Seismic vulnerability assessment of power transmission networks using complex-systems based methodologies

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ABSTRACT:
This paper develops a methodology for seismic vulnerability assessment of power transmission systems. The analysis is carried out from the perspective of system's *form* (i.e., topological importance of elements) and system's *strength* (i.e., probability of failure). The *form* combines the electrical properties of the network (e.g., electrical distance, power flow) with the *systems approach* via hierarchical network decomposition. On the other hand, the *strength* focuses on evaluating the probability of failure by means of the physical consequences of multiple earthquakes scenarios. The results are compared with different complex-systems vulnerability assessment techniques. As a result, it can be concluded that the proposed approach exhibit features that provide a better understanding of the vulnerability than traditional approaches.

Keywords: Vulnerability assessment, power networks, complex-systems theory

1. INTRODUCTION

The electric power system is one of the principal lifelines of a country that influences proper operation of the industry sector and principal facilities, such as hospitals, governmental institutions and education institutions. Major disturbances into the electrical power system have resulted in severe economic and physical consequences for the regions affected (Force 2004). The incorporation of vulnerability and risk analysis of lifelines into decision making process has been widely explored (e.g., Xingbin and Singh 2004, Koonce et al. 2008, Pinar et al. 2010). Quantifying the possibility of major disturbances in lifeline systems is of importance to various levels of decision makers.

A power transmission system consists of a group of sub-systems (e.g., power generating facilities, substations, and supervisory control and data acquisition facilities) which are inter-connected through transmission lines arranged within a high dimensional network (i.e., large amount of edges and nodes). The electrical interaction between the system components and its dynamics can be modelled in different fashions, thus several techniques for vulnerability analysis have been proposed. These techniques can be categorized into probability based (Xingbin and Singh 2004, Ma et al. 2010, Wenyuan and Choudhury 2007), complex system theory (Watts and Strogatz 1998, Barabasi and Albert 1999, Casals 2009, Bompard et al. 2011) and energy-function analysis (Fouad et al. 1994, Lu et al. 2008, Liu et al. 2009). In Table 1.1, the description of each technique is provided and their differences are highlighted.

Complex systems theory has been used as a tool for analyzing features of the interacting elements that has common characteristics and the dynamics (e.g. power system), and can be extracted analytically (Barabasi and Albert 1999). Complex systems theory uses indicators such as betweenness, centrality or degree distribution (Casals 2009), to detect vulnerabilities of the system. However, topological measurements do not consider any of the electrical properties of the network such as voltage capacity, load demand, or power flow. Some work have been done to integrate the electrical information into the topological analysis of power systems, and new definitions of betweenness and distances has been proposed (Bompard et al. 2011, Wang et al. 2011).

Table 1.1. Description of methodologies for vulnerability analysis of power systems

Methodology / **Feature**	**Probability**	**Energy function**	**Complex-systems theory**
Power system representation	Random variables	Electrical features	Graph (lines, buses)
Power system dynamics	Probability distribution functions	Stability power and trend	Topological features
Attack	Random variables	Injection of energy	Physical node/edge removal
Vulnerability measure	Probability of the system to work out of its limits	Distance between current state and stable state	Topological changes
Algorithm	Monte Carlo / Analytical algorithms	Branch potential energy (BPE) / Energy function (TEF)	Graph theory

In this work, a framework for seismic vulnerability analysis of power transmission systems is proposed. Novel concepts of vulnerability using complex systems are applied to model seismic failure scenarios using fragility curves, which integrate both the probability of failure and the consequences (topological and electrical) of a power transmission system. This paper is organized as follows: Section 2 discusses the seismic vulnerability model and its theoretical background. A complex system approach for vulnerability analysis using hierarchical decomposition of networks is explained. Following, the reformulation of topological measures to integrate electrical properties are discussed. Section 3 presents the results of applying our seismic vulnerability approach to the IEEE-118 test system. Section 5 show a comparison of the seismic vulnerability model using pure topological and topological-electrical vulnerability measures, to highlight the advantages of the reformulated topological concepts. Finally, in Section 6 conclusions and future work are presented.

2. PROPOSED APPROACH

2.1. Hierarchical representation of networks

Complex networks have been studied using system thinking representations for dealing with complexity, effectiveness in decision-making processes, and inside of the system's behaviour (e.g. Barabasi and Albert 1999, Bompard et al. 2011). Hierarchical representation of networks can be used to represent the internal network dynamics at different levels of resolution.

