An Innovative Response Surface Procedure to Estimate the Seismic Reliability of the Steel Moment-Resisting Structures

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SUMMARY:

Performance of a structure could be evaluated by computing the probability of occurrence for a specific state in the structure. Mostly, simulation techniques such as Monte Carlo simulation (MCS) and approximate approaches such as first-order reliability methods (FORM) are used to find the failure probability of the structural elements. Since the simulation methods are computationally expensive and the FORM could show inefficiency in reliability analysis of the building structures under earthquake excitation, here an innovative response surface method has been proposed. Linear response surface function has been used to approximate the performance function, based on the calculated damage index for the structural elements. The proposed method has been implemented on a nine-story steel moment-resisting building frame and the results indicate that the failure probability of the structural elements could be approximated with appropriate accuracy using the proposed method.

Keywords: reliability analysis; response surface method; damage index; Monte Carlo simulation

1. INTRODUCTION

The behavior of the structural elements and the effects of the surrounding environment are accompanied by inevitable uncertainties. In this regard, accurate performance estimation of the structure should be carried out with respect to these uncertainties. In the context of the reliability analysis, failure probability of a structural system under randomness of its characteristic parameters and applied loads could be expressed mathematically in its simplest form as in Eqn. 1.1 (Ditlevsen and Madsen, 1996).

$$p_f = \int_{g(\mathbf{X})\leq 0} f(\mathbf{X})\, d\mathbf{X} \tag{1.1}$$

Where, p_f is the probability of failure, X is the vector of considered random variables, $f(\mathrm{X})$ is the joint probability density function of the random variables, and $g(\mathrm{X})$ is the performance function which is used to define the state of the system. In structural reliability analysis, the performance function is defined in its simplest form as: *performance = threshold – response of the structure*. Threshold is a predefined constant value and the structure is considered safe if the response of the structure does not exceed this value. The limit in which the performance function is equal to zero ($g(\mathrm{X})=0$) is called limit state function (LSF). The LSF separates the space of the random variables into two domains: safe and failure domain. Since calculating the integral in Eqn. 1.1 is not feasible for most of real-world structural systems (Haukaas and Der Kiureghian, 2004), simulation techniques such as Monte Carlo simulation (MCS) and approximate methods such as response surface methods could be used to find the solution for structural reliability problems. The simulation techniques are highly efficient methods to compute the reliability of complex engineering structures. However, they involve obtaining hundreds of samples for the desired responses of the structure, in order to maintain an acceptable accuracy (Ditlevsen and Madsen, 1996; Nowak and Collins, 2000; Baecher and Christian, 2003). Since the finite element analysis of the building structures under earthquake excitation with nonlinear behavior could be highly demanding, implementing the simulation techniques in many circumstances does not seem as an easy solution to these types of reliability problems. In these regard, approximate approaches such as response surface methods could be of practical value. In this study, a new response surface procedure has been proposed to estimate the reliability of the building structures under earthquake excitation. The emphasis has been placed on reliability analysis of steel moment-resisting frames. The performance of the structure has been evaluated through the calculated cumulative response at the end of the nonlinear time history analysis.

2. GENERAL ASPECTS OF THE RESPONSE SURFACE METHOD

First introduced by Box et al. (Box and Wilson, 1951; Box, Hunter, and Hunter, 1978), the purpose of response surface method (RSM) is to establish an approximate model to estimate the desired parameter, based on the input variables. As mentioned before, reliability analysis of the building structures could be computationally demanding and the RSM could facilitate an easy and fast approach to obtain the failure probability of these strcutures. In this regard, many researchers have done numerous studies on developing response surface methods and improving their efficiency (for example; Faravelli, 1989; Bucher and Bourgund, 1990; Rajashekhar and Ellingwood, 1993; Yao and Wen, 1996; Kim and Na, 1997; Tandjiria, Teh and Low, 2000; Guan and Melchers, 2001; Gayton, Bourinet and Lemaire, 2003; Gupta and Manohar, 2004; Kaymaz and McMahon, 2005; Wong, Hobbs and Onof, 2005; Gavin and Yau, 2008; Allaix and Carbone, 2001; to name a few).

