Infrastructure Performance Oriented Reliability Assessment Using Weighed Stochastic Petri Net

M. Faraji & J. Kiyono
Kyoto University, Japan

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SUMMARY:
It is now apparent that our nation's infrastructures and essential utilities have been optimized for reliability in benign operating environments. As such, they are susceptible to cascading failures induced by relatively minor events such weather phenomena, accidental damage to system components, and major events such earthquakes. This paper presents a new kind of integrated modeling method for simulating the reliability of critical infrastructure spatially lifelines for a hazard and the subsequent interdependencies among the interconnected infrastructures. In previous studies, usually the weight of network components is not defined or in some case is assigned by expert idea or complicated network analysis. The new Weighed Stochastic Petri Net (WSPN) modeling approach which is proposed in this paper is based on the graph theory, fragility curves, Stochastic Petri net (SPN) modeling and Markov Chain analysis. In this model by using the simple graph theory parameters, weight of each component is defined in stochastic Petri nets. Therefore the cascading impacts throughout the network and reliability can be assessed based on weighed stochastic Petri nets.

Keywords: infrastructure, reliability, stochastic Petri Net, graph theory, Markov Chain analysis

1. INTRODUCTION

Elements of infrastructure, which include electric power, natural gas and petroleum production and distribution, telecommunications (information and communications), transportation, education, water supply, banking and finance, emergency and government services, agriculture, and other fundamental systems and services, are highly interconnected and mutually dependent in complex ways, both physically and through a host of information and communications technologies (Rinaldi et al., 2001).

Understanding and analyzing infrastructure performance and interdependencies is essential for the effective response and management of resources for rescue, recovery, and restoration. Currently, there are two main ways of representing relationships between infrastructure elements: graphic and matrix representations (Dunn et.al, 2004). Graphic representation of infrastructure components interaction is often intuitive and easy to understand while matrix representation is more organized and can be further extended to quantitative analysis and modeling.

Unlike graphic and matrix representations, the simulation of a cascading process is a technique that dynamically represents relationships between infrastructure elements. The technique allows the coupling of multiple interdependent infrastructure elements to address infrastructure protection, mitigation, response, and recovery issues.

In this study, it is tried to obtain the weight of the network elements by using the graph theory and calculate the geographical network parameters. Then Weighed Stochastic Petri Net modeling is done by using the weighted network elements, fragility curves, Stochastic Petri net theory and Markov Chain analysis.

2. STOCHASTIC PETRI NET THEORY

Standard models used for reliability analysis are Reliability Block Diagrams, Fault Trees, Markov Chains, and Petri Nets. Since the first three are not capable of modeling discrete events caused by trigger events, e.g. cascading effects, it is addressed modeling infrastructure performance using Petri Nets. Petri Nets is suitable to formalize and simulate dynamic aspects of complex systems, describing the semantics and activity of workflow systems.

Petri net had been applied to study the behavior of concurrent, asynchronous, distributed, parallel, non deterministic, and/or stochastic systems (Murata, 1989). A basic Petri net structure *C* can be described as a seven-tuple, $C = (P, T, I, O, A, w, B)$, where, *P* stands for place, *T* for transition, *I* for input function, *O* for output function, *A* for arc connecting *P–T*, *w* for arc weight expressing the number of arcs, and, *B* for inhibitory place. The existence of the characteristic of a place is indicated by the presence of token.

One important structural property of the basic Petri net enables the determination of the place invariant by the incidence matrix, *C*. A place invariant is the set of places in which the weighted sum of the tokens remain constant for all markings (Murata, 1989). It was shown that the minimal place invariants of a Petri net are capable of representing the interdependencies among the interconnected infrastructures (Gursesli et.al, 2003). The incidence matrix has the dimension $m \times n$ if the numbers of places and transitions are *m* and *n*, respectively. If the place invariant *y* is a $m \times 1$ column vector, then, solution of *y* is given by the Eqn. 2.1.

$$C^T * y = 0 \tag{2.1}$$

where, C^T is the transpose matrix of *C*.

Transition expressing the occurrence of an event is characterized by instantaneous time in basic Petri net model whereas it is more realistic for such problems to be characterized by stochastic time distribution which can be addressed by applying a SPN model (Bobbio, 1990). The SPN model states that, in a timed Petri net, each transition takes a positive time to fire (occurrence of an event) and the firing time is an exponential random variable. This paper adapts the related theory of SPN analysis from Zuberek (1991). SPN analysis is performed to depict the reachability graph which indicates all the possible markings for a specific initial marking condition. The resulting reachability graph from this analysis can be used to generate the corresponding Markov Chain, analysis of which simulates the steady state of the Petri net.

Generalized Stochastic Petri Nets (GSPN) differ from regular Petri Nets in that two types or transitions exist, i.e. immediate transitions and timed transitions, (Krings, 2003). As an extension to Petri Nets, arc multiplicity is a convenient way to represent the case when more than one token is to be created or absorbed. The multiplicity is denoted next to the arc. When a transition fires, a token is consumed for each arc incident to the transition and a new token is created for each arc incident from the transition. It should be noted that tokens are not moved, but they are consumed and created, thereby not necessarily keeping the number of tokens in a net constant.

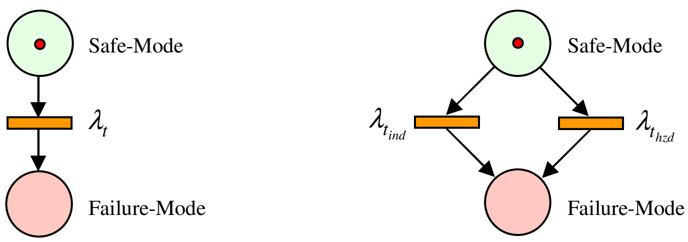

Figure 2.1. Simple Generalized Stochastic Petri Nets, (a) single-mode Model, (b) Multi-mode Model

Now, it is defined GSPN primitives useful in modeling common mode faults and cascading effects. The GSPN shown in Fig. 2.1.a models a simple system and is the simplest of the proposed GSPN modeling primitives. Places the safe-mode and failure-mode represent the state of the infrastructure which is initially functional, as indicated by the token in place safe-mode. The infrastructure is failing with fail rate λ_t in a single mode fault model. Petri nets are useful in determining the reliability *R(t)* of a system, where *R(t)* is defined as the probability that the system is functioning during the entire time interval *[0,t]*, given it was functioning at *t=0*. The simple system of Fig. 2.1.a produces $R(t) = e^{-\lambda t}$.

Introducing common mode failure models partitions the fail rates, resulting in rates for faults obeying the independence of faults assumption, and those that do not. Partitioning the fail rate in the simple model of Fig. 2.1.a results in the GSPN primitive is shown in Fig. 2.1.b. The aggregate fail rate is given by $\lambda = \lambda_{ind} + \lambda_{hzd}$, where the subscripts indicate the fail rates contributable to independent and common model faults respectively. Thus λ_{ind} is the fail rate for components obeying the independence of fault assumption.

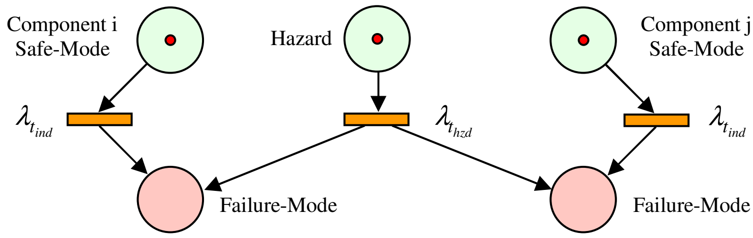

Figure 2.2. Common Mode Generalized Stochastic Petri Nets

The multi-mode GSPN primitive can be used to derive a common mode failure GSPN primitive as shown in Fig. 2.2 for a two system scenario. The common mode fault affecting both systems is modeled by the subnet in the centre, consisting of place com and its associated timed transition with fail rate λ_{hzd} . Whereas each system may fail independently as the result of the firing of their timed transition with rate λ_{ind} , both systems fail if the centre transition fires. Note that the fail rate of the centre transition does not depend on the markings of places sys-i-up and sys-j-up. That is, the transition does not fire twice as fast since it represents the common mode failure of two systems. The reason is that by the definition of common mode failure λ_{hzd} implies that both systems are subjected to the same input.

In the GSPN primitives above it is differentiated between independent failures and common mode failures. In real systems, the separation of independent and dependent failures can be extended to smaller granularities. Rather than having simply an independent and common mode portion of the system, each set of reused components represent a potential source for common mode failure for the systems having those components. Each set can be modeled by a timed transition having its respective fail rate. Not only allow a more accurate model, but also allows sets of reused components to span over subsets of systems.

3. GRAPH THEORY

Every networked system exhibits a distinct topology or physical layout. Concepts from modern graph theory are fundamental to enable measuring of these observable differences in network topology and flow type. A graph or network is a pair $G = (V, E)$ of sets. The elements of V are the vertices (nodes) of the graph G , whereas the elements of E are its edges (links). The graph components can have different properties within the network in terms of their function, ability to connect with others, preferential attachment and importance (Diestel, 2000). They also have different probabilities of failure given natural and man-made hazards. Using graph theory, several parameters such as vertex degree, clustering coefficient, connectivity and redundancy ratio can be calculated to characterize the network topology and assign weight to the network components as a component importance factor.

4. WEIGHTED STOCHASTIC PETRI NET

As described in previous section, the critical infrastructure systems can be represented as nodes in a network where they are connected through a set of links representing the logical relationships among them. In this network system, failure of one node affects the functioning of the interconnected nodes. By using graph theory concept in stochastic Petri nets, the cascading impacts throughout the network and reliability can be assessed.

4.1. Modeling Framework

The reliability of a component requires the determination of its resistance capacity. Considering various uncertainties and the random factors involved, the capacity should be described based on a probabilistic model. The uncertainties stem from the uncertainties in the material properties, dimensions and the models used for the evaluation of the capacity. In this study, a set of fragility curves will be developed for the infrastructure components using the seismic structural modeling associated with Monte Carlo simulation to convey the information about the vulnerability of infrastructure components for different seismic intensities. Considering the most probable hazard condition as a common mode fault, the cascading impacts on the interconnected infrastructures can be captured through the development and analysis of a network based model, such as graph theory and SPN; further analysis of the corresponding Markov Chain simulates long term probability of the infrastructure failure.

4.2. Fragility Curves

Fragility curve is defined as a mathematical expression that represents the conditional probability of reaching or exceeding a certain damage state of an infrastructure at a given hazard level. Fragility curves can be developed empirically with damage database and analytically with structural failure modeling. The steps of the analysis include; classifying the damage states of the infrastructure components, e.g., critical strain, stress, drifts, seismic structural modeling of the components with the Monte Carlo simulation of the uncertain design parameters for a certain value of seismic intensity, calculate the exceeding probabilities of the damage states, repeat the same analysis for different seismic intensities, develop the analytical fragility curves.

4.3. Transition Probability and Markov Chain

For SPN analysis, if state s_j is directly t_k reachable from state s_i, the transition probability is given by Eqn. 4.1.

$$q(s_i, s_j) = \frac{r(t_k) * n_i(t_k)}{\sum_{t \in T} r(t) * n_i(t)} \tag{4.1}$$

where, $r = a$ firing rate function which assigns firing rate $r(t)$ to each transition t; $n = a$ firing rank function indicating the number of active firings for each transition (Sultana, 2009). The transition probability matrix is used to generate the corresponding Markov Chain. Theory of Markov Chain can be found elsewhere (Kemeny et al., 1974); therefore, only a brief description is given here. Markov Chain is developed with a transition probability matrix, T_r with probability entries, summing up to 1.0. It represents a sequence of probability vectors $p_0, p_1, p_2, ..., p_n$ and, it can be written by Eqn. 4.2.

$$p_n = p_0 * T_r^n; \qquad n = 1,2,3,... \qquad (4.2)$$

where, p is called a steady state vector if the state vectors p_n get closer and closer to p as n increases. The entries of p are the long term probabilities of the Markov Chain states.
In summary, the fragility curves of the infrastructure components is shown the probability matrix of its various hazard conditions, the seismic frequency analysis is predicted the seismic probability; the graph model of the network is simulated to obtain the importance factor of the components which is used in transition matrix generation. Then SPN model will simulate the cascading impacts for infrastructure disruption and the corresponding Markov Chain analysis will show the long term failure trend of the vulnerable infrastructures.

5. CASE STUDY

As a case study, a simple lifeline system is assumed with 9 nodes and 10 links, Fig. 5.1. The failure probability of the components is illustrated in Table. 5.1. Also the result of the proposed WSPN model for the network, spatially the graph theory parameters and performance oriented reliability of the components, is gathered in Table 5.1.

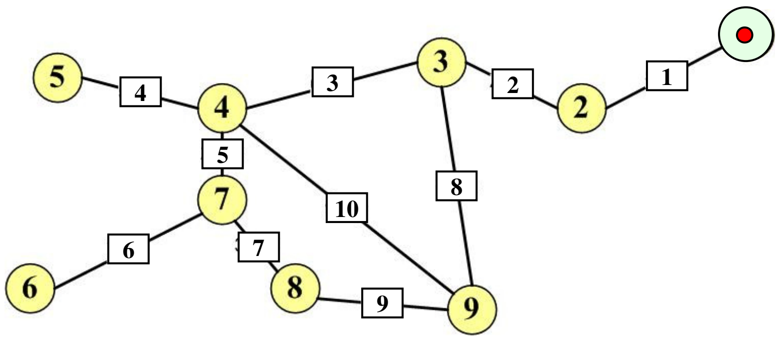

Figure 5.1. The simple network system with 9 nodes and 10 links

Table 5.1. Network Characterization

Node	Degree	Mean	Clustering	Redundancy	Efficiency	Reliability
1	1	1.44	0.000	0.250	0.857	1.00
2	2	1.05	0.000	0.250	0.832	0.74
3	3	0.83	0.333	0.265	0.906	0.61
4	4	0.71	0.166	0.224	0.966	0.54
5	1	1.10	0.000	0.187	0.865	0.23
6	1	1.30	0.000	0.222	0.805	0.17
7	2	0.75	0.000	0.194	0.905	0.39
8	2	0.86	0.000	0.200	0.871	0.45
9	3	0.96	0.333	0.333	0.867	0.52

6. CONCLUSION

In this paper, it is presented a simple WSPN modeling for identifying and quantifying performance of lifeline systems. In this model, the fragility curves of the infrastructure components is presented the probability matrix of its various hazard conditions and the seismic frequency analysis is predicted the seismic probability; the graph model of the network is simulated to obtain the importance factor of the components which is used in transition matrix generation. Then SPN model will simulate the cascading impacts for infrastructure disruption. Analysis of the Markov Chain generated from the reachability graph rendered the probability matrix of the steady state or long term condition of the network infrastructures. The extended analysis of the developed Markov Chain tracked the extended Petri net analysis accurately. Integration of these modeling techniques provides a useful and significant tool for predicting the overall probability matrix of infrastructure damage states.

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