Assessment of an Energy-Based Method for Pushover Analyses

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LISBOA 2012

ABSTRACT:
A nonlinear static analysis based on a energy criterion has been presented in the last years by the same authors and represents a consistent tool to understand the response of real structures excited by seismic input. In order to apply this methodology a specific parameter called pseudo-energy has been introduced. In this study the proposed energetic method is assessed making reference to several structural schemes. The responses of 2D r/c frames extracted from real tridimensional r/c framed structures are considered. The results are presented in terms of floor displacement, interstory drift and base shear. For all the studied cases, in order to assess the reliability of the energy-based method, a comparison with the results of the nonlinear static analysis method suggested by EC8 and the time-history dynamic analysis is performed. Aimed at this comparison an error parameter representative of global fitting of the computed response is introduced.

Keywords: Energy-based Design, Pushover, Pseudo-Energy Spectra, Nonlinear Static Analyses

1. INTRODUCTION

Current seismic design is still mostly based on the use of linear static and dynamic analysis methods. In recent years, seismic codes have received – more or less explicitly – new design philosophies, such as Capacitive Design (CD) and Performance-Based Seismic Design (PBSD), shifting the focus from the concept of resistance to that of performance. This has led researchers and designers to the development and adoption of analysis methods able to better assess the actual behavior of earthquake-resistant systems, even in the post-elastic range. Therefore they have extensively developed nonlinear, static and dynamic analysis methods, even in relation to the availability of computational tools suitable for the complexity of the calculations. Currently, the nonlinear time-history analysis represents the most advanced tool for the correct interpretation of seismic structural behavior. However, the difficulties inherent in its use make it remain a method restricted mainly to the field of research or to situations of structural configurations relatively simple or highly significant. On the contrary, Nonlinear Static Analysis (NLSA) is a powerful and relatively easy to use tool to evaluate the response of systems characterized by nonlinear behavior. In literature several methodologies have been proposed to conduct this type of analysis. Various methodologies have also been proposed by various codes (Eurocode8 2003; ATC-40 1994; FEMA-356 2000; FEMA-440 2005). The NLSAs are based on the comparison of the structural capacity with the seismic demand. The capacity of the structure is represented by the pushover curve that, through suitable operations, is transformed into the capacity curve of an equivalent SDOF system. The capacity curve concisely represents the behavior of the actual MDOF system through an equivalent SDOF system and is directly comparable with the response spectrum, which represents the seismic demand. The comparison between system capacity and seismic demand allows to define the performance point that represents the required performance of the equivalent SDOF system in terms of force and displacement. The response of the complex MDOF system can then be computed. The various methods proposed in codes and literature provide for different procedures to arrive at the definition of the performance point.

The assessment of the seismic response associated with nonlinear behaviors has also reawakened interest in assessments based on an energy approach (Anderson & Bertero, 2006). An NLSA method based on the concept of energy seems to be a powerful and reliable tool for the correct evaluation and interpretation of the actual behavior of systems subjected to seismic input. An NLSA method based on an energy criterion has been initially formulated in (Mezzi et al. 2006; Parducci et al. 2006; Mezzi et al. 2007) and then refined by Tomassoli & Mezzi (2010a; 2010b) and represents a consistent tool to evaluate the response of real structures excited by seismic input. In those papers it has been observed how the methodology of NLSA based on an energy approach makes it possible to overcome many of the problems associated with the application procedures of the current methods of NLSA and – similarly – it shows remarkable reliability.

2. PSEUDO-ENERGY RESPONSE SPECTRA

Conventionally the equilibrium point between the capacity of the structure and the seismic demand in an NLSA is defined by the intersection of the capacity response spectrum in the ADRS plan. When the capacitive curve is expressed in terms of energy (work of external forces as a function of the displacement of the energetically equivalent SDOF system), the demand must also be congruently formulated. For this purpose an energy magnitude was introduced, referred to as Pseudo-Energy (Mezzi et al., 2006) computed as the area under the envelope curve of the cyclic response of the non-linear SDOF oscillator, correspondingly to the maximum displacement. The pseudo-energy spectra to be employed as demand spectra can be calculated in various ways (Tomassoli & Mezzi 2010). In the present study the following scenarios were considered. In the first case the spectra (SpeCod) of displacement and pseudo-energy at constant ductility were calculated starting from the elastic displacement spectrum formulated by EC8 for soil type B, suitably transformed by means of the relationships proposed by Newmark and Hall (1982) for the force reduction factor, and by Miranda (2002) to switch from the displacement of the elastic system to the nonlinear one. In the second case, the pseudo-energy spectra at constant ductility were calculated for two groups of seven spectrum-fitting accelerograms. The first group (SpeRec) of accelerograms consisted of natural recorded accelerograms, scaled by a single factor 1.2. The second group (SpeGen) consisted of generated accelerograms. Fig. 1 shows the three families of spectra in the Pseudo-Energy/Displacement/Ductility (PsEDD) space.

