On the use of a hyperbolic model in calibrating earthquake magnitude distributions

Jui-Pin Wang & Duruo Huang
The Hong Kong University of Science and Technology, Hong Kong

SUMMARY:
From more than 54,000 earthquakes around Taiwan, the magnitude cumulative probability is found in a good fit to the hyperbolic function, which has been widely used in modelling nonlinear relationships in some engineering analysis, such as soil stress-strain behaviour developed into a well-known hyperbolic soil constitutive model. The details of this application to the seismicity around Taiwan are provided in this paper. The result shows that both new and conventional approaches suggest equally satisfactory fit to the observed magnitude distribution around Taiwan. As a result, the new method provides an alternative to model the earthquake magnitude distribution when it is needed in seismic hazard analysis.

Keywords: magnitude probability function, hyperbolic model, Taiwan

1. INTRODUCTION

Probabilistic seismic hazard analysis (PSHA) has been becoming the prescribed approach in site-specific earthquake resistant design for critical structures (USNRC, 2007; IAEA, 2002). The essentials of PSHA are to estimate mean rate (e.g., 10^{-4} / year) of design motion from seismicity data and geological evidences in a probabilistic framework, in which the uncertainty of earthquake magnitude, source-to-site and ground motion attenuation are accounted for. For developing the magnitude distribution function the conditional-probability approach with the earthquake recurrence parameters (known as *a*-value and *b*-value) developed by McGuire and Arabasz (1990) are commonly used. Basically, the probability for certain magnitude is the ratio of the frequency of this earthquake to total earthquakes considered, expressed as follows:

$$\Pr(m_1 \le M < m_2 \mid m_0 \le m_1, m_2 \le m_{\max}) = \frac{N(m_1, m_2)}{N(m_0, m_{\max})} \tag{1.1}$$

where m_0 and m_{max} are threshold magnitude and maximum magnitude; N denotes the number. Since earthquake rate can be modeled by the Gutenberg-Richter law (1944):

$$N(M \ge m^*) = 10^{a-bm^*} \tag{1.2}$$

Eqn. 1.1 becomes:

$$\Pr(m_1 \le M < m_2 \mid m_0 \le m_1, m_2 \le m_{\max}) = \frac{10^{a-bm_2} - 10^{a-bm_1}}{10^{a-bm_{\max}} - 10^{a-bm_0}} \tag{1.3}$$

Apparently, the cumulative magnitude probability F_M can be extended as the following Eqn. 1.4. Such a magnitude cumulative function is governed by *b*-value.

$$F_M(m^*) = \Pr(M < m^* \mid m_0 \le m^* \le m_{\max}) = \frac{10^{a-bm^*} - 10^{a-bm_0}}{10^{a-bm_{\max}} - 10^{a-bm_0}}$$

$$= \frac{10^{-bm^*} - 10^{-bm_0}}{10^{-bm_{\max}} - 10^{-bm_0}} \quad (1.4)$$

2. SEISMICITY AROUND TAIWAN

Fig. 2.1 displays the spatial distribution of declustered earthquakes from 1973 to 2009 around Taiwan, and Fig. 2.2 shows the observed theoretical cumulative probability with m_0 and $m_{\max}$ equal to 5.0 and 8.0, respectively. Note that this earthquake catalog has been used for earthquake statistics studies (Wang et al., 2011), and that its characteristics, such as incompleteness, have been discussed. Also, m_0 5.0 is considered the magnitude threshold possibly causing damage on engineered structures, and $m_{\max}$ 8.0 is an estimate based on the historic seismicity.