In reliability analysis, RSM intends to approximate the LSF with a function of considered random variables. This function is referred to as response surface function (RSF). The RSF could be used instead of the LSF to estimate the probability of the failure, using promising methods such as MCS or approximate methods such as first- and second-order reliability methods (FORM and SORM). Mostly a polynomial function of the random variables is used as the RSF. The polynomial degree of RSF is one of the important aspects of the RSM. The accuracy of the RSF and also the computational cost for establishing the RSF, are dependent on the degree of RSF (Rajashekhar and Ellingwood, 1993; Allaix and Carbone, 2001). In order to set up the RSF, the LSF should be evaluated in the selected sampling points to establish a system of equations to determine the constant coefficients of the RSF. The selection of the sampling points has also crucial effect on the results of the RSM. The sampling points are selected around a center point within a specific distance from it. The range in which the sampling points are selected could be defined as $[\mu_i - f\sigma_i \, , \mu_i + f\sigma_i]$, where μ_i and σ_i are mean value and standard deviation of i*th* random variable (Bucher and Bourgund, 1990). The parameter f controls the distance around the center point, in which the LSF is being approximated by RSF.

3. PROPOSED RESPONSE SURFACE METHOD

3.1. Cumulative Response Function

Damage indices could be used to check the performance of a designed structure, to assess the damage in the structure after earthquake, or to study the reliability of an existing structure to estimate the performance of the structure in pre-earthquake evaluation (Kappos, 1997). In this study the damage index (DI) of the structural elements are calculated based on the model developed by Mehanny and Deierlein (Mehanny and Deierlein, 2001). Positive and negative cumulative inelastic deformation of the element has been used to calculate the DI, according to Eqn. 3.1.

$$D_\theta^+ = \frac{(\theta_p^+ \mid_{current\,PHC})^\alpha + (\sum_{i=1}^{n^+} \theta_p^+ \mid_{FHC,i})^\beta}{(\theta_{pu}^+)^\alpha + (\sum_{i=1}^{n^+} \theta_p^+ \mid_{FHC,i})^\beta} \tag{3.1}$$

where the θ_p^+ is the inelastic component deformation in the positive direction, θ_{pu}^+ is the capacity of the element under monotonic loading, α and β are the calibration coefficients which are equal to 1 and 1.5 in steel elements, respectively. The damage index is calculated based on the deformation in primary half cycle (PHC) and follower half cycle (FHC). More explanation on this DI could be found in Mehanny (1999). D_θ^+ is the amount of damage cause by positive loading and the damage caused by reversed (negative) loading should also be calculated. Then these two values are used to obtain the damage index of the element in the monitoring section, using Eqn. 3.2. The calibration coefficient γ is equal to 6 in steel elements. The amount of the DI could vary from 0~0.1 (not damaged) to 1~1.1 (failed) in a specific section of an element.

$$D_\theta = \sqrt[\gamma]{(D_\theta^+)^\gamma + (D_\theta^-)^\gamma} \tag{3.2}$$

In this study; in order to estimate the damage in a specific element of the building; the DI calculated in the sections of the element is used to calculate the overall damage measure of the element. In this regard the weight of the damage in each section of an element is calculated through Eqn. 3.3.

$$W_{DI_i} = \left(\frac{DI_i}{\sum_{i=1}^{N} DI_i}\right) \tag{3.3}$$

where DI_i is the damage index in the i*th* section of the element, N is the number of the monitoring sections in the element and W_{DI_i} is the weight of the damage in the i*th* section. Then the measure of damage in the element ($DI_{element}$) is calculated by summation of the damage in the sections of the element with respect to the weight of damage in each section (Eqn. 3.4). This representative for the damage of an element could be used to estimate the failure probability of that element in the structure.