In Gomez et al. (2010), hierarchical representation of a transportation network is provided. The hierarchy is formulated recursively by applying a clustering algorithm to the system and its derived sub-systems. The clustering technique used in Gomez et al. (2010) is the Markov Clustering Algorithm (MCL), which uses the affinity matrix and random walks to simulate flow through the network and identify communities. The hierarchy is then obtained as follows:

1. The network is represented by a graph ($G = \{\mathcal{V}, \mathcal{E}\}$), where power lines are edges ($\mathcal{E}$), and substations ($\mathcal{L}$) and generation ($\mathcal{G}$) plants are nodes (i.e., $\mathcal{V} = \{\mathcal{L}, \mathcal{G}\}$). Note that, the entire network is represented by one fictitious node in the first level of the hierarchy (see Figure 2.1a), and the first fictitious network is represented only by this node, i.e., $\mathcal{V}^{(1)} = \left\{V_1^{(1)}\right\}$ (see $V_1^{(1)}$ in Fig. 2.1a).

2. The clustering algorithm is applied to the network and a next level of the hierarchy is obtained. Consequently, a fictitious network is created, this time composed by three fictitious nodes $\left(\mathcal{V}^{(2)} = \left\{V_1^{(2)}, V_2^{(2)}, V_3^{(2)}\right\}\right)$ (Fig. 2.1b) and its corresponding edges ($\mathcal{E}^{(2)}$). This procedure is repeated to each sub-system (i.e., fictitious networks) until the real network is obtained, which implies having the original nodes in the bottom of the hierarchy.

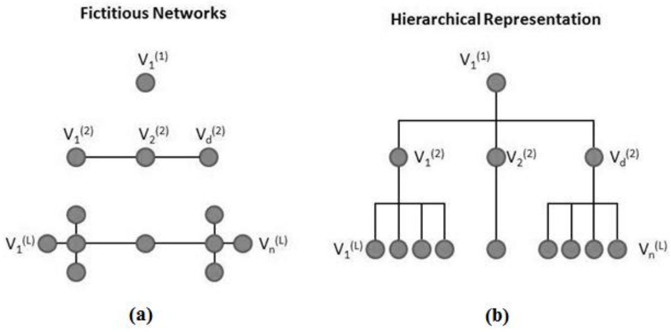


Figure 2.1: Hierarchical decomposition of networks

2.2 Electrical properties of power systems

A power system of $N + 1$ buses can be described in terms of the flow equations that represent it as follows:

$$U_B = Z_B I_B \tag{2.1}$$

where U_B is a vector that represents the voltage in each bus (node), Z_B is the impedance matrix of $N \times N$, where each entrance $z_B(i,j)$ is the impedance between bus i and bus j, and I_B is a vector that represents the injection current. The equivalent impedance between two pair of buses is described in terms of a real part called resistance (R) and the complex part, reactance (X). The first assumption for this approach is that the line losses are ignored, which indicates that the resistance of a line is zero, and in consequence, the impedance is calculated only in terms of the reactance (Arianos et al. 2009).

Efficiency of power networks has been studied as a measure of performance. Moreover, changes on the network efficiency after a disturbance are considered as a measure of the vulnerability of the network (Latora and Marchiori 2005, Arianos et al. 2009, Chen et al. 2009). The definition of efficiency from complex networks theory is how effective is the communication from one node to another when interchanging information. Efficiency is calculated in terms of the inverse of the geodesic distance (i.e., the shortest path) between all pair of nodes (Latora and Marchiori 2005). Nevertheless, in electrical power systems, the power does not flow exclusively for a certain path (e.g., shortest path) as in some infrastructure networks (e.g., transportation network), but it affects all the lines that belong to any path in between. For this reason, the calculation of the network efficiency should be reformulated in terms of the effectiveness of the power to flow between a pair of nodes.

