

# Calculation Methods for Inter-Story Drifts of Building Structures

**J. Zhou & G.B. Bu**

*School of Civil Engineering and Mechanics, Xiangtan University, Xiangtan 411105, China*

**K.N. Li**

*Canny Seismic-Control Technology, Si-Park, Wuxi 214000, China*



## SUMMARY:

Inter-story drift is an important indicator of structural behaviour in performance-based seismic analysis. This paper describes the definition and contents of the inter-story drift. Existing calculation methods for harmful and harmless inter-story drift are revisited first. Their advantages and shortcomings are compared. Finally, the generalized shear deformation of region, which seems to be suitable for structures predominated by flexural-shear deformation, is introduced and evaluated.

*Keywords: Seismic Analysis, Harmful Inter-story Drift, Generalized Shear Deformation, Building Structure*

## 1. INTRODUCTION

Inter-story drift is one of the particularly useful engineering response quantity and indicator of structural performance, especially for high-rise buildings. However, many researchers and engineers do not notice the difference between inter-story drift and harmful inter-story drift. Also, few programmers have considered the harmful inter-story drift in their structural analysis procedure. So they may unreasonably use the inter-story drift as unique standard for structural behaviour judgment, which may eventually lead to unacceptable results and relatively conservative conclusions. As a result, considerable work has been done to investigate on the calculation methods and reasonable limit values of the inter-story drift for different structure types (Zhen and Xie, 2010; Deng et al., 2008; Wei and Wang, 2006; Xu, 2005; Xin et al., 2000; Zhang et al., 1999). However, these methods focus on calculating the harmful displacement of structures predominated by global flexural or flexural shear deformation, such as shear wall and frame-shear wall structures. In addition, there are still no provisions in current codes in China about a reasonable and formal calculation method for the harmful displacement (GB50011, 2010). Therefore, it's urgent for researches to further study the inter-story deformation and put forward a simple and effective approach on harmful inter-story drift calculation.

## 2. INTER-STORY DRIFT

Current codes (GB50011, 2010) in China define the inter-story drift of building structures as relative translational displacement between two consecutive floors. It consists of the following three parts.

- (1) Inter-story shear drift induced by vertical members;
- (2) Inter-story flexural drift induced by vertical members in calculated story  $\Delta u_b$ ;
- (3) Inter-story flexural drift induced by vertical members in inferior story  $\Delta u'_b$ .

So the inter-story drift of  $i$ -th story can be denoted as

$$\Delta u_i = (\Delta u_{si} + \Delta u_{bi}) + \Delta u'_{si} \quad (2.1)$$

where  $i$  is the story number. Note that the torsion rotation of story is not considered in this study.

The inter-story drift is divided into two parts including harmful and harmless drift by structural researchers in China (Xu, 2005; Zhang et al., 1999). The harmful inter-story drift refers to the drift that can directly make damages to the calculated story and is induced by deformation of the vertical members of the calculated story. So it can be obtained from Eqn. 2.1, and written as

$$\Delta \tilde{u}_i = \Delta u_{si} + \Delta u_{bi} \quad (2.2)$$

The harmless inter-story drift is caused by the flexural deformation of vertical members in inferior story, or the rotation of the inferior floor. It is considered to be harmless to the calculated story as no internal drift but rigid body deformation occurs. So the harmless inter-story drift of  $i$ -th story equals to  $\Delta u_b'$ . Thus, the relationships between harmful and harmless inter-story drift can also be expressed as

$$\Delta \tilde{u}_i = \Delta u_i - \Delta u_b' \quad (2.3)$$

For shear-type buildings, such as frame structures, the displacement induced by floor rotation is much smaller than translational displacement, so the harmful inter-story drift is usually ignored in engineering analysis nowadays. However, with the increment of height, this rotation effect may be rise up and can't be ignored, which need to be further studied in future work. In addition, the inferior floor rotation will induce large rigid body displacement for shear wall and frame-shear wall structures, based on plane-section of wall bending. The following sections focus on evaluating the current methods for inter-story deformation.

### 3. CALCULATION METHODS FOR THE HARMFUL INTER-STORY DRIFT

Four current methods for calculating harmful inter-story drift are introduced in this paper. It should be noted that these methods are just suitable for shear wall structures, predominated by global flexural deformations and based on plane-section assumption. The following sections describe these methods and compare their advantages as well as shortcomings.

#### 3.1. Secant Method

One of the first formal provisions for calculating harmful inter-story drift of buildings is the Guangdong provincial additional regulations (JGJ3, 2005). In this code, the harmful inter-story drift can be calculated by

$$\Delta \tilde{u}_i = u_i - u_{i-1} - \theta_{i-1} h_i = \Delta u_i - \theta_{i-1} h_i \quad (3.1)$$

where  $\Delta \tilde{u}_i$  denotes harmful inter-story drift of the  $i$ -th story,  $u_i$  and  $u_{i-1}$  refer to the  $i$ -th and  $i-1$ -th story horizontal displacement,  $\theta_{i-1}$  is the horizontal displacement angle of the  $i-1$ -th story,  $h_i$  is the  $i$ -th story height. This method essentially shows the idea that the harmless inter-story drift, induced by the inferior floor rotation (rigid body rotation), should be subtracted from the total deformation. However, as can be seen from Fig. 3.1, this method assumes that the inferior floor rotation angle is equal to  $\theta_{i-1}$ , the secant angle of the horizontal deformation of  $i-1$ -th story. But in actual practice, they may be unequal. So we call it secant method. As a result, it may bring some errors and obtain relatively larger harmful inter-story drift theoretically, which should be evaluated in future work.

#### 3.2. Improved Secant Method

Based on the same principles of the secant method, Deng (2008) derived a simple expression of the harmful inter-story drift based on recursion method, denotes as

$$\Delta \tilde{u}_i = \Delta u_i - \Delta u_{i-1} \quad (3.2)$$

Actually, we can directly obtain Eqn. 3.2 assuming  $h_{i-1}$  equals to  $h_i$  in Eqn. 3.1. By this assumption, Eqn. 3.2 can be also derived easily from Xu's study (Xu, 2005), in which harmful inter-story drift angle is defined as

$$\tilde{\theta}_i = \theta_i - \theta_{i-1} = \frac{u_i - u_{i-1}}{h_i} - \frac{u_{i-1} - u_{i-2}}{h_{i-1}} = \frac{\Delta u_i}{h_i} - \frac{\Delta u_{i-1}}{h_{i-1}} \quad (3.3)$$

This approach provides an easy and convenient way to calculate harmful displacement. However, it actually is equivalent to the first method based on the same principles and assumptions. So it will produce the same errors as the first method.

### 3.3. Tangent Method

To reduce these errors, the tangent method is put forward. Just as the name implies, this method replaces  $\theta_{i-1}$  in Eqn. 3.1 by  $\bar{\theta}_i$ , the average value of the tangent angle at the bottom end of all vertical members in the whole story (see Fig. 3.2). By comparison, the tangent angle is much approach to the practical floor rotation than secant angle mentioned above. However, the tangent angle is a mean value, which is averaged by deformation of vertical structural members. It proves in practical engineering that the inter-story drifts are sometimes predominated by the deformation of critical member, so it is still need to be further studied for the relationships between the global inter-story drift and local member deformation.

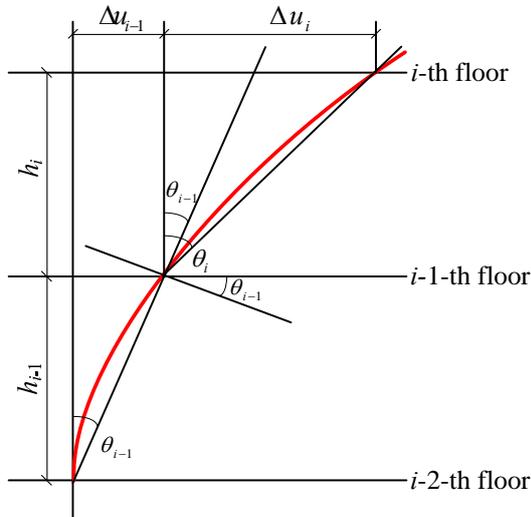


Figure 3.1. The secant method principle

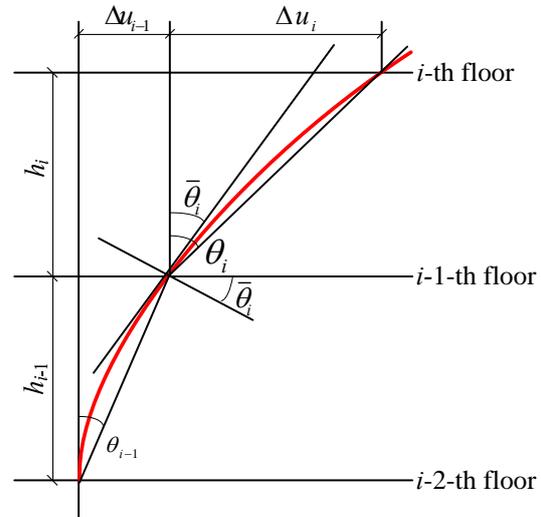


Figure 3.2. The tangent method principle

### 3.4. Fixing Floor Method

When calculating  $\Delta \tilde{u}_i$ , fix the  $i-1$ -th floor, by limiting all the rotation and translational degrees at the bottom ends of all vertical members, and apply the same static loads at floors beyond the  $i-1$ -th floor as the case without fixing floor. As shown in Fig. 3.3,  $\theta_{i-1}$  equals to zero. It means that there is no harmful drift induced by the rotation of inferior floor, and the total calculated deformation is considered to be harmful to structures. So we can easily obtain

$$\Delta \tilde{u}_i = \Delta u_i \quad (3.4)$$

This method provides a direct way to calculate harmful inter-story drift. However, there are such shortcomings as follows. Firstly, this method consumes too much time to get the harmful drift at each story, as it needs to build  $n$  structural models and fix an identical floor at each case. It seems to be too complicated in practical analysis. Secondly, the boundary conditions, which are changed when fixing a floor, are different from the origin model and may make some errors. Finally, this method is just suitable for the static analysis cases, because in dynamic analysis, it becomes impossible to predict the dynamic load carried by each floor due to the randomness of earthquakes.

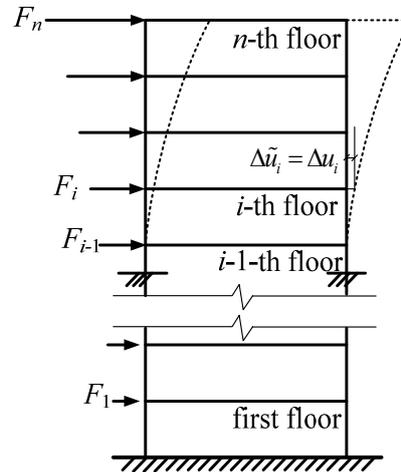


Figure 3.3. The fixing floor method principle

### 3.5. Numerical Example

In order to evaluate the above methods, a shear wall model is analysed by structural static/dynamic analysis program CANNY (Li, 2010) and the harmful drifts are calculated. For brevity, the shear wall structure has been established as a simplified plane model (Fig. 3.4), and the wall elements are modelled based on prescribed moment-rotation relationships. The harmful drifts obtained from different methods are compared in Fig. 3.5. As shown in the figure, the harmful inter-story drift angle is much smaller than the total inter-story drift angle, with larger deformation in lower stories and smaller deformation in higher stories. The harmful drifts calculated by the tangent method are much approach to that by the fixing floor method, which is considered as a relatively accurate but much complicated approach. However, the harmful drifts calculated by the secant method are larger in comparison to another two methods, especially in lower stories.

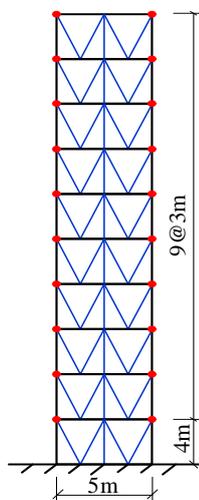


Figure 3.4. Shear wall model

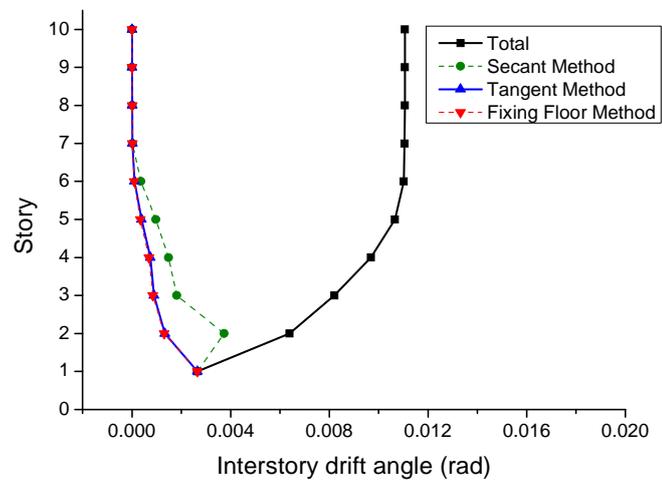


Figure 3.5. Harmful drift angle comparison

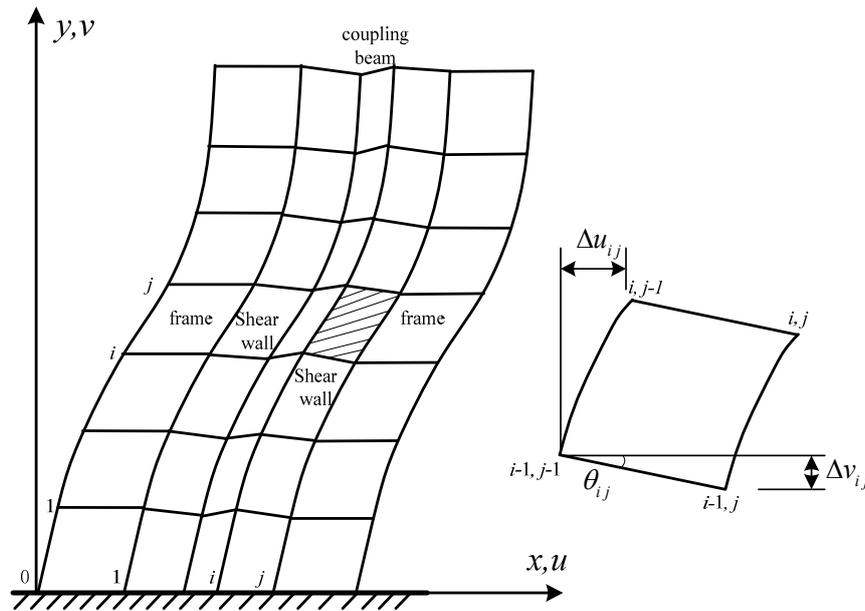
#### 4. GENERALIZED SHEAR DEFORMATION OF REGION

Since the high-rise buildings, especially for frame-shear structures, are usually predominated by flexural-shear deformation rather than pure flexural or shear deformation and may not be strictly confirm to the plane-section assumption, a new parameter named generalized shear deformation was put forward by Xin (2000). In his study, the structure is divided into three different regions of wall, frame and coupling beam as shown in Fig. 4.1, assuming each region has different generalized shear deformation as

$$\gamma_{ij} = \frac{\Delta u_{ij}}{h_i} - \frac{\Delta v_{ij}}{l_{ij}} \quad (4.1)$$

where  $i$  is the story number,  $j$  is the member number,  $l_{ij}$  is the span of  $j$ -th member,  $\Delta u_{ij}$  and  $\Delta v_{ij}$  are the horizontal and vertical displacement of the  $j$ -th member, respectively. Considering the incompatibility of different members, this method offers a way to calculate the harmless displacement induced by each region. The rigid body rotation of  $i$ - $j$  region is defined as the ratio of vertical displacement difference of members and the corresponding span, denoted as

$$\theta_{ij} = \frac{\Delta v_{ij}}{l_{ij}} \quad (4.2)$$



**Figure 4.1.** Definition of generalized shear deformation of region

To demonstrate the general shear deformation of different regions, a simplified plane shear-wall model is established (Fig. 4.2) and pushover analyse has been conducted. The general shear drifts in terms of wall, coupling beam, and frame regions are presented in Fig. 4.3. As shown in the figure, the general shear drifts in shear wall region are much smaller than total inter-story drifts. However, the general shear drifts in frame region are much approach to total inter-story drifts, and that in coupling beam region are somehow larger than the total inter-story drifts.

This general shear deformation of region is based on the rigid floor assumption in the vertical direction. However, in frame members, if the node of vertical member has no rotation but only vertical displacement, then it results in no floor rotation but only vertical shear deformation because the floor

slab is thin and usually treated as no stiffness in the direction of out floor plane, as shown in Fig. 4.4. In this case, such definition is not acceptable. Therefore, the CANNY program author (Li, 2010) insists that the floor rotation must be accompanied by the rotation of the structural nodes with relevant effective connection elements and has included it in the CANNY program, which is much approach to the practical analysis.

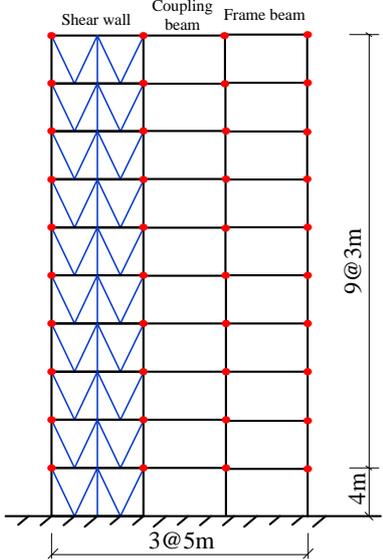


Figure 4.2. Frame hear wall model

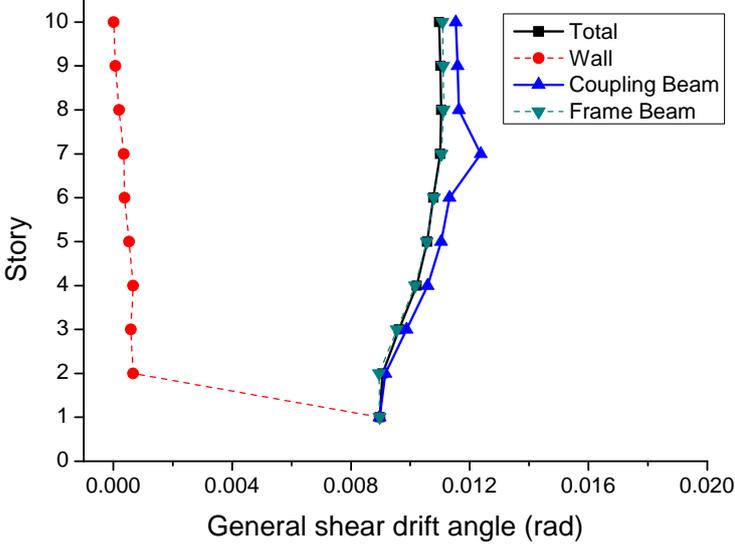


Figure 4.3. Harmful drift angle comparison