$$DI_{element} = \sum_{i=1}^{N} W_{DI_i} \times DI_i \tag{3.4}$$

Using DI to evaluate the performance of the structure facilitates considering the effects of cumulative nonlinear deformation of structural elements in probabilistic seismic evaluation of the building. Also the DI-based performance function provides the option of calculating the reliability of a dynamic system with time-invariant reliability approach (Koduru and Haukaas, 2010). Since the cumulative damage index is calculated at the end of the nonlinear time history analysis of the structure, reliability of the building structure under the earthquake action could be computed at the end of the structural analysis, using the time-invariant reliability methods.

In this study the performance of a structural element or a story of the structure is evaluated by the corresponding damage representative. In order to approximate the Damage-based limit state function of an element, a linear function of the considered random variables is proposed (Eqn. 3.5).

$$\tilde{g}(\mathrm{X}) = a + \sum_{i=1}^{n} b_i x_i \tag{3.5}$$

where $\tilde{g}(\mathbf{X})$ is the RSF of the considered random variables x_i , $i = 1,...,n$. a and b_i are the coefficients of the RSF, which should be calculated based on the evaluation of the LSF. Once these coefficients are found, the RSF is used to find the probability of failure of the structure.

3.2. Determining the Sampling Center

In order to generate the sampling points required to establish the RSF, center of the sampling should be specified. In this study, the center point of sampling is chosen based on the results of the three deterministic analyses of the structural system. These analyses are performed based on three different realizations of the random variables. First structural analysis is performed for the realization of the random variables in which the load-type random variables are set to $\mu_i - f\sigma_i$ and the resistance-type random variables are set to $\mu_i + f\sigma_i$ (μ_i and σ_i are the mean and standard deviation of i*th* random variable), resulting in the lower bound value for the response of the structure (R_{lower}). Second analysis is performed with the mean values assigned to all of the random variables. This analysis results in the assumed mean response (R_{mean}) of the structure. Finally the last analysis is performed for the realization of the random variables in which the load-type random variables are set to $\mu_i + f\sigma_i$ and the resistance-type random variables are set to $\mu_i - f\sigma_i$, resulting in the upper bound value for the response of the structure (R_{upper}). In order to categorize the parameters into load-type and resistance-type variables, forward finite difference approach could be used:

$$\frac{\partial g}{\partial x_i} = \frac{g(\mu_i + \Delta x_i) - g(\mu_i)}{2\Delta x_i} \tag{3.6}$$

where μ_i is the mean value of the random variable x_i and Δx_i is the perturbation factor which should be reasonably small with respect to each random variable. A random variable with positive gradient value is considered as a resistance-type variable. On the other hand the gradient value of load-type variable is negative. Table 3.1 contains the realization of the random variables for these three deterministic analyses.

Table 3.1. Realization of each type of the random variables for deterministic analyses

	load–type variables	resistance–type variables
Analysis 1 (R_{lower})	$\mu_i - f\sigma_i$	$\mu_i + f\sigma_i$
Analysis 2 (R_{mean})	μ_i	μ_i
Analysis 3 (R_{upper})	$\mu_i + f\sigma_i$	$\mu_i - f\sigma_i$

Then a linear regression is used to set up a relation between the f parameter and the calculated responses from the deterministic analyses. Table 3.2 contains the values of f parameter and the response values for which the linear regression should be applied. The established relation between the f parameter and the response values are shown in Figure 3.1, separately for the load-type and resistance-type variables. p and q are the constant value resulted from the linear regression analysis. This established relation is used to define the center of the sampling, with respect to the desired threshold in the performance function.