Arianos et al. (2009) introduced the definition of *electrical distance* (δ_{ij}) as the cost of the energy to transit from bus i to bus j, and it is formulated in terms of the impedance of the line as follows:

$$\delta_{ij}^{k} = Z_{ij} = z_B(i,i) + z_B(j,j) - z_B(i,j) \tag{2.2}$$

where k is any path, and the distance between a pair of nodes i,j is calculated in terms of the equivalent impedance Z_{ij} (for details, see Arianos et al. 2009). Using this definition, the efficiency is reformulated in terms of electrical distance and the power transmission capacity. The new efficiency is called net-ability, and is defined as (Arianos et al. 2009):

$$A = \frac{1}{N_{\mathcal{G}} N_{\mathcal{L}}} \sum_{g \in \mathcal{G}} \sum_{d \in \mathcal{L}} \frac{C_{dg}}{Z_{dg}} \tag{2.3}$$

where $N_{\mathcal{G}}$ is the number of generation buses, $N_{\mathcal{L}}$ is the number of load buses, C_{dg} is the power transmission capacity between a pair of nodes, and Z_{dg} is the equivalent impedance between a pair of nodes. Likewise, the power transmission capacity is defined as (Bompard et al. 2011):

$$C_{ij} = \min_{l \in \mathcal{E}} \left(\frac{p_{max}^{l}}{|f_{ij}^{l}|} \right) \tag{2.4}$$

where p_{max}^{l} is the maximum power transmitted through line l, and f_{ij}^{l} is the change of the power on line l ($l \in \mathcal{E}$) for injection at generation bus i and withdrawal at load bus j, obtained by the difference between the entries f_{lj} and f_{lj} in the Power Distribution Factor matrix (P).

These concepts are part of the topological representation of networks, and are useful to aggregate electrical properties with topological analysis of complex systems. Consequently, the idea of replacing the geodesic distance for the electrical distance, and use net-ability in order to include the electrical properties and flow behavior of the electrical system is adopted in this work.

2.3 Seismic hazard representation

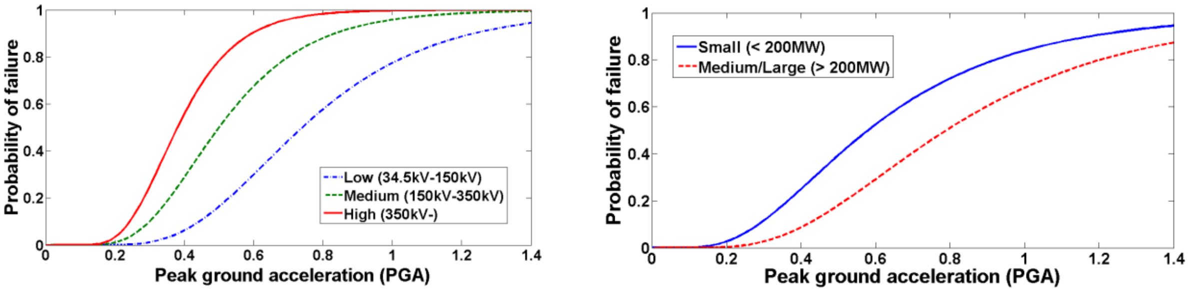
In order to model earthquake scenarios and response of the transmission power system, the probability of failure of a substation subject to different magnitude of earthquake load should be quantified. The probability of reaching or exceeding different states of damage given peak building response can be characterized using the earthquake fragility curves (e.g. NIBS 2003). However, this measurement depends in the peak ground acceleration (PGA) at the location of the substation, which differs on the source and magnitude of earthquake. To develop the earthquake scenario and the *strength* of the system against seismic failure, the following procedure is developed: (i) the magnitude and epicentre of the earthquake scenario are chosen, and (ii) the attenuation equation is then used to calculate the PGA at the desired location of every system. The following attenuation equation is used in this analysis (Sanchez-Silva and Rackwitz 2004):

$$PGA = h(m, r) = b_1(r) e^{b_2 m} \tag{2.5}$$

where m is the magnitude of the earthquake, b_2 is 0.573, r is the distance from the site to the earthquake epicentre, and $b_1(r)$s a function of distance describing the energy dissipation. $b_1(r)$ is calculated as follows (Sanchez-Silva and Rackwitz 2004):

$$b_1(r) = \frac{9.81 \times 0.0955}{\sqrt{r^2 + 7.3^2}} e^{-0.00587r} \tag{2.6}$$

Following NIBS (2003), the fragility curves used for substations (HAZUS ESS2, ESS4, ESS6) and generation facilities (HAZUS EPP2, EPP4) are shown in Figs. 2.2(a) and 2.2(b), respectively. Different parameters are used according to the classification of the substation (low, medium and high voltage), assuming unanchored substations and a scenario of complete damage. Likewise, the fragility curves considered for the generation facilities assume a complete damage of the facility and are classified according to its generation capacity as small and medium/large.