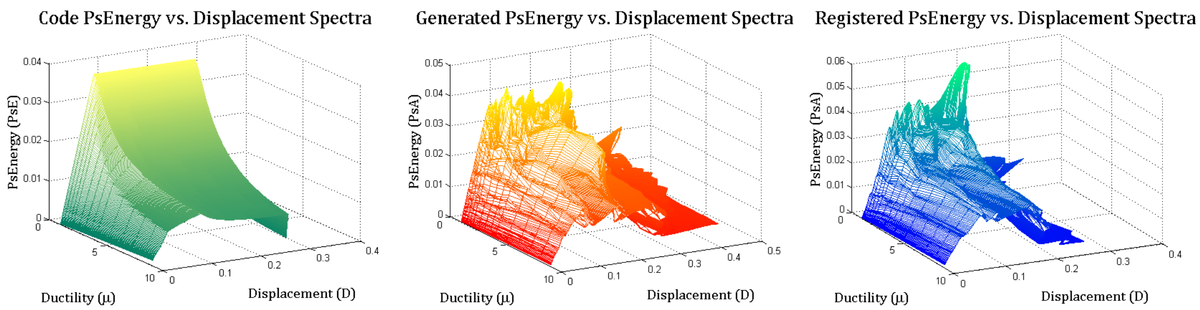


Figure 1. Pseudo-Energy/Displacement/Ductility response spectra. (A) from EC8 elastic spectrum, (b) from spectrum-fitting generated accelerograms; (c) from spectrum-fitting recorded accelerograms

3. ENERGY-BASED NON LINEAR STATIC ANALYSIS

In an energy-based NLSA method, the performance point can be determined directly by the intersection of the capacity curve of the structure, expressed in terms of energy, with the demand curve or, better yet, surface represented by the energy response spectra. The procedure of non-linear static analysis based on an energy approach is briefly illustrated here.

The first step consists of a pushover analysis with a pre-assigned distribution of lateral forces. In the present study, four different distributions of forces are adopted. The modal distribution (Mod) involves the application of floor forces proportional to the shape of the fundamental vibration mode. The uniform distribution (Uni) involves the application – at each floor – of forces proportional to the mass of the floor. The multimodal distribution (Mul) involves the definition of a system of forces on the basis of an equivalence multimodal criterion (Valles & al., 1996). The adaptive distribution (Ada) entails the change in the distribution of the floor forces, as the displacement of the control point grows, as a function of plasticization of the structure. The analyses were carried out using the code Ruaumoko (2005). Knowing, at every step, the lateral plane forces and the associated displacements, the total work done by external actions can be calculated. Dividing this work by the base shear, the displacement of the energy-equivalent SDOF system subjected to a force equal to the base shear is evaluated. It is therefore known the energy pushover curve from which, by dividing the ordinates by the effective mass, the capacity curve in terms of energy can be obtained. It is therefore a matter of defining what effective mass is to be used in the evaluations. In this study, to test the influence of this definition, different hypotheses have been formulated taking as effective mass: (i) the mass of the SDOF equivalent system as determined by the method provided by EC8 (2003) $m^* = \Sigma\Phi_i \cdot m_i$; (ii) the participating mass of the first mode m_I; (iii) the total mass of the structure m_{tot} . In the PsEDD space, the structural capacity is described by a curve that is generally crooked. The intersection of this curve with the surface of the demand spectra defines the performance point. Knowing the performance point and going backwards also all the structural parameters of interest (plane displacements and sliding, base shear, dissipated energy, etc.) are known. Since the equation of the capacity curve is not known, the solution can be found by proceeding in a different way. For this purpose, two procedures have been formulated (Mezzi & al. 2006).

3.1. Procedures A

In the first method the pushover curve is transformed into a bilinear curve as indicated in EC8, and this defines a yielding displacement value $u_{k,y}$. At each point of the curve, characterized by a displacement u_k, a displacement ductility value $\mu_k = u_k/u_{k,y}$ can be associated. In this way it is univocally defined the capacity curve PsEDD. The intersection of this curve with the demand spectrum represents the performance point sought. The critical aspect of the method is represented by the bilinear transformation of the capacitive curve: various assumptions are possible for the behavior (EPP, ESH, etc.) controlling the bi-linearization (Faella et al. 2004). In the present case, reference is made to an EPP model congruent with the non-linear SDOF models used for the calculation of constant ductility spectra. For different cases of non-linear behavior, the demand spectra should also be assessed with congruent behavioral models. Fig. 2a reports a graphical representation of the Procedure A.