Table 3.2. Regression variables for the load-type and resistance-type variables

	Regression for load –type variables			Regression for resistance –type variables		
Center of Sampling	$-f$	0	$+f$	$-f$	0	$+f$
Damage-based Response	R_{lower}	R_{mean}	R_{upper}	R_{upper}	R_{mean}	R_{lower}

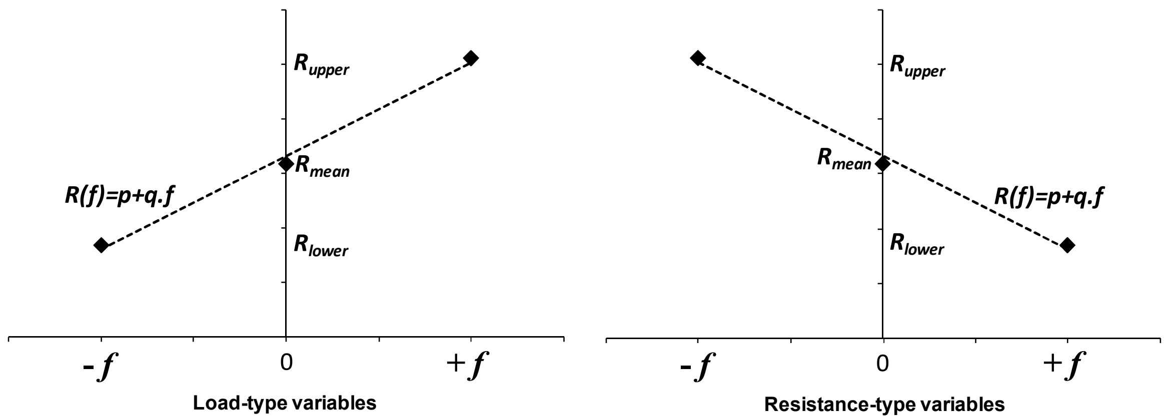

Figure 3.1. Relation between the center point of sampling and the damage-based response of the structure

Since the RSF is intended to approximate the LSF in the portion of the random space that the sampling points are selected; by using the derived relation between the parameter f and the damage-based response of the structure; the sampling points are localized around the point which is close to the LSF. To this end, the distance of the sampling center from the mean value of load-type and resistance-type variable are determined from Eqn. 3.7 and Eqn. 3.8, respectively.

$$f' = (threshold - p) / q \tag{3.7}$$

$$f'' = -(threshold - p) / q \tag{3.8}$$

By placing the sampling around the point in which the load-type variables are equal to $\mu_i + f'\sigma_i$ and the resistance-type variables are equal to $\mu_i + f''\sigma_i$, the sampling points will be generated in a portion of the random space that is close to LSF. Therefore the LSF could be appropriately approximated by the established RSF.

3.3. Sampling Method and the Failure Probability

In this study the random sampling technique has been used in each step of the proposed procedure to generate the sampling points. The established RSF in each step is considered as one of the possible RSFs to approximate the LSF, which is used to calculate the failure probability. The corresponding failure probability is also considered as one of the random estimations of the actual failure probability.

Therefore instead of finding a specific value, n random samples would be calculated for the failure probability. Then these n samples are used to calculate the ultimate failure probability in the system. Since establishing the proposed linear RSF for large number of the random variables is practically feasible, this procedure could be repeated as many times as required to reach the desired accuracy. The ultimate probability of failure is obtained as the mean value of the calculated failure probabilities, up to the current iteration of the procedure. Convergence of the obtained failure probability could be to check by Eqn. 3.9, in each iterative step of the procedure.

$$\left| \frac{p^{c}_{f,ultimate} - p^{p}_{f,ultimate}}{p^{c}_{f}} \right| < \varepsilon \tag{3.9}$$

where $p^{c}_{f,ultimate}$ and $p^{p}_{f,ultimate}$ are the ultimate failure probability which are calculated at current and previous iterations of the procedure, respectively. The accuracy of the calculated ultimate failure probability (ε) is assumed to be 1% in this study. The proposed method will be referred to as random response surface (RRS) method, hereafter.