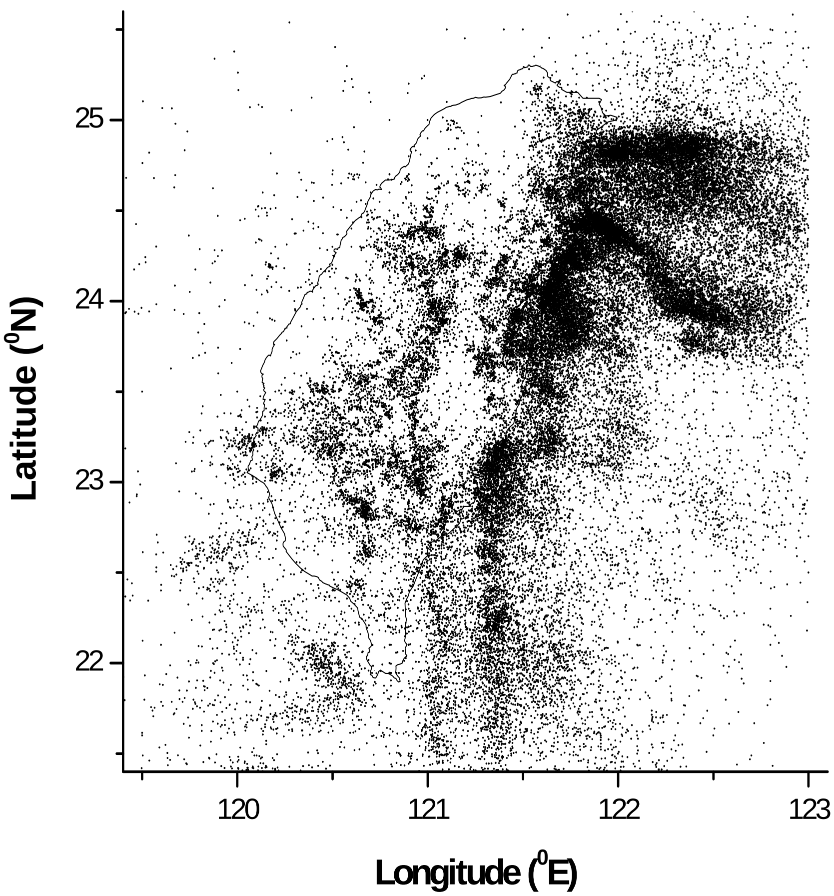

Figure 2.1. The spatial distribution of more than 54,000 earthquakes around Taiwan since 1973 (Wang et al., 2011)

Accordingly, Fig. 2.3 shows the Gutenberg-Richter relationship for this seismicity around Taiwan, with *a*-value and *b*-value equal to 5.83 and 0.92, respectively. From the two parameters, Fig. 2.4 shows the expected magnitude cumulative probability through Eqn. 1.4.

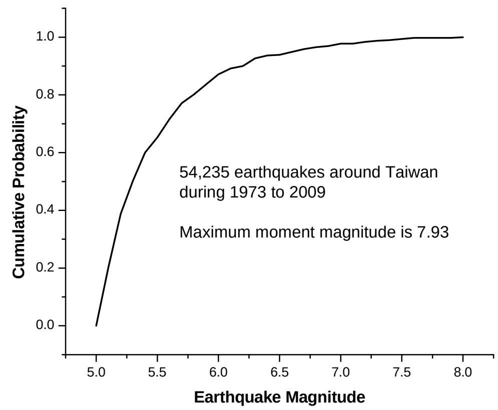

Figure 2.2. Observed magnitude cumulative probability for the seismicity around Taiwan

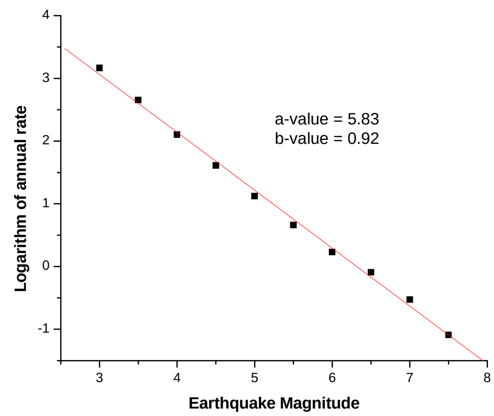

Figure 2.3. The Gutenberg-Richter relationship for the seismicity around Taiwan

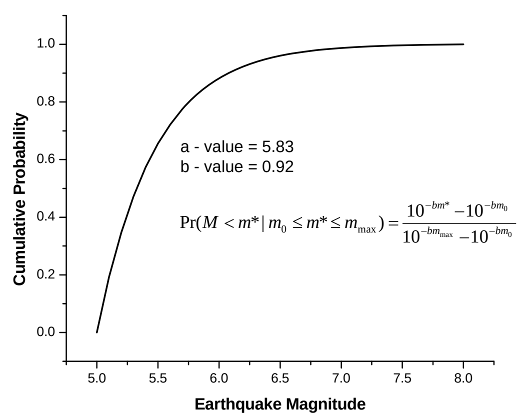

Figure 2.4. The expected magnitude cumulative probability through the conditional probability method