3.2. Procedures B

A second approach relies on the fact that only one point of the demand spectrum represents the solution to the problem. This point in the PsEDD space is univocally determined from knowledge of the fundamental period of the structure and the ductility of the system. The definition of these parameters is not immediate and takes place in two stages. The graphical representation of the procedure is shown in Fig. 2b. a preliminary analysis of the structure assuming a perfectly linear elastic behavior is performed. The capacitive curve of the elastic structure lies on the plane at constant ductility $\mu = 1$ and intersects the elastic spectrum at an elastic performance point (E). It is also possible to go backwards to arrive at the associated period value T_1. Starting from the point just found and moving on the surface of the spectrum while maintaining the period T_1 fixed, a curve (CT) with a constant period is constructed. The intersection of this curve with the capacity curve (CP) provides the point of inelastic performance (P) which represents the solution to the problem.

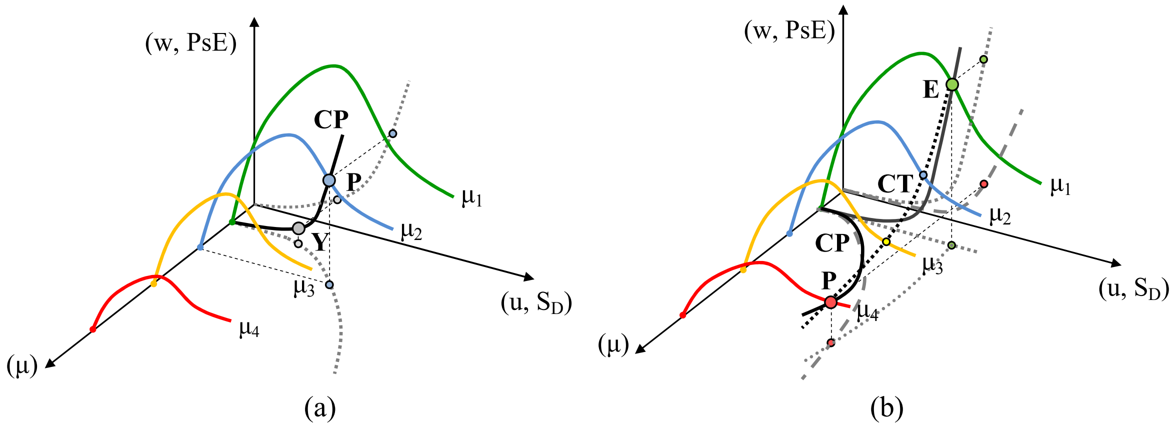


Figure 2. Graphic representation of the energy-based NLSA methods: (a) procedure A; (b) procedure B

4. SAMPLE CASES

To verify the validity of the proposed energy-based NLSA methodology, an analysis is performed on case studies consisting of 2D reinforced concrete frames (Fig. 3), considered extracted from real 3D structures. Each system is designed according to EC8 (2003), for ductility class high (DCH), for a soil type B, for peak ground acceleration on rigid soil $a_g = 0.35 \cdot g$ to which a PGA = $a_g \cdot S = 0.438 \cdot g$ corresponds. The materials are concrete C25/30 and steel with a characteristic yield tension of 450 MPa. The following loads were considered: dead load of structural elements, dead load of floors G_1 = 3.00 kN/m^2, permanent loads of floors G_2 = 2.00 kN/m^2, accidental loads Q_k = 2.00 kN/m^2, roof snow load Q_n = 1.50 kN/m^2. Table 4.1 shows the main characteristics of the five analyzed models (A, B, C, D, E). For each structure the following parameters are given: the number of floors and bays, the value of the total mass m_{tot}, the percentages of equivalent mass m*, and the percentage of mass of the first mode m_I, the value of the fundamental period of vibration T, the relevant coefficient of participation Γ, and the design force reduction factor R.

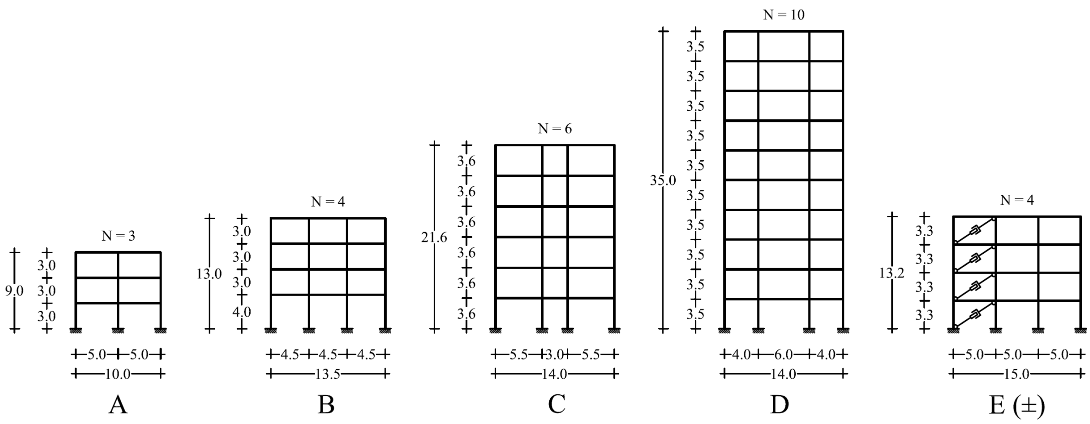


Figure 3. Structural schemes of the analyzed cases

Table 4.1. Main properties of the analyzed model

Case	Stories	Bays	m_{tot} (t)	m* (%)	m_I (%)	Period (s)	Factor Γ	DC	R
A	3	2	83.4	66.5	85.3	0.462	1.282	High	5.85
B	4	3	144.6	64.8	86.6	0.509	1.336	High	5.85
C	6	3	327.6	50.2	72.7	0.624	1.448	High	5.85
D	10	3	602.9	48.7	71.8	1.064	1.476	High	5.85
E (+)	4	3	160.9	68.8	86.4	0.500	1.257	High	5.85
E (–)	4	3	160.9	68.8	86.4	0.500	1.257	High	5.85

In the present study the accuracy of the solution is evaluated as a function of the main classes of control parameters: distribution of lateral forces; assumption of the effective mass; representation of the seismic demand. To assess the validity of the pushover method based on energy, it was assumed, as conventionally exact, the solution of nonlinear dynamic analysis obtained as the average of the maximum values of the response corresponding to two groups of accelerograms, generated (AccGen) and recorded (AccRec). The response parameters taken into consideration are the base shear, the floor displacements, the interstory drifts.

In order to have a synthetic factor to evaluate the error of the solution found with the NLSA methodologies with respect to the one conventionally assumed as exact, a global error ε is defined as

$$\varepsilon = 100 \cdot \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} \left(\frac{P_{push}^{(i)} - P_{time}^{(i)}}{P_{time}^{(i)}} \right)^2} \tag{5.1}$$

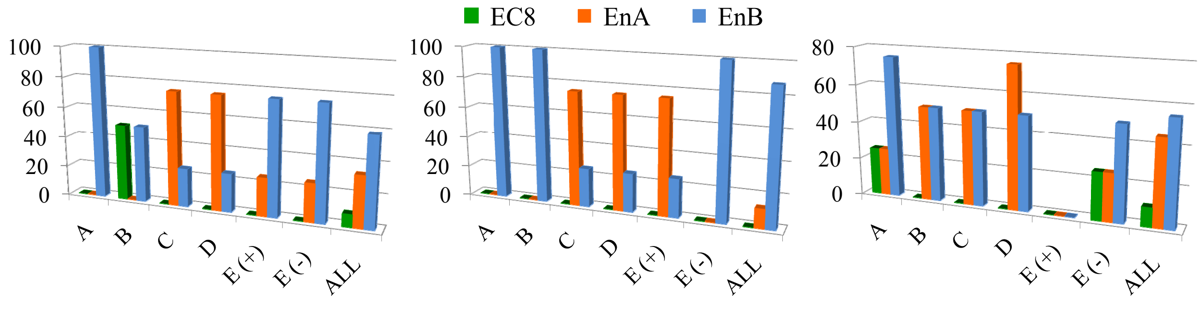
$P_{push}^{(i)}$ and $P_{time}^{(i)}$ are the response values (displacement, sliding plane, base shear) evaluated with the pushover analysis and the time-history analysis respectively and relative to the i-th floor of the building. N is the number of floors in the structure. In the specific case of the base shear the parameter degenerates into a simple percentage difference. This way a total error estimate given by the NLSA method taken into account, normalized according to the number of floors, is got. As a term of comparison, the response is also evaluated using the N2 method proposed by EC8 (2003). In the following the global error is evaluated for displacements (ε_d), interstory drifts (ε_{dr}), base shear (ε_V). Ultimately, each case of analysis is characterized by the variability of the following parameters: demand spectrum (three cases: SpeCod, SpeGen, SpeRec); group of accelerograms for the control of the solution (two cases: AccGen, AccRec); distribution of lateral forces (four cases: Mod, Uni, Ada, Mul); effective structural mass (three cases: m^*, m_I, m_{tot}); procedure for finding the solution (two cases: En-A and En-B). For each case there are 96 scenarios of analysis relative to the energy methods. Moreover, 16 other cases of analysis correspond to the application of the N2 method. In conclusion, for each case study, there are 112 analysis scenarios that, in view of the six models analyzed, provide an investigation database of 672 scenarios.

5. ANALYSIS OF RESULTS

The considered NLSA methodologies are compared with each other. In each analysis scenario, the methodology of success is the one associated with the minimum value of global error previously defined, i.e. the one that best approximates the result of the conventionally exact solution. Table 5.1 shows the percentages of success of the EC8 standard method and the two procedures defined for the energy method with respect to the total. The same results are represented in the graphs in Fig. 4. Specifically, reference is made to four meta-scenarios applicable to each case:
1) NLSA with demand spectra derived from the elastic ones provided by the code (SpeCod) and dynamic analyses with generated accelerograms (AccGen);
2) NLSA with demand spectra derived from the elastic ones provided by the code (SpeCod) and dynamic analyses with recorded accelerograms (AccRec);
3) NLSA using as demand spectra the response spectra of the generated accelerograms (SpeGen) and dynamic analyses with generated accelerograms (AccGen);
4) NLSA using as demand spectra the response spectra of the recorded accelerograms (SpeRec) and dynamic analyses with recorded accelerograms (AccRec).
In each meta-scenario, among the 28 scenarios investigated, only that whose combination of parameters leads to the best approximation is considered.

Table 5.1. Percentage of successes of the NLSA procedures

Case	Displacement Error ε_d			Drift Error ε_{dr}			Base Shear Error ε_d			Meta Scenarios
	EC8-N2	En-A	En-B	EC8-N2	En-A	En-B	EC8-N2	En-A	En-B	
A	0.0	0.0	100.0	0.0	0.0	100.0	25.0	25.0	75.0	4
B	50.0	0.0	50.0	0.0	0.0	100.0	0.0	50.0	50.0	4
C	0.0	75.0	25.0	0.0	75.0	25.0	0.0	50.0	50.0	4
D	0.0	75.0	25.0	0.0	75.0	25.0	0.0	75.0	50.0	4
E (+)	0.0	25.0	75.0	0.0	75.0	25.0	-	-	-	4
E (–)	0.0	25.0	75.0	0.0	0.0	100.0	25.0	25.0	50.0	4
All	8.3	33.3	58.3	0.0	12.5	87.5	10.0	45.0	55.0	24

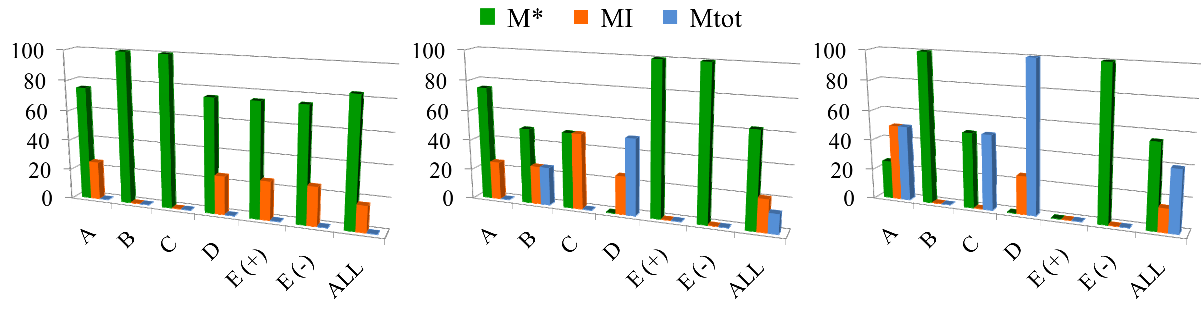


(a) (b) (c)

Figure 4. Percentage of successes of the NLSA procedures: (a) displacement global error; (b) interstory drift global error; (c) base shear global error.

When the comparison is made on the capability to approximate the displacements (Fig. 4a), the application of the En-B methodology shows better results on the entire sample of cases. The same result, with a more pronounced trend, is obtained when using as a benchmark the ability to approximate the interstory drifts. In addition, to approximate the base shear, the same trend is also evidenced, although less clearly. With regard to the single case study, it is not possible to say which is the best method in a statistically significant way, having to refer to only four meta-scenarios of analysis. It appears in general, however, that the methodologies based on an energy approach lead to better results, in the individual cases as in the entire sample, than does the EC8 method.

The analysis of the results continues by examining the influence of the choice of effective mass used in the evaluations. The graphs of Fig. 5 show the percentages of success corresponding to the three different hypotheses of assuming the effective mass: (i) mass of the SDOF equivalent system as determined by the methodology provided by EC8 $m^* = \Sigma\Phi_i{\cdot}m_i$; (ii) participating mass of the first modal shape m_I; (iii) total mass of the structure m_{tot}. Reference is made to the four meta-scenarios of investigation previously defined, each of which comprises 28 analysis scenarios, among which only the one whose combination of parameters leads to the best approximation is considered.



(a) (b) (c)

Figure 5. Percentage of successes associated with the choice of the effective mass in the NLSA procedures: (a) displacement global error, (b) interstory drift global error, (c) base shear global error.

Referring to the displacement global error (Fig. 5a), it is quite clear that the use of the mass m* of the SDOF equivalent system defined by the EC8 methodology practically always leads to the best results, for the individual cases and for the whole sample. This trend, although less pronounced, can also be found by considering the interstory drift global error. For this parameter, even the use of the first mode participating mass can lead to better results, and in case D even the total modal mass. Finally, referring to the global error of the base shear, it is not possible to define a clear trend for the individual models. On the entire sample (ALL), the adoption of equivalent mass m* still gives the greatest number of positive cases, while adoption of the total mass gives a number of appreciable and numerically comparable events. It should be remind, however, that the base shear appears to be a less significant parameter than the displacements and the interstory drifts.

The graphs in Fig. 6 show the success rate corresponding to the previously described four different hypotheses assumed for the distribution of forces along the height of the building: modal distribution; uniform distribution; multimodal distribution; adaptive distribution. Reference is made to the four meta-scenarios of investigation previously defined, each of which comprises 28 analysis scenarios, among which only the one whose combination of parameters leads to the best approximation is considered.

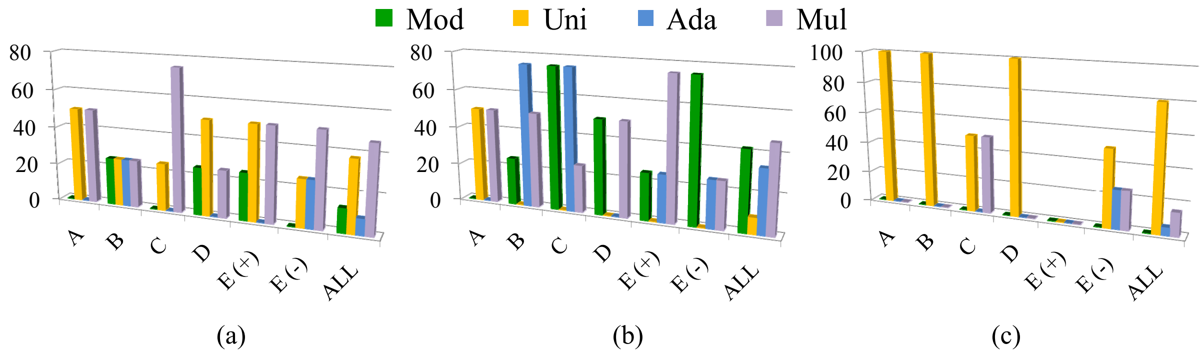


Figure 6. Percentage of successes associated with the choice of the lateral force distribution in the NLSA procedures: (a) displacement global error; (b) interstory drift global error; (c) base shear total error.

In all the considered situations, a clear tendency to define an optimal distribution of forces cannot be found, neither by analyzing the individual models nor by considering the whole sample. Making reference to the total displacement error of the entire sample, it can be observed that the assumption of the multimodal distribution gives the highest number of successes, even if the result associated with using the uniform distribution is comparable. This performance remains substantially true, even for individual cases. Considering the global error of interstory drift, this trend is lost. Using multimodal distribution still leads to good results, in the majority of cases (A, D, E^+) and in the whole sample. However, there is a collapse of the uniform distribution performance – in the entire sample – which is the one characterized by the minimum number of positive events. On the contrary, by making the comparison with reference to the global error of the base shear, it is precisely the uniform distribution for which there is always the largest number of positive events, both in the individual cases and in the whole sample. Note that for model E^+ it was not possible, in terms of base shear, to determine a preferential distribution of lateral forces. This problem is attributable to the constant threshold of plasticization of the pushover curve, which tends to be independent of the type of distribution of lateral forces. Ultimately, for the purposes of a correct estimate of the base shear it would be advisable to adopt the uniform distribution of lateral forces, whereas for the estimation of displacements and interstory drifts, clear indications cannot be formulated. For these response parameters, considering the whole sample, the multimodal distribution (Mul) is the one that leads to a greater number of positive evaluations in any event.

It should be noted that if the assumption of a value of one of the parameters of influence, taken individually, leads to the occurrence of the greater number of positive events, it does not necessarily mean that it is absolutely the best choice when considering the combined effect of all the parameters.

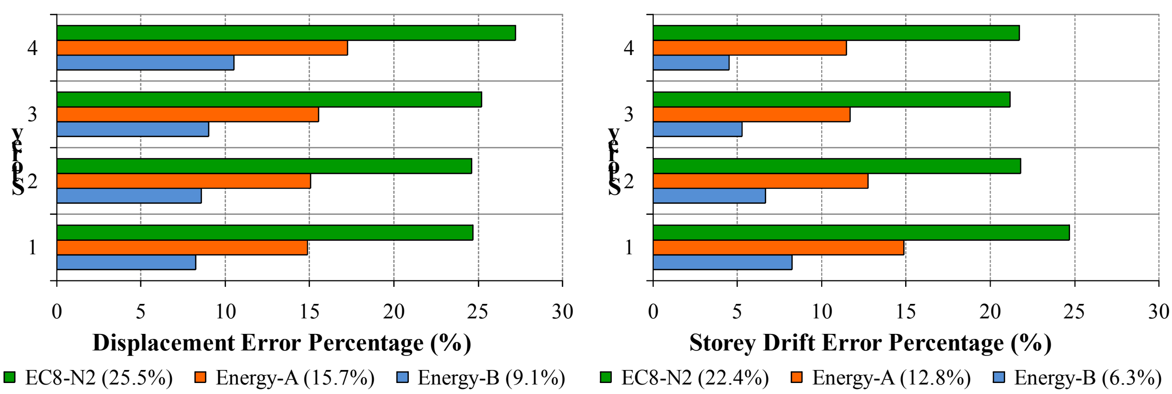
It is therefore necessary to consider the number of positive events related to the combination of the three parameters (lateral forces, effective mass, procedure). Table 5.2 shows the combinations of the three factors of influence in decreasing order of successes on a total of 24 scenarios, with reference to the three control parameters considered, i.e. global errors of displacements, interstory drifts and base shear.

Table 5.2. Percentage of successes of the best performing combination of the influence parameters of the NLSA

Displacement Error ε_d				Drift Error ε_{dr}				Base Shear Error ε_d			
Force	Meth.	Mass	%	Force	Meth.	Mass	%	Force	Meth.	Mass	%
Uni	EnB	m*	25.0	Mul	EnA	m*	25.0	Uni	EnB	m*	30.0
Mul	EnA	m*	20.8	Mod	EnB	m*	20.8	Uni	EnB	m_{tot}	20.0
Mul	EnB	m*	12.5	Ada	EnA	m*	12.5	Combinations with Percentage <10%			
Mul	EnB	m_1	12.5	Ada	EnA	m_1	12.5				
Combinations with Percentage <10%				Mul	EnA	m_1	12.5				

Critical examination of the table shows that in order to obtain the best possible estimate of displacements of the floor, the multimodal distribution of forces seems to be the most appropriate, despite the fact that the occurrence of the greatest number of positive events is attributable to the uniform one. However, uniform distribution appears to be the best for the purposes of estimating the base shear value. Considering the overall error of interstory drift, the multimodal distribution of forces still seems to be the most suitable. The number of positive events associated with the Modal and Adaptive distribution is appreciable and, since the number of floors is not high, they tend to give the same approximations. On the contrary, it seems quite clear that the most suitable choice of the effective mass to use in the analysis is that of the equivalent SDOF system, as proposed by EC8. Notwithstanding the best estimate of the solution associated with an NLSA methodology based on energy, it is not possible to define which of the two application procedures formulated is the best. Procedure B leads to good estimates both in terms of floor displacement and base shear. The result is inverted, making procedure A more reliable when referring to the global error of the interstory drift.

In order to better emphasize the benefits obtained from the use of NLSA methods based on energy approaches, the results of some analyses extracted from singular cases of the performed investigations are presented. In particular, the following scenarios are referred: NLSA solutions based on the use of seismic demand spectra deduced by the spectrum-fitting generated accelerograms; dynamic analyses performed with generated accelerograms; multimodal distribution of forces; effective mass corresponding to the equivalent SDOF system. Fig. 7 shows the global errors, with reference to model B, measured for the displacements and interstory drifts respectively. The legend also indicates, in parentheses, the value of the global errors associated with the various methods. Fig. 8 and Fig. 9 show the same results obtained for models C and D respectively.



(a) (b)

Figure 7. Case B. Performances of the different NLSA methods: (a) global error of displacements; (b) global error of interstory drifts

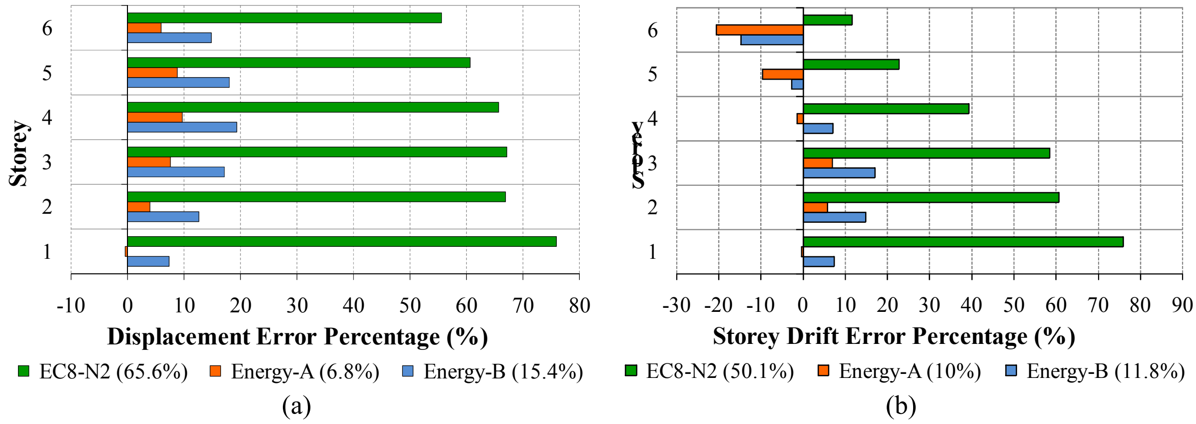


Figure 8. Case C. Performances of the different NLSA methods: (a) global error of displacements; (b) global error of interstory drifts

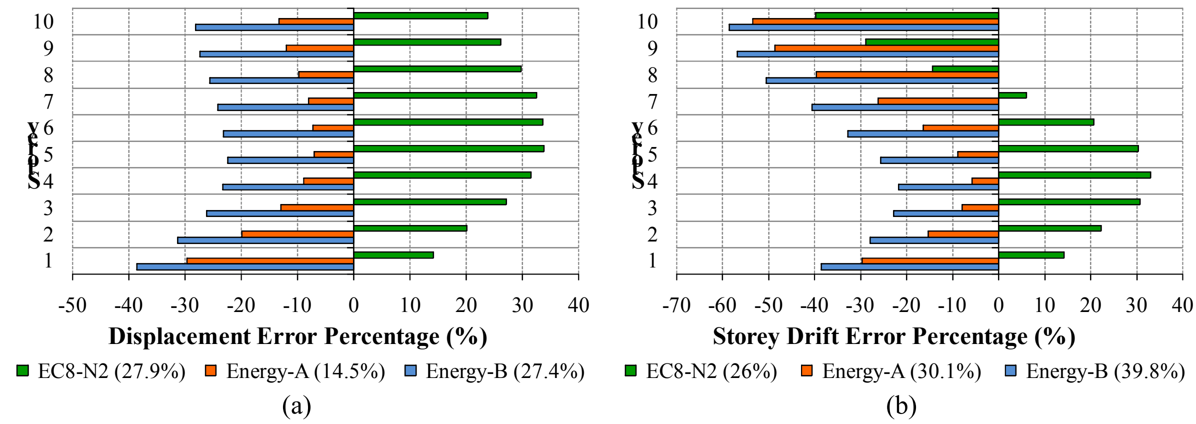


Figure 9. Case D. Performances of the different NLSA methods: (a) global error of displacement; (b) global error of interstory drift

As clearly shown by comparison with the errors obtained depending on the different methodologies, the nonlinear static analysis based on energy has always provided a good approximation of the solution given by the time history analysis, and in general better than that obtained by the N2 method of EC8 (2003). It is observed that as the number of floors increases, the estimate of the solution tends to be less accurate for all the methods. This result can be reasonably attributed to the inherent limitation of the pushover analysis that, at higher levels, fails to discern the effects of the contribution of the higher modes. The problem could be overcome by adopting more appropriate distributions of lateral forces (Fajfar & Fischinger 1989; Fajifar & Gaspersic 1996; Freeman 1978) or alternative methodologies of pushover taking into account all modal contributions (Chopra & Goel 2002).

6. CONCLUSIONS

The use of NLSA methods based on energy criteria for the seismic analysis of the structures represents – first of all – the application of a principle of fundamental physical significance for which the input energy is equal to the dissipative capacity of the structure. The adoption of the energy-based NLSA method requires to express both the seismic capacity and demand in terms of energy. The introduction of an energy magnitude defined pseudo-energy, allows to represent the demand in terms of pseudo-energy spectra at constant ductility that can be effectively used in an energy-based NLSA. The sample of the case studies analyzed shows that the energy method leads to assessments of the response that are notably better than those made in the methods proposed by the regulations. But there are still many aspects connected to the choice of the various factors that influence the accuracy of the results of the

NLSA methods. The choice of the distribution of lateral forces to adopt in the pushover analysis is still a very delicate aspect and it is impossible to define an optimal distribution. Regarding the effective mass to be considered in the analyses, the mass of the SDOF equivalent system proposed in the EC8 methodology appears to be the optimal choice.

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