4. VALIDATION STUDY

In order to implement the proposed method to determine the reliability of the building structures under earthquake excitation, the failure probability of a nine-story steel moment-resisting frame denoted as SAC-9 building is calculated. This building has been designed according to the 1994 UBC seismic design code specifications for Los Angeles, California region. Detailed characteristics of the SAC-9 building can be found in Gupta and Krawinkler (1999). The 2-D moment-resisting frame of this building; used in numerical analyses; is presented in Figure 4.1.

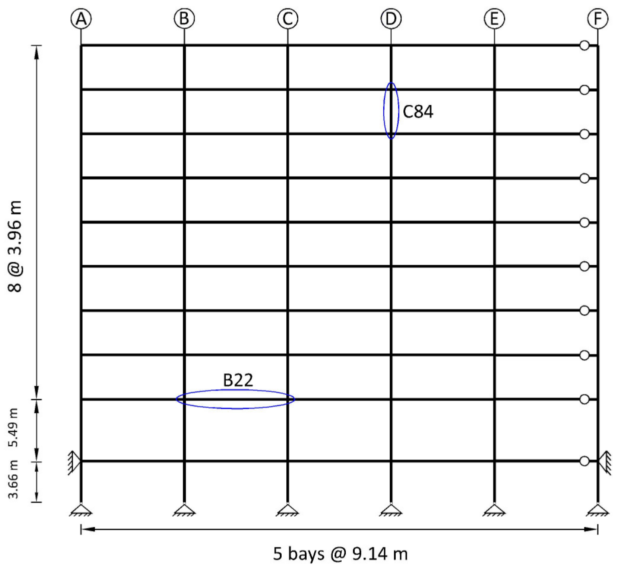

Figure 4.1. Moment-resisting frame of the SAC-9 building

The OpenSees finite element platform (McKenna, Fenves and Scott, 2003) has been used to perform the nonlinear time history analysis of the structure. By assigning a material with random characteristics to the fiber sections of the elements, probabilistic capacity and stiffness of the structural elements has been incorporated into the finite element model. Modulus of the elasticity and the yield stress of the constructional steel have been taken as the probabilistic characteristics of the steel. Also the uncertainty in the seismic mass of the stories and the damping ratio of the first and third modes of vibration (to assign rayleigh damping) are considered in creating the probabilistic model of the structure. The probabilistic parameters considered in reliability analysis of the structure, along with their mean value, coefficient of variation (COV), standard deviation (SD) and distribution type are shown in Table 4.1.

In order to consider the nonstationary stochastic characteristics of the ground motion, the model proposed by Rezaeian and Der Kiureghian has been used to create a filtered white-noise process with both temporal and spectral nonstationary characteristics (Rezaeian and Der Kiureghian, 2008). The created nonstationary process has been used along with random peak ground acceleration (PGA) to model the probabilistic characteristics of the seismic loading.

Table 4.1. Characteristics of the considered random variables

Parameter	Mean	COV (%)	SD	Distribution Type
Yield Stress in Columns	3620.4 *kgf/cm^2*	15%	543.1×10^4	Lognormal
Yield Stress in Beams	2606.1 *kgf/cm^2*	15%	309.9×10^4	Lognormal
Modulus of Elasticity	2.1×10^{10} *kgf/cm^2*	3%	63×10^7	Lognormal
PGA	1*g* *m/sec^2*	30%	0.3 *g*	Lognormal
Story Seismic mass	50410 *kgf-sec^2/m*	20%	10082	Lognormal
Damping Ratio	4%	25%	1%	Lognormal

In order to evaluate the accuracy of the RRS method in estimating the failure probability of the SAC-9 building, MCS technique has been used to obtain the benchmark results. In order to have sufficient number of simulation, 1000 runs of the finite element model of the SAC-9 has been performed. The number of the required simulation to maintain a specific level of accuracy for MCS results can be found through Eqn. 4.1, (Soong and Grigoriu, 1993; Nowak and Collins, 2000).