3. HYPERBOLIC FUNCTION AND PARAMETER CALIBRATION

Hyperbolic functions have been utilized in correlating two variables in engineering. The well-known

Duncan & Chang soil stress-strain model (Duncan and Chang, 1970) was developed on the basis of this type of nonlinear functions. Under the hyperbolic form, the relationship between magnitude probability function (f_M) and earthquake magnitude can be expressed as follows:

$$f_M(m) = \frac{m - m_0}{c + d(m - m_0)} \tag{3.1}$$

Note that *c* and *d* are function parameters and m_0 is a constant. Therefore, the derivative of f_M against *m* becomes:

$$\frac{\partial f_M}{\partial m} = \frac{c}{(c + d(m - m_0))^2} \tag{3.2}$$

Since $\frac{\partial f_M}{\partial m}$ presents the slope of the curve at $M = m$, when *M* approaches m_0, $\frac{\partial f_M}{\partial m}$ is equal to $1/c$. As a result, parameter *c* can be calibrated from the initial slope (S_{int}) of the curve as follows:

$$c = \frac{1}{S_{\text{int}}} \tag{3.3}$$

Substituting Eqn. 3.3 into Eqn. 3.1, parameter *d* can be expressed as follows:

$$d = \frac{1}{f_M} - \frac{1}{S_{\text{int}}(m - m_0)} \tag{3.4}$$

Using the boundary condition, $f_M = 1$ at $M = m_{max}$, parameter *b* becomes:

$$d = 1 - \frac{1}{S_{\text{int}}(m_{\max} - m_0)} \tag{3.5}$$

Through these relationships, parameters *c* and *d* can be calibrated from the observed cumulative probability curve shown in Fig. 2.2. For this magnitude distribution, *c* and *d* are 0.25 and 0.84, respectively. Accordingly, the cumulative magnitude probability through the hyperbolic relationship in Eqn. 3.1 is shown in Fig. 3.5.

4. DISCUSSIONS AND CONCLUSIONS

Fig. 4.6 shows the comparison of the three cumulative probabilities. Both models are equally providing a good fit to this observed seismicity around Taiwan, with the hyperbolic function slightly over-estimating and under-estimating the cumulative probabilities in low and high magnitude ranges. In contrast, the conventional prediction over-estimates and under-estimates those in high and low magnitude ranges.

The earthquake magnitude cumulative function is found to follow a hyperbolic function, which has been widely used in modeling nonlinear relationships in engineering analysis. Through the hyperbolic relationship, the two parameters for the seismicity around Taiwan are calibrated as 0.25 and 0.84, which can provide a good fit for the more than 54,000 earthquakes. The new method is considered useful for seismic hazard analysis in need of magnitude probability distribution, providing an alternative to the conventional conditional-probability approach.

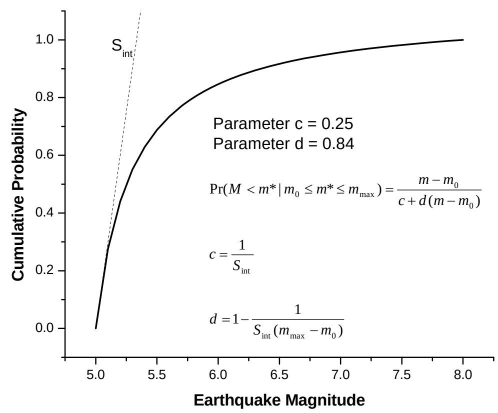

Figure 3.5. The hyperbolic prediction on the magnitude cumulative probability calibrated from the observed function

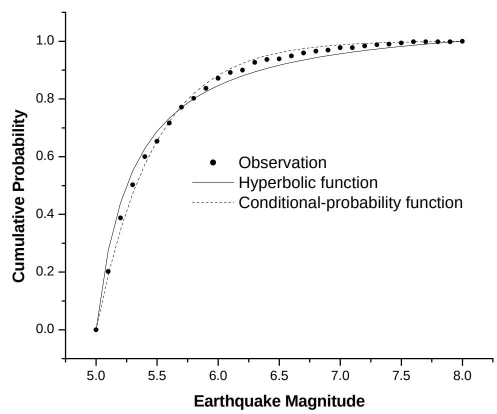

Figure 4.6. The comparison of the three earthquake magnitude functions; both new and conventional methods suggest equally satisfactory fit to the observation

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