Two Degrees of Freedom PID Control for Active Vibration Control of Structures

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SUMMARY:
Two degrees of freedom PID control algorithm consists of two PID systems that have different signal errors. The input signal error can be the output response, expected response or difference between them. In this article, single degree of freedom PID control and two degrees of freedom PID control are used as control algorithms. An active tuned mass damper is used for the reduction of vibration of a single degree of freedom system subjected to base excitation. Simulations are performed in MATLAB environment and the results for these two methods obtained and compared to each other. The results reveal the advantages of two degrees of freedom PID algorithm to the classical PID control.

Keywords: Two degrees of freedom PID control, Active Tuned Mass Damper.

1. INTRODUCTION

The proportional-integral-derivative (PID) controller is the most common industrial control algorithm (Åström and Hägglund, 1995). Modified forms of PID control have been proposed to overcome the limitations of PID controllers. Two degrees of freedom (2DOF) PID control algorithm is a control method which, in addition to the omission of an abrupt change in the control force, eliminates the steady state response owing to the input slope and acceleration. Furthermore, employing this control method, the response of the system to unit step disturbance has such a low amplitude that approaches to zero (Ogata, 1997).

Tuned mass damper (TMD) is one of the most simple and practical vibration reduction devices. The modern concept of TMDs for structural applications can trace its roots in dynamic vibration absorber invented by Frahm (1909). The objective of incorporating a TMD into a structure is to reduce energy dissipation demand on the primary structural members under the action of external forces. This reduction is accomplished by transferring some of the structural vibrational energy to the TMD and dissipating the energy at the damper of the TMD (Soong and Dargush, 1997). The detailed theory and working principles of undamped and damped TMD to control the displacement of an undamped single degree of freedom system subjected to a harmonic force have been described by Den Hartog (1956). Active tuned mass dampers (ATMD) were introduced by inclusion of an active control mechanism to the TMD to improve its effectiveness (Chang and Soong, 1980).

In this paper, the effectiveness of classical PID control and 2DOF PID for active vibration control of a structure is investigated. An ATMD is used as the active control device and is fitted to the roof of the structure. The results for uncontrolled structure and structure fitted with passive and active TMD are obtained and comparisons are made in terms of defined evaluation criteria. The paper ends with a discussion.

2. EQUATIONS OF MOTION

2.1. Equations of motion of uncontrolled structure

The equation of motion for an n DOFs structure subjected to base excitation can be written as

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=-\mathbf{M}\mathbf{r}\ddot{x}_g(t) \tag{2.1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively; $\mathbf{X}(t)$ is an n-dimentional displacement vector; $\ddot{x}_g(t)$ is the ground acceleration; $\mathbf{r}$ is an n-dimentional unit vector and the dot indicates a derivative with respect to time.

From specified damping ratios ξ_n for l modes $(n=1,2,\ldots,l)$, the damping matrix, $\mathbf{C}$, is expressed as (Chopra, 2007)

$$\mathbf{C}=\mathbf{M}\left(\sum_{n=1}^{l}\frac{2\xi_n\omega_n}{M_n}\phi_n\phi_n^T\right)\mathbf{M} \tag{2.2}$$

where ω_n is the n th mode natural frequency; ϕ_n is the n th mode shape and $M_n=\phi_n^T\mathbf{M}\phi_n$.

2.2. Equations of motion of structure fitted with ATMD

It is assumed that an ATMD whose mass, damping and stiffness are denoted by m_T, c_T and k_T, respectively, is connected to the i th DOF of the structure. The structure with the attached ATMD can be treated as an $n+1$ DOF structure and its equations of motion can be written as

$$\bar{\mathbf{M}}\ddot{\bar{\mathbf{X}}}(t)+\bar{\mathbf{C}}\dot{\bar{\mathbf{X}}}(t)+\bar{\mathbf{K}}\bar{\mathbf{X}}(t)=-\bar{\mathbf{M}}\bar{\mathbf{r}}\ddot{x}_g(t)+\mathbf{p}u(t) \tag{2.3}$$