$$N = \frac{1-p_f}{\mathrm{C.O.V}^2[p_f].(p_f)} \tag{4.1}$$

Where $\mathrm{C.O.V}\ [p_f]$ is the coefficient of variation of the failure probability (p_f), which indicates the accuracy of the MCS results. Based on Eqn. 4.1, for probability of failure equal to 28%, 1000 simulations would result in 5% accuracy. Since the designed building structures are supposed to enter the nonlinear phase and dissipate the applied excitation of the earthquake by plastic deformation, probability of failure in most of the cases is greater than 28% and the 5% accuracy is assured by 1000 simulations.

Here, the proposed procedure has been used to estimate the failure probability of the structural elements of the SAC-9 building. The Damage index of each element in its end sections with higher seismic demand has been calculated through Eqn. 3.1 and Eqn. 3.2. The failure probability in the critical section of the each element is calculated by the damage-based performance function. The results of the numerical analysis are presented only for two of the structural elements, denoted as B22 and C84 in Figure 4.1. The obtained results from the MCS and RRS method are represented in Figure

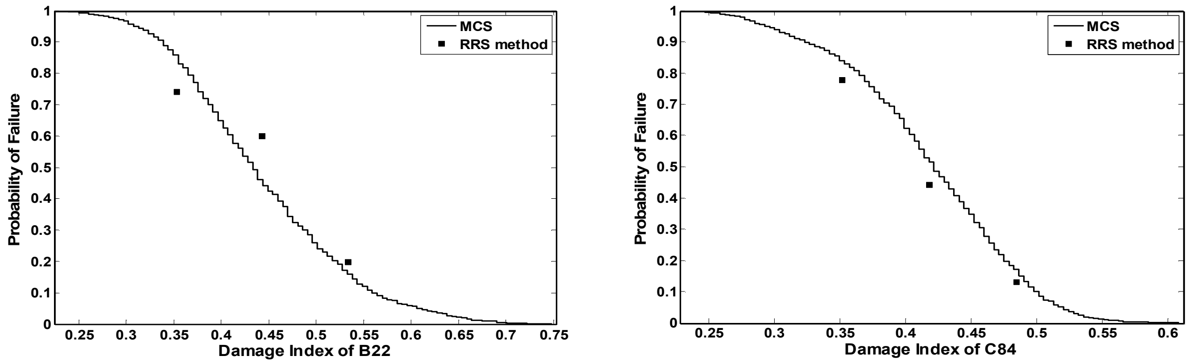

Figure 4.2. Results of the MCS and RRS method for B22 and C84 elements

4.2. The estimations of the RRS method are obtained for the Mean + SD, Mean and Mean − SD damage thresholds, resulted from the MCS. These results indicate that the RRS method could estimate failure probability in the sections of the structural elements with appropriate accuracy. The difference between the estimations of the RRS method and the MCS results in the Mean + SD threshold is less than 5% in most of the structural elements of the SAC-9 building.

5. CONCLUSION

In this study, a new response surface procedure has been proposed to estimate the failure probability of the building structures under earthquake excitation. The emphasis has been placed on reliability analysis of steel moment-resisting frames. The performance of the structure has been evaluated through the calculated damage representatives for the elements of the structure. A linear function of the considered random variables has been used to approximate the limit state function. The sample points used to obtain the coefficients of the response surface function are chosen randomly in each step of the analysis. Since the resulted failure probability is obtained based on the random pairs of sampling points, this procedure should be repeated for several times to assure that the desired accuracy on the estimated failure probability has been achieved. The proposed method has been implemented on a nine-story steel moment-resisting building frame and the results are compared with those from the Monte Carlo simulation (MCS). The results indicate that with significantly less computational effort, the proposed method could be used in probabilistic seismic performance evaluation of the steel moment-resisting frames. The failure probability of a single structural element of the structure could be calculated by the RRS method, based on the corresponding damage measures.

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