(a) (b)

Figure 2.2. Fragility curves for complete damage
a) unanchored substations, and b) unanchored generation facilities

2.4 Seismic vulnerability analysis of electrical transmission networks by hierarchical decomposition

In most approaches, the vulnerability is measured in terms of the consequences/changes of the power network after a disturbance. In order to achieve this assessment, three main elements should be obtained: 1) the representation of the *form* of system (e.g., flow equations, probability distribution function, hierarchy, etc.); 2) the system *strength*, which represents the relevance of the disturbance's consequences in each element; 3) information regarding the disturbance behavior (e.g., deletion of nodes, earthquake pattern equations, hurricane pattern equations, or other hazard representation). In this work, a seismic vulnerability analysis of electrical transmission networks is implemented by using hierarchical decomposition and electrical-topological representation of the power network. Gomez et al. (2011) quantified vulnerability through hierarchical representation as follows:

$$W^{(l)}(e_j) = \Upsilon^{(l)}[e_j] \times c^{(l)} \times F_1(e_j) \times F_2(e_j) \tag{2.7}$$

where $W^{(l)}(e_j)$ is the vulnerability measure of element e_j in level (l) of the hierarchy; $\Upsilon^{(l)}[e_j]$ is an indicator function of the presence of element e_j in level (l); $F_1(e_j)$ is the *form* of element e_j relative to the system (i.e., relative importance); $F_2(e_j)$ is the strength measurement of element e_j (i.e., failure probability); and, $c^{(l)}$ is a weighting factor related to the level of resolution of the hierarchy in (l). The seismic vulnerability assessment methodology for power transmission networks using hierarchical decomposition is summarized in the Algorithm 1. A specific implementation of Eqn. 2.7 is adopted to assess vulnerability in power systems:

- The drop in net-ability (see Eqn. 2.3) is introduced as a measure of the importance (i.e., *form*) of an element in the system (see Eqn. 2.8). This replacement follows the original idea to measure the change in the minimum distance (from Gomez et al. 2011), but includes the power flow equations of the system.

  $$F_1(e_j) = \Delta A(e_j) = \frac{A - A_j}{A} \tag{2.8}$$

  where $A(e_j)$ is the net-ability of the network without the element e_j , and A is the total net-ability of the network.

- The measure of the *strength* of an element $(F_2(e_j))$ is evaluated for different earthquakes scenarios using the procedure mentioned in previous section. This work is focused on giving a general estimation of the vulnerability of the system, thus a Monte Carlo simulation is utilized for obtaining a measure in a representative number of earthquakes scenarios. Consequently, the vulnerability is calculated for each iteration and a final vulnerability curve is presented. As well, in each iteration, the strength is calculated as:

$$F_2{}^t\left(e_j\right) = P\left(z \le PGA_t(e_j)\right) \tag{2.9}$$

where $PGA_t(e_j)$ is the resulting peak ground acceleration in the location of element (e_j), when earthquake scenario t is simulated; and, $P(z)$ is the probability of failure of a substation in an earthquake scenario.

Algorithm 1. Seismic vulnerability analysis using hierarchical decomposition

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1:  $G = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{\mathcal{L}, \mathcal{G}\}$
2:  Calculate $\delta_{ij}$for all $\{i, j\} \in \mathcal{V}$ (see Eqn. 2)
3:  Calculate $F_1(e_j)$ for all $j \in \mathcal{V}$ (see Eqn. 9)
4:  Obtain the hierarchy H=recursiveClustering($G, \delta_{ij}$)
5:  Calculate $\Upsilon^{(l)}[e_j]$, and $c^{(l)}$, for all $l \in L$ and $j \in \mathcal{V}$
6:  For $t = 1$ to T do
7:      Obtain earthquake scenario $ES_t = \{m_t, r_t\}$
        where $m_t$ and $r_t$ are the magnitude and location of earthquake source at time t
8:      Obtain $PGA_t(e_j)$ for all $j \in \mathcal{V}$
9:      Obtain $F_2^t(e_j)$ (see Eqn. 10)
10:     Actualize $W_t^{(l)}(e_j) = \Upsilon^{(l)} \times c^{(l)} \times F_1 \times F_2^t$
11: end for
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3. CASE OF STUDY

The IEEE 118 Bus Test Case (Fig. 3.1) is a representation of 1962 Midwestern US power system. This power system has been widely used as a standard test case since its publication. The system consists of 118 buses (i.e., nodes), 186 branches (i.e., edges), 91 load sides (circles) and 54 thermal units (squares).

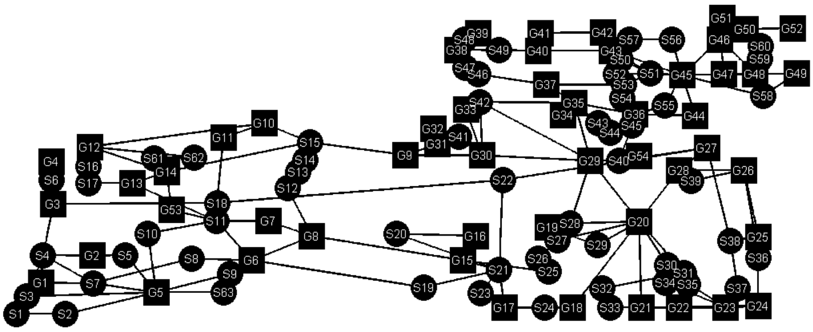


Figure 3.1. IEEE-118 Test Case network representation

The seismic vulnerability methodology summarized in algorithm 1 is applied for this network. First, the transmission system of the IEEE-118 is represented by a graph (G), where the set of generation facilities $(\mathcal{G})$ and substations $(\mathcal{L})$ composes the vertex set of the graph (i.e., $\mathcal{V}$); and, electrical lines of the power system are the edges between nodes $(\mathcal{E})$, as shown in step 1 of algorithm 1. Then, the electrical distance of the 186 branches is calculated using equation 2 and a new distance matrix representation of the network is formulated with this calculation (see step 2 of the algorithm 1). Likewise, the physical location of every node and connections (adjacency matrix) of the power system are obtained. Using the drop in net-ability to represent electrical-topological properties of the network, the *form* $(F_1(e_j))$ of each element of the network is calculated as shown in Eqn. 2.8. The cumulative distribution of this measure is shown in Fig. 3.2. From this distribution, it can be seen that most of the nodes have a small drop in net-ability, but few of them are highly important in the electrical dynamics of the network.

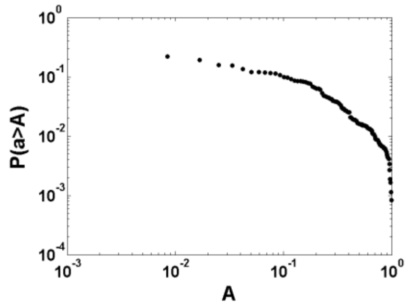


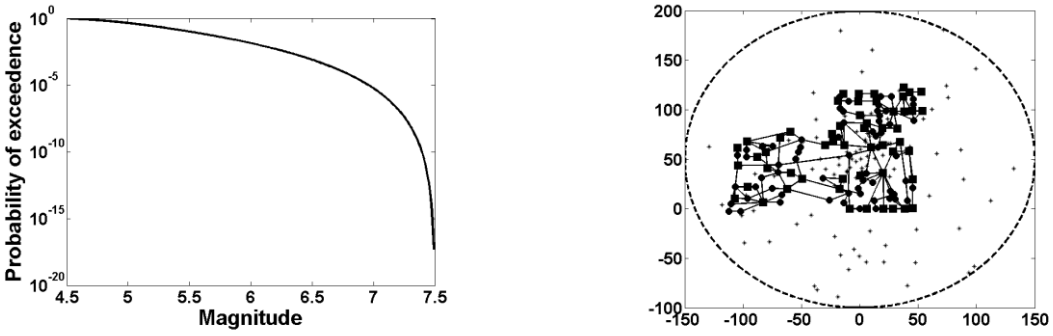
Figure 3.2. Cumulative distribution of drop in net-ability in the IEEE-118 bus system

Through recursive Markov clustering (MCL), the hierarchy of the IEEE-118 is acquired using 2RNet software[1]. For applying the clustering algorithm, the physical distance (d_{ij}) of the network was replaced by the electrical distance (δ_{ij}). The resulting hierarchy is composed by four different levels: one single unit in the top of the hierarchy, represented by its centroid, node 30; in the second level, nine clusters were found; in the third level, the clustering identified 97 clusters; and finally, in the fourth level there is a cluster per node, i.e., 118 clusters. Once the hierarchy is obtained, the hierarchy-dependent terms of the vulnerability equation (see Eqn. 2.7) are obtained ($\Upsilon^{(l)}[e_j]$, and $c^{(l)}$). Finally Monte Carlo simulation is carried out (steps 8 to 12 of algorithm 1).

3.1 Seismic vulnerability by hierarchical decomposition

The vulnerability of the system is calculated, including all the levels of the hierarchy and for every node of the system. It consists of a Monte Carlo simulation for earthquake scenarios (ES). For the magnitude of each earthquake scenario, only magnitudes within the range of 4.5-7.5 were considered, according to the seismic hazard curve shown in Fig. 3.3(a).The simulation assumes a spatially uniform distribution for the epicenter in a circular area of 150km of diameter, as shown in Fig. 3.3(b).

The resulting distribution of the vulnerability measure for the nodes is shown in Fig. 3.4. The simulation was developed 1000 times, and all the curves obtained are plot in the figure, where the yellow line represents the mean of the results. It can be seen that the resulting measure differs from a fragility curve due to the influence of the *form* measure. It is also relevant to notice that around 90% of the nodes have a vulnerability measure less than 3.193E-3, which correspond to around 3% of the maximum vulnerability, from which we can conclude that our methodology highlights the elements that are topologically and electrically relevant to the network dynamics.



(a)

(b)

Figure 3.3. Earthquake scenarios
a) Probability of exceedence of magnitudes, and b) Area of simulation, 100 earthquake scenarios

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[1] Developed by Risk and Reliability Research group of Universidad de los Andes, Bogota, Colombia. Available in http://www.2rsoft.tk/

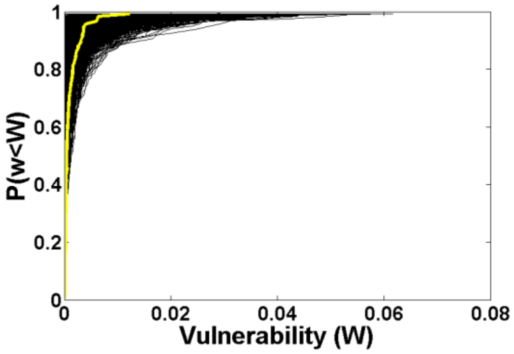

Figure 3.4. Seismic vulnerability of IEEE-118 network

For better understanding of the results, different type of substation and generation facilities were studied. In Table 3.1, some of the topological and electrical parameters are specified, with the vulnerability measure of each node. The nodes S-41 and G-30 are the most vulnerable substation and generation facility, respectively; while nodes S-9 and G-15 have the highest vulnerability measure less than 3.193E-3 (where 90% of the nodes are). Comparing the substations, it is relevant to notice that even though both nodes have similar degree and are low-voltage substations, the drop in net-ability capture intrinsic information from the network, which is directly reflected in the vulnerability measure. Now, if we compare the generation facilities, the difference between drop in net-ability is even higher, reflecting a higher difference in the vulnerability measure. Finally, the seismic activity is reflected in all the vulnerability results, since the location of the nodes are all different, the final vulnerability becomes a trade-off between the strength and the form of it. The influence of the seismic activity strength can be observe comparing nodes G-30 and S-41; even though the substation has a higher drop in net-ability, the vulnerability measure of the generation facility is higher, reflecting a higher seismic influence.

Table 3.1. Electrical and topological parameters of selected substations

Node number	Vulnerability measure	Degree	Drop in net-ability	Bus load (MW)	Maximum power generation
S-9	3.20E-03	6	8.13E-02	11.8	N/A
S-41	6.30E-03	5	1.13E-01	30	N/A
G-15	2.60E-03	4	6.88E-02	N/A	30
G-30	1.23E-02	5	1.06E-01	N/A	80

From the results obtained, two major conclusions can be drawn. First, the seismic vulnerability approach clearly introduces a trade-off between the probability of failure of a specific node and the relative importance of it. Second, the *form* of the system completely represents the topological and the electrical dynamics through the integration of the hierarchical representation and net-ability measure of each bus of the power network.

4. COMPARISON BETWEEN VULNERABILITY MEASURES

In this section, a comparison of the proposed approach is developed, alternating the algorithm for obtaining the *form*, i.e., the relative importance of the node in the network (F_1 in line 6 of algorithm 1). Three criteria are selected for this purpose, random, degree, and drop in net-ability (used in this approach). In the random assignment of importance, a uniform distribution is used to obtain different values of importance between 1 and 10, this simulation is repeated 20 times and the result is introduced as the measure of form. For the degree distribution, the level of importance is assigned according to the degree of each element, the nodes with the highest degree (i.e., 8-9) has the highest level of importance and so on, following the degree distribution (see Fig. 4.1). Finally, the drop in net-ability is considered as explained in the previous section.

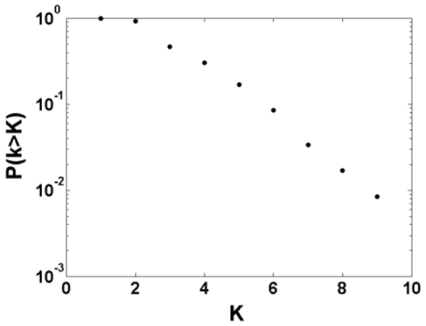

Figure 4.1. Cumulative distribution of node degree in the IEEE-118 bus system

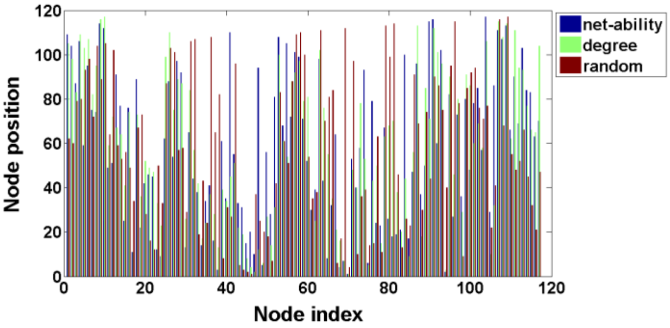

Figure 4.2. Comparison of form calculation algorithms

The resulting ranks are shown in Fig. 4.2. In order to compare the results obtained using the different techniques, a quantification of the difference should be done. In this work, the differences are measured using the differences of the ranking. In Table 3.2 the distances measures are exposed, where Δ is the distance between the ranking obtained with net-ability (n) and, whether degree (d), or random (r) (i.e., $\Delta_{a,b} = R_a - R_b$). The second column represents the number of nodes where the ranking of the first methodology is greater than the second, the third column represents the opposite situation; the fourth column represents the maximum positive distance. The last column represents the minimum distance.

Table 3.2. Electrical and topological parameters of selected substations

Difference	**$\Delta > 0$**	**$\Delta < 0$**	**max(Δ)**	**min(Δ)**	**avg(Δ)**
$\Delta_{n,d}$	54	58	82	-52	~17
$\Delta_{n,r}$	60	57	83	-105	~31

First, let us compare the results from net-ability and degree. It can be seen that approximately half of the nodes are ranked in different positions, which means that the resulting vulnerability using net-ability includes information related to the electrical properties, that a topological measure cannot capture. It is also important to note that the maximum difference (i.e., vulnerability given by net-ability is higher than the one given by using degree) is so much higher than the minimum difference, showing that using the degree measure the vulnerability tend to be underestimated. Finally, comparing the results from net-ability with random form, the minimum and maximum difference are very high, this is because the methodology does not have any information related to the importance of each node (i.e., form) and therefore the resulting ranking is very different. Finally, the average distance between net-ability and degree is much smaller than using random measures, which shows the importance to represent the vulnerability in terms of the probability of failure and also the relative importance of the node.

5. CONCLUSIONS

A novel seismic vulnerability assessment using hierarchical decomposition of networks is used to detect vulnerabilities in the system. The proposed approach integrates the electrical and topological importance of each element, with its probability of failure, obtaining a complete representation of the system behaviour. Results show the relevance between topological and electrical integration in the representation of the network, changing the distribution of the vulnerability measure. Results also show a trade-off between system strength and system form, given a vulnerability measure that includes the seismic context of the network, compared to previous approaches that focus only on topological or electrical vulnerabilities. Therefore, the proposed approach allows a better understanding and representation of the seismic vulnerability behaviour of power systems.

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