SUMMARY:
Past earthquakes have shown the susceptibility of skewed bridges to exhibit more significant damage in comparison to straight bridges. In particular, widespread damage of skewed bridges was observed in Chile during the 2010 Maule earthquake. The damage observed and analytical studies indicate an influence of the skew angle in the displacement response of the piers. This paper proposes a simplified 3-DOF nonlinear model to study the displacement demands of columns of skewed bridges with seat type abutments. The uncoupled simplified 3-DOF nonlinear model captures the peak demands predicted by detailed three-dimensional nonlinear finite element models. The 3-DOF simplified model is appealing when conducting parametric studies of multi-span skewed bridges and contributes to the development of displacement-based methods for these structures.

Keywords: Skewed Bridges Displacement Demands, Simplified Model, Seismic Damage to Skewed Bridges.

1. INTRODUCTION
Displacement based design and retrofit of bridges requires a clear understanding of the seismic demands undergone by these structures. Given their deck geometry, skewed bridges are classified as irregular structures and the evaluation of their displacement demands is challenging as it requires a combination of several modes of vibration. In particular skewed bridges tend to rotate due to the pounding between the deck and its abutments or adjacent frames, leading to different levels of seismic damage.

A comprehensive survey of the damage of skewed bridges during different earthquakes is presented in the following paragraphs.

1.1 Seismic Damage of Skewed Bridges
The susceptibility of skewed bridges to exhibit more significant seismic damage than straight bridges was identified first during the 1971 San Fernando earthquake (Jennings et al, 1971) and has been clearly observed in each major earthquake since then. The performance of 21 bridges that were damaged during nine major earthquakes since the 1971 San Fernando earthquake was investigated as part of this study. In general terms, damages have occurred to short and medium multi-span skewed bridges with: more than two spans, equal and unequal skew angles greater than 30°, seat type abutments, concrete piles, poor transverse restraint, and rocker or elastomeric bearings. The primary cause of collapse was failure of columns and the second cause was unseating of superstructure. Some of these bridges had been retrofitted with longitudinal restrainers.
The damages observed include unseating of the superstructure, failure of bearings, breakdown of transversal and longitudinal restrainers, cracking of girders, shear failure of piers, displacement of abutments, slumping of the backfill, cracking of embankments and failure of piles (Figure 1). The consequences of failure of skewed bridges vary from disruption of bridge serviceability due to large permanent displacements at expansion joints, to bridge closure due to collapse of superstructure or loss of gravity load capacity in columns. A summary of some relevant failures is provided in the following paragraphs.

In bridges designed prior to the dissemination of ATC-6-2 (1983), which contains comprehensive seismic provisions, the considerable deck rotations led to permanent deck offset or unseating of the superstructure. For instance, although retrofitted with longitudinal restrainer cables to supplement its short support length (200 mm), the Gavin Canyon Undercrossing fell down during the 1994 Northridge earthquake (Moehle et al 1995, Klosek et al 1995), which might indicate the importance of the transverse displacement demand in the seismic assessment of skewed bridges.

Brittle failures have occurred in rocker and roller bearings on skewed bridges; the I-5/I-605 overpasses during the 1987 Winter-Narrows earthquake (Priestley, 1988) and the Mukogawa bridge during the 1995 Hyogo Ken Nambu earthquake are good illustrations (NIST, 1996). During the 2010 Maule earthquake permanent offset and unseating of the superstructure occurred in short span skewed bridges with laminated elastomeric bearings and poor transverse restraint (MAE, 2010). Las Mercedes bridge and Route 5 overpass are examples of this failure.

Damage in the substructure of skewed bridges has been clearly evidenced in past earthquakes. Columns of the Foothill Boulevard and the Northbound Truck Route Undercrossings suffered extensive shear damage during the 1971 San Fernando earthquake (Jennings et al, 1971). Although columns at that time had inadequate confinement and insufficient transverse reinforcement, the damage was aggravated by the increment in displacement demand due to skewness. Similarly, the increasing demand at the base of the architectural flares in the columns of the Mission Gothic Undercrossing exacerbated the shear damage during the 1994 Northridge earthquake (Moehle et al, 1995). Shear failure caused by torsion due to deck skewness was also observed in the wall piers of the Kawaraginishi Bridge during the 1995 Hyogo Ken Nambu earthquake (NIST, 1996).
During the 1989 Loma Prieta earthquake, the Struve Slough Bridge, a skewed bridge supported on extended pile shafts, collapsed mainly due to large displacement demand at the top of the columns that was attributed to foundation flexibility. The skewness of the bridge most likely worsened the foundation flexibility effects. For example, the additional displacement demand induced by skewed decks could have further increased the P-Delta effect and further reduced the column capacity.

1.2 Displacement Demand of Skewed Bridge Piers

Seismic damage due to past earthquakes illustrates that skewed bridges tend to rotate during earthquakes. The rotations increase the probability of transverse unseating at joints. This problem is particularly relevant when considering existing skewed bridges, in which, given their support details, rotation of the superstructure due to pounding between the deck and its abutment is more likely to happen. The understanding of this Embankment-Abutment-Structure-Interaction (EASI) was progressed by research conducted by Shamsabadi and Kapuskar (2010). Furthermore, a minimum required seat length, which is estimated by an empirical equation that accounts for the skew angle of the bridge, is explicitly defined in most seismic design specifications.

The rotation of the deck increases the displacement demands of skewed bridge piers (Kavianijopari, 2011, Tirasti and Kawashima, 2008). This displacement demand seems to be particularly important in the case of skewed bridges with seat type abutments, as illustrated in the damage observed during past earthquakes. However, the parameters which drive the displacement demands of the piers are not fully defined or understood.

In addition, a good understanding of the displacement demands of the piers is needed in the current displacement-based design procedure for new bridges (AASHTO, 2009), in which the displacement demand is directly compared with the provided displacement capacity to ensure the desirable seismic performance. This paper presents a simplified nonlinear 3-DOF model to calculate the seismic demands of skewed bridge piers.

2. SKEWED BRIDGE MODELS

The bridge considered in the analysis is a continuous three-span structure with a skew angle of 45 degrees (Figure 2a). The bridge is 120 m long, 12 m wide, and is supported at the two ends on seat type abutments with one-inch expansion gaps (Figure 2b). The superstructure consists of a concrete deck slab supported on six 1.7 m deep, longitudinal reinforced concrete I-girders. The substructure consists of two bents supported by two piers per bent (D = 1.2 m) rigidly connected to the base. The deck is rigidly connected to the superstructure by the cap-beams.

Figure 2. General sketch of the bridge model
2.1 Proposed 3-DOF simplified model

The proposed 3-DOF simplified nonlinear model aims to estimate the seismic demand of the piers of skewed bridges with seat type abutments, and is mainly intended to capture the peak demand responses. The model consists of a rigid bar that represents the bridge deck with three DOFs (transverse translation, longitudinal translation, and in-plane rotation). The transverse direction is assumed to be in the direction of the skew and the longitudinal direction is normal to this (Figure 3). This coordinate system is adopted here as it is convenient when developing the uncoupled equations of motion for the system. Also, experimental evidence indicates that the predominant direction of the first transverse mode of vibration of skewed bridges seems to be in the azimuth of the skew bents (Catacoli et al, 2012).

The model considers multi-support excitations that represent the primary seismic load paths of the bridge (Figure 2a). Orthogonal ground motions in the directions of the selected coordinate system are applied at ground level ($\ddot{E}_{1T}$ and $\ddot{E}_{1L}$) and at abutment level ($\ddot{E}_{2T}$ and $\ddot{E}_{2L}$). The transverse far field ground motion at the abutments ($\ddot{E}_{2T}$) is transferred to the bridge by the shear keys, and hence after failure of the shear keys this component is not longer taken into consideration. The longitudinal far field ground motion at abutments ($\ddot{E}_{2L}$) is transferred to the bridge after the closure of the expansion gap.

The equation of motion in the transverse direction is given in Eqn. 2.1. Static condensation is used to take into consideration the contribution of the columns and the shear keys. The utilized formulation considers the reduction in the stiffness of the system and the change of the input ground motion after the failure of the shear keys. The failure of the shear keys is assumed to be brittle, which is illustrated by the backbone shown in Figure 4. The columns modeled use sectional crack properties.

Eqn. 2.2 represents the equation of motion in the longitudinal direction. The kinematic interaction at the near field embankment is included by springs at abutment level. The behavior of the springs is assumed to follow a bilinear elastic backbone curve (Figure 5). The formulation considers the increment of stiffness and seismic load in the system after gap closure. The increment in the system damping provided by the energy dissipated by the abutment backfill is incorporated using equivalent viscous damping.

The equation of motion for the in-plane rotation of the deck is given in Eqn. 2.3. The contribution of the columns, shear keys and abutment passive pressure to the rotational stiffness is accounted for by static condensation (Figure 6). In order to reduce the nonlinearity in the equation for this degree of freedom, the gap is assumed to be closed for the entirety of the shaking. The developed equations of motion are solved using the Newmark method.

\begin{align}
\begin{align*}
m\ddot{U}_T + c\dot{U}_T + \left\{ K_{\text{columns}} - T + 2 * K_{\text{shk}} (\ddot{r}_U) \right\} U_T &= -m * \ddot{E}_{1T} - 2 * m(\ddot{r}_U) * \ddot{E}_{2T} \tag{2.1} \\
m\ddot{U}_L + c_{(\text{gap})}\dot{U}_L + \left\{ K_{\text{columns}} - L + K_{\text{abt}} (\ddot{E}_g) \right\} U_L &= -m * \ddot{E}_{1L} - m(\ddot{E}_g) * \ddot{E}_{2L} \tag{2.2} \\
I_{\theta} \ddot{\theta} + c\dot{\theta} + K_{\text{rot}} \dot{\theta} &= -m_a * \ddot{E}_{2L} * \sin(\phi) * \left( \frac{L}{2} \right) \tag{2.3}
\end{align*}
\end{align}

where: 
\begin{align*}
K_{\text{rot}} &= K_{\text{columns}} - \text{Rot} + K_{\text{shk}} (r_0) * \frac{L^2}{2} + K_{\text{abut}} * \frac{L^2}{4} * \sin(\phi)
\end{align*}
**Figure 3.** Proposed 3-DOF simplified model for the seismic response of skewed bridges

**Figure 4.** Load path in the transverse direction

**Figure 5.** Load path in the longitudinal direction
2.2 Detailed Elastic and Inelastic Three Dimensional Finite Element Models

Three dimensional Finite Element (FE) models similar to those proposed by Shamsabadi and Kapuskar (2010) have been developed in order to compare the accuracy of the proposed 3-DOF simplified model with respect to state of the practice: elastic and inelastic models used in time history analysis. The models are developed using the computer program SAP 2000.

In the elastic FE model, the shear keys are represented by linear springs in the direction of the skew and the abutment backfill is represented by a set of linear springs perpendicular to the face of the skewed abutment. Shell elements are used to represent the deck slab and the concrete I-girders, cap beams and columns are modeled using beam elements. The columns are modeled as being rigidly connected (fixed) to the deck and to the foundation and use cracked section properties. An elastic FE model with the deck supported on ideal rollers at both ends is also explored. The modal use in the elastic FE models was 5% of the critical damping.

In the inelastic FE model, the backfill abutment and the one-inch expansion joint are represented by a set of nonlinear springs perpendicular to the skew angle. The nonlinear spring uses a multi-linear plasticity model capable of including in a single element, the gap and the backbone curve of the abutment with the tension side of the curve set to zero. The shear keys are modeled using multi-linear elastic elements and the effective stiffness was used to account for the gap in the transverse direction. The recommendations of Shamsabadi and Kapuskar (2010) were used to define the abutments and the shear keys stiffnesses.

3. RESULTS

Table 3.1 compares the period of vibration in the transverse, longitudinal and rotational directions of the elastic FE model with two different boundary conditions, one assuming ideal rollers at both ends and the other assuming shear keys at the abutments, to those obtained for the 3-DOF simplified model under similar conditions. For the three degrees of freedom assessed (longitudinal translation, transverse translation and in-plane rotation), the results show good agreement for the periods of vibration of both models.
Table 3.1. Comparison of periods of vibration in the transverse, longitudinal and rotational directions.

<table>
<thead>
<tr>
<th>Boundary Condition</th>
<th>$T_{\text{transverse (s)}}$</th>
<th>$T_{\text{longitudinal (s)}}$</th>
<th>$T_{\text{rotational (s)}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Rollers</td>
<td>1.20</td>
<td>1.30</td>
<td>2.05</td>
</tr>
<tr>
<td>Shear Keys</td>
<td>0.91</td>
<td>0.9</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The ground acceleration time history recorded during the 1989 Loma Prieta earthquake at Lower Crystal Spring station was selected as input ground motion. The record was scaled to match the Uniform Hazard Spectrum (UHS) for a Vancouver site class C using a probability of exceedance of 2% in 50 years. The record was applied in the longitudinal and transverse directions of the bridge.

Figure 7 shows the displacement at the deck center of mass in the transverse direction. The displacement response predicted by the 3-DOF simplified model is in good agreement with the response predicted by the nonlinear FE model. The 3-DOF simplified model captures both, the peak displacement demands and the transverse demands after the failure of the shear keys. The elastic FE model, which is a model commonly used in practice, underestimates the column demands as it cannot capture the changes in the transverse demand after the failure of the shear keys that occurs after 7.5 seconds of shaking.

Figure 9 shows the in-plane rotations of the deck center of mass. To some extent, the 3-DOF simplified model captures the time history trend and the resultant residual rotation predicted by the nonlinear FE model. This residual rotation in skewed bridges has been also reported by Tirasti and Kawashima (2008), and Shamsabadi and Kapuskar (2010). The physical model assumed to describe the rotation mechanism in the case of the 3-DOF simplified model requires further improvement in
order to obtain a better prediction of the peak rotational response. It is also noted that the elastic FE model is unable to capture the shifting and the in-plane rotation of the system.

4. Discussion

Bridge piers are usually designed for peak displacement values. The proposed 3-DOF simplified model captures the peak transverse and longitudinal demands of the deck accurately. The trend and shifting of the in-plane rotation expected for skewed bridges with seat type abutments is also captured to some extent. These demands are underestimated or unpredicted by elastic FE models.

Maragakis and Jennings (1987) developed a coupled set of the equations of motion and an analytical model for a three span skewed bridge with seat type abutments. In contrast, this paper develops a set of uncoupled equations of motion. In addition, the proposed 3-DOF simplified model is applicable to short and medium multi-span skewed bridges with seat type abutments and continuous decks. As the
model is mainly intended to capture the peak demand responses, it uses simplified approaches to account for nonlinear effects due to shear key failure, gap opening and closure, near field abutment interaction and multi-support excitations. The proposed 3-DOF simplified model is appealing for parametric studies of skewed bridges and after further validation will be used to study the effects of Soil-Foundation-Structure Interaction (SFSI).

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