where $\mathbf{p}$ is an $n+1$- dimentional allocation vector of the control force whose i th component is -1 and $n+1$th component is 1 and the other components are 0; $u(t)$ is the control force; x_T is the relative displacement of the ATMD with respect to the i th DOF of the structure; $\bar{\mathbf{X}}(t)=\{\mathbf{X}(t),x_T(t)\}^T$ is an $n+1$-dimentional displacement vector; $\bar{r}$ is an $n+1$-dimentional vector whose $(n+1)$ th component (ATMD location) is 0 and the other components are 1 and $\bar{\mathbf{M}}$, $\bar{\mathbf{C}}$ and $\bar{\mathbf{K}}$ are $(n+1)\times(n+1)$ mass, damping and stiffness matrices of the structure fitted with the TMD, given, respectively, by:

$$\bar{\mathbf{M}}=\left[\begin{array}{ccccc|c} & & & & & 0\\ & & & & & \vdots\\ & & \mathbf{M}_{n\times n} & & & 0\\ & & & & & \vdots\\ & & & & & 0\\ \hline 0 & \cdots & m_T & \cdots & 0 & m_T\end{array}\right]\begin{array}{c}1\\ \vdots\\ i\\ \vdots\\ n\\ n+1\end{array}$$
$$\begin{array}{cccccc}1 & \cdots & i & \cdots & n & n+1\end{array} \tag{2.4}$$

$$\bar{\mathbf{C}} = \begin{bmatrix} & & & & & 0 \\ & & & & & \vdots \\ & & \mathbf{C}_{n\times n} & & & -c_T \\ & & & & & \vdots \\ & & & & & 0 \\ 0 & \cdots & 0 & \cdots & 0 & c_T \end{bmatrix} \begin{matrix} 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{matrix} \tag{2.5}$$
$$\begin{matrix} 1 & \cdots & i & \cdots & n & n+1 \end{matrix}$$

$$\bar{\mathbf{K}} = \begin{bmatrix} & & & & & 0 \\ & & & & & \vdots \\ & & \mathbf{K}_{n\times n} & & & -k_T \\ & & & & & \vdots \\ & & & & & 0 \\ 0 & \cdots & 0 & \cdots & 0 & k_T \end{bmatrix} \begin{matrix} 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{matrix} \tag{2.6}$$
$$\begin{matrix} 1 & \cdots & i & \cdots & n & n+1 \end{matrix}$$

Letting $\mathbf{f}(t) = -\bar{\mathbf{M}}\bar{\mathbf{r}}\ddot{x}_g(t) + \mathbf{p}u(t)$, Eqn. 2.3 can be expressed in state-space as (Åström and Hägglund, 1995, Soong, 1990)

$$\dot{\bar{\mathbf{Z}}} = \mathbf{A}\bar{\mathbf{Z}} + \mathbf{B}\mathbf{f} \tag{2.7}$$

where

$$\bar{\mathbf{Z}} = \begin{Bmatrix} \bar{\mathbf{X}} \\ \dot{\bar{\mathbf{X}}} \end{Bmatrix}; \quad \mathbf{A} = \begin{bmatrix} \mathbf{0}_{(n+1)\times(n+1)} & \mathbf{I}_{(n+1)\times(n+1)} \\ -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} \end{bmatrix}_{2(n+1)\times 2(n+1)}; \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{(n+1)\times(n+1)} \\ \bar{\mathbf{M}}^{-1} \end{bmatrix}_{2(n+1)\times(n+1)} \tag{2.8}$$

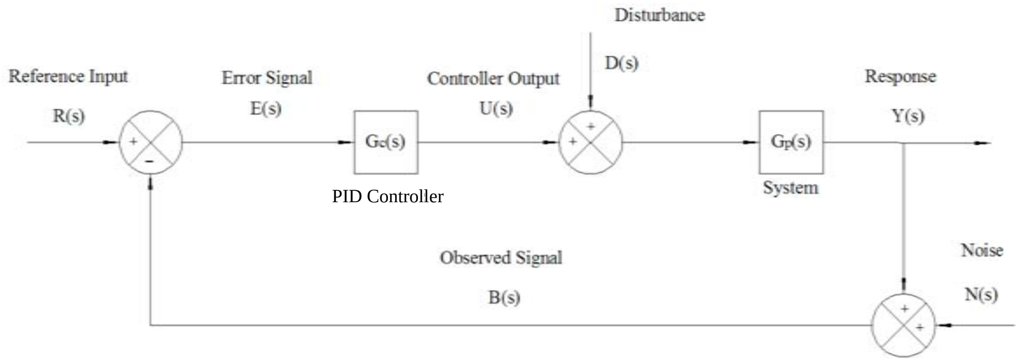
3. ACTIVE CONTROL STRATEGY

3.1. Classical PID control system

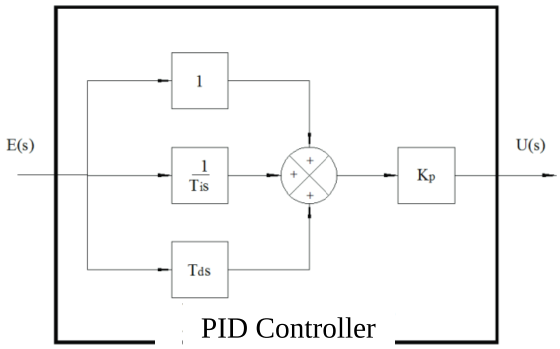
PID control is the must common industrial control algorithm. The PID algorithm has the following form (Åström and Hägglund, 1995)

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt + K_d \frac{de(t)}{dt} \tag{3.1}$$

where $u(t)$ is the controller input; $e(t)$ is the control error which is the difference between desired response and actual response and K_p, K_i and K_d are the PID control parameters. The control variable is thus a sum of three parts (Figure 3.1).



(a)



(b)

Figure 3.1. Block diagram of (a) Controlled structure, (b) PID control (Ogata, 1997).

1. P-term which is proportional to the error.
2. I-term which is proportional to the integral of the error.
3. D-term which is proportional to the derivative of the error.

Correlations between the three parameters may not be exactly accurate, because K_p, K_i and K_d are dependent on each other. In fact, changing one of these variables can change the effect of the other two (Åström and Hägglund, 1995).

3.2. 2DOF PID control system

A general form of 2DOF PID control system is demonstrated in Figure 3.2. The system has the reference input $R(s)$, disturbance input $D(s)$ and noise input $N(s)$. $G_{c_1}(s)$ and $G_{c_2}(s)$ are the transfer functions of the PID controllers and $G_p(s)$ is the transfer function of the system.

For this system, three closed-loop transfer functions, i.e., G_{yr}, G_{yd} and G_{yn}, may be derived (Ogata, 1997).

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c_1} G_p}{1 + \left(G_{c_1} + G_{c_2}\right) G_p} \tag{3.2}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1+\left(G_{c_1}+G_{c_2}\right)G_p} \tag{3.3}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{\left(G_{c_1}+G_{c_2}\right)G_p}{1+\left(G_{c_1}+G_{c_2}\right)G_p} \tag{3.4}$$

Hence:

$$G_{yr} = G_{c_1}G_{yd} \tag{3.5}$$

$$G_{yn} = \frac{G_{yd}-G_p}{G_p} \tag{3.6}$$

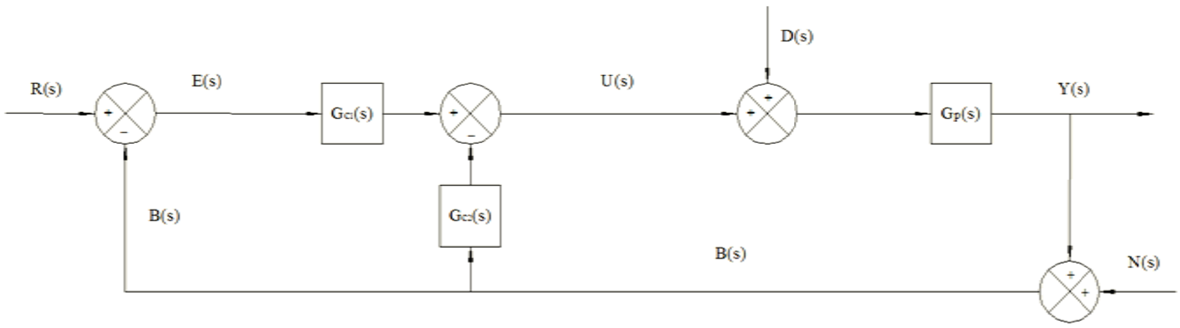


Figure 3.2. Control strategy with 2DOF PID (Ogata, 1997).

In this case, if G_{yd} is given, then G_{yn} is fixed, but G_{yr} is not fixed, because G_{c_1} is independent of G_{yd}. Thus, two closed-loop transfer functions among three closed-loop transfer function G_{yr}, G_{yd} and G_{yn} are independent. Hence, this system is a 2DOF control system.

3.3. Control design

Because often the parameters of PID controllers will be set in place, different tuning rules has been proposed in the literatures for this purpose. Automatic tuning methods have been developed as well.

In this paper, to adjust the PID parameters, genetic algorithm constrained optimization method is employed. Accordingly, the system response to unit step excitation must satisfy some strict conditions. Herein, the rise time of less than 0.05 sec, overshoot less than 10%, and the settling time of less than 1 sec is selected.

4. EXAMPLE

A 3-story frame with the following mass, damping and stiffness matrices is assumed:

$$\mathbf{M}=\begin{bmatrix}98.3 & 0 & 0\\ 0 & 98.3 & 0\\ 0 & 0 & 98.3\end{bmatrix}\mathrm{kg};\quad \mathbf{C}=\begin{bmatrix}175 & -50 & 0\\ -50 & 100 & -50\\ 0 & -50 & 50\end{bmatrix}\frac{\mathrm{N.s}}{\mathrm{m}}$$
$$\mathbf{K}=10^5\begin{bmatrix}12.0 & -6.84 & 0\\ -6.84 & 13.7 & -6.84\\ 0 & -6.84 & 6.84\end{bmatrix}\frac{\mathrm{N}}{\mathrm{m}} \tag{3.7}$$

This system is a simple model of an experimental three-story frame in Structural Dynamics and Control/Earthquake Engineering Laboratory (SDC/EEL) in the University of Notre Dame (Shakib, 2010).

In this paper, four ground motion records (two near field and two far field) are selected: El Centro 1940, Hachinohe 1968, Northridge 1994 and Kobe 1995.

5. CONTROL EFFECTIVENESS

To evaluate the control system performance the following eight evaluation criteria are considered (Table 5.1). In naming these criteria, $|\square|$ represents the absolute value and $\|\square\|$ denotes the RMS response.

Table 5.1. Evaluation criteria

Index	*Description*
$J_1=\dfrac{\max_t \lvert x_i^c(t)\rvert}{\max_t \lvert x_i^u(t)\rvert}$	Normalized peak floor displacement
$J_2=\dfrac{\lVert x_i^c(t)\rVert}{\lVert x_i^u(t)\rVert}$	Normalized RMS floor displacement
$J_3=\dfrac{\max_t \lvert \ddot{x}_i^c(t)\rvert}{\max_t \lvert \ddot{x}_i^u(t)\rvert}$	Normalized peak floor acceleration
$J_4=\dfrac{\lVert \ddot{x}_i^c(t)\rVert}{\lVert \ddot{x}_i^u(t)\rVert}$	Normalized RMS floor acceleration
$J_5=\dfrac{\max_t \lvert F^c(t)\rvert}{W_{tot}}$	Normalized peak control force ($W_{tot}=2892\ \mathrm{N}$)
$J_6=\dfrac{\max_t \lvert F^c(t)\rvert}{\max_t \lvert F(t)\rvert}$	Normalized peak control force
$J_7=\dfrac{\lVert F^c(t)\rVert}{\lVert F(t)\rVert}$	Normalized RMS control force
$J_8=\dfrac{\max_t \lvert d_i^c(t)\rvert}{\max_t \lvert d_i^u(t)\rvert}$	Normalized peak floor drift

In Table 5.1, $x_i^c(t)$ and $\ddot{x}_i^c(t)$ are time histories of the displacement and acceleration of the i th story ($i = 1,2,3$) of the controlled system, respectively; $x_i^u(t)$ and $\ddot{x}_i^u(t)$ are time histories of the displacement and acceleration of the i th story ($i = 1,2,3$) of the uncontrolled system, respectively; $F^c(t)$ is the time history of control force; W_{tot} is the total weight of the structure; $F(t)$ is the excitation force and $d_i^c(t)$ and $d_i^u(t)$ are time histories of the drift of the i th story ($i = 1,2,3$) of the controlled and uncontrolled system, respectively.

5.1. Simulation results

The system's responses are obtained through numerical integration of the equation of motion in three cases: system fitted with passive TMD, system fitted with ATMD and classical PID control strategy is used, system fitted with ATMD and 2DOF PID control strategy is used.

It has been assumed that the mass ratio of TMD is 1% and the controller operates with a time delay of 0.02 sec. To obtain the frequency and damping ratios of ATMD, instead of using the empirical formulas available to the literature (Ioi and Ikeda, 1978) a sensitivity analysis is performed using white noise excitation.

For example, the displacement and acceleration response for the 3rd story for both uncontrolled and controlled structure with 2DOF PID control strategy subjected to Kobe earthquake are shown in Figures 5.1 and 5.2. Table 5.2 shows the values of evaluation criteria for different control strategies. The results show that ATMD is more effective than the passive TMD in reducing the system's responses. Moreover, 2DOF PID control shows better performance compared to classical PID control.

Table 5.2. Evaluation criteria

Excitation	**Control strategy**	**Evaluation criteria (percent)**							
		J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
El Centro	Passive	78.0	51.4	81.5	47.8	0.0	0.0	0.0	77.4
	Active PID control	6.0	3.4	9.6	2.0	28.5	9.5	73.8	21.0
	Active 2DOF PID control	2.0	1.1	6.1	2.6	31.6	11.0	85.9	22.4
Kobe	Passive	60.4	50.0	43.2	34.0	0.0	0.0	0.0	63.3
	Active PID control	7.9	6.3	6.5	1.7	51.3	8.1	65.0	20.7
	Active 2DOF PID control	2.0	1.6	2.7	1.8	57.2	9.3	74.2	15.9
Northridge	Passive	57.5	33.7	59.4	32.8	0.0	0.0	0.0	57.6
	Active PID control	2.2	1.7	1.6	0.7	88.8	10.1	70.8	9.5
	Active 2DOF PID control	1.0	0.5	1.8	0.9	94.9	11.3	78.9	10.1
Hachinohe	Passive	79.2	48.6	66.4	40.6	0.0	0.0	0.0	79.2
	Active PID control	5.2	4.0	14.7	1.4	16.9	12.1	68.9	17.3
	Active 2DOF PID control	1.9	1.2	2.9	1.5	19.0	13.3	75.6	16.5

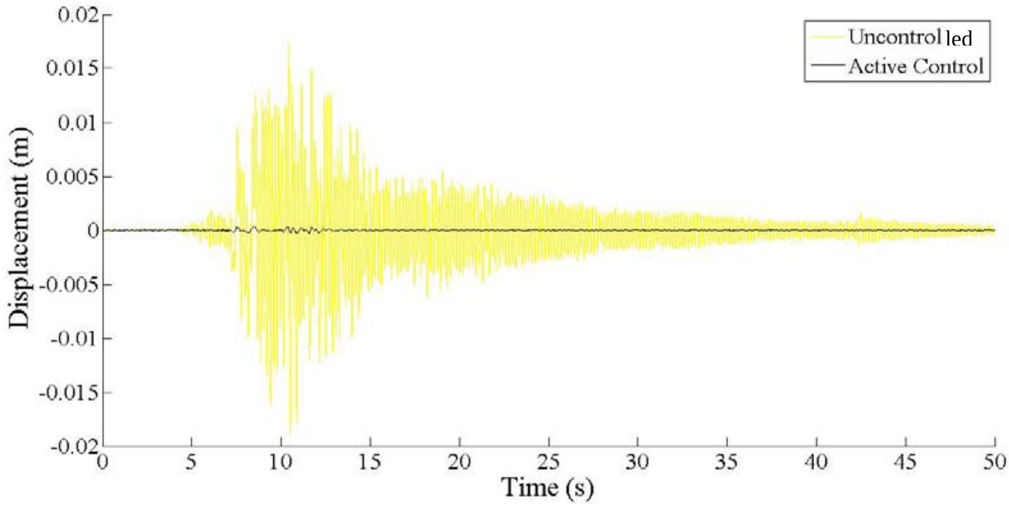


Figure 5.1. Control effectiveness on displacement response of the 3rd story subjected to Kobe earthquake.

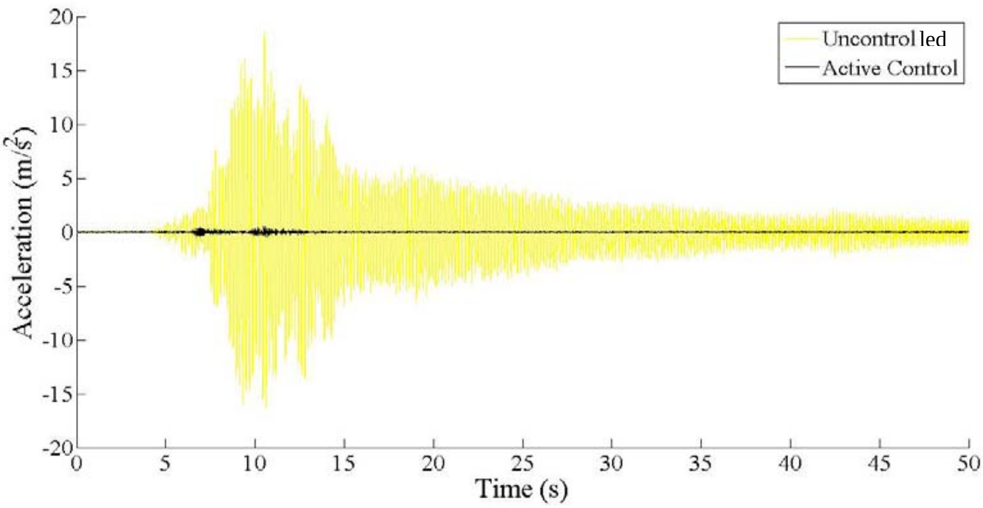


Figure 5.2. Control effectiveness on acceleration response of the 3rd story subjected to Kobe earthquake.

6. CONCLUSIONS

The effectiveness of classical PID and 2DOF PID control algorithms for active vibration control of a structure is investigated. A three story structures is considered as an example. It was shows that 2DOF PID exhibits more effectiveness in reducing system's response composed to classical PID control algorithm.

Although active control strategies demonstrate very high effectiveness, it should be noted that the values of active control force maybe so high that its realization be non-economic. However, by the compromise between response levels and active control force values, the desired results can be achieved.

REFERENCES

Åström, K.J. and Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning*, 2nd Edition. Instrument Society of America .

Chang, J.C.H. and Soong, T.T. (1980). Structural control using active tuned mass dampers. *Journal of Enineering Mechanics* **106:6,** 1091-1098.

Chopra, A.K. (2007). *Dynamics of Structures, Theory and Applications to Earthquake Engineering*, Pearson Education.

Ioi, T. and Ikeda, K. (1978). On the dynamic vibration damped absorber of the vibration system. *Bulletin of The Japan Society of Mechanical Engineers* **21:151,** 64-71.

Ogata, K. (1997). *Modern Control Engineering*, Prentice-Hall.

Shakib, A.(2010). *Comparison of Semi-Active Control Strategies for Seismic Response Reduction of Structures Using MR Dampers*. MSc Thesis, University of Tehran, Tehran, Iran (In Persian).

Soong, T.T. (1990). *Active Structural Control: Theory and Practice*, John Wiley and Sons, New York, NY.

Soong, T.T. and Dargush, G.F. (1997). *Passive Energy Dissipation System in Structural Engineering*, John Wiley and Sons, New York, NY.