismic Tests of Precast Beam-to-Column Joint Subassemblages with Unbonded Tendons. *PCI Journal*. January-February; 64-80.

Priestley, M.J.N. (1996). The PRESSS program - current status and proposed plans for Phase III. *PCI Journal*. Mar.-Apr, **41(2)**:22-40.

Roke, D., Sause R., Ricles J.M. and Gonner, N. (2009). Damage-free seismic-resistant selfcentering steel concentrically-braced frames. *STESSA*, Philadelphia, August 2009.

Shenton, H.W., III. and Jones, N.P. (1991). Base excitation of rigid bodies. I: Formulation, *Journal of Engineering Mechanics*. **117**, 2286–2306.

Somerville, P., Smith, N., Punyamurthula, S. and Sun, J. (1997). Development of ground motion time histories for phase 2 of the FEMA/SAC steel project. SAC/BD-97/04, SAC Joint Venture. Sacramento, CA.

Tilby, C. (1981). South Rangitikei Railway Bridge Construction. Transactions of the Institution of Professional Engineers New Zealand, Civil Engineering Section, **8(2)**, 16.

Whittaker, A.S. and Soong, T.T. (2003). A Overview of Non-structural Component Research at Three U.S. Earthquake Engineering Research Centers. *Proceedings of Seminar on Seismic Design, Performance, and Retrofit of Nonstructural Components in Critical Facilities (ATC-29-2)*. pp. 271–280. Applied Technology Council, Redwood City, California.

WWW. (2010). http://upload.wikimedia.org/wikipedia/commons/7/79/Motorbike_rider_mono.jp and http://neilhellmanlibrary.files.wordpress.com/2007/07/wall.jpg.

Yu, P., Gutschmidt, S. and MacRae, G.A. (2011). Modelling of Rigid Rocking of Structures during Earthquake Using Linear Functions. *Proceedings of the Ninth Pacific Conference on Earthquake Engineering, Building an Earthquake-Resilient Society*, 14-16 April, 2011, Auckland, New Zealand. Paper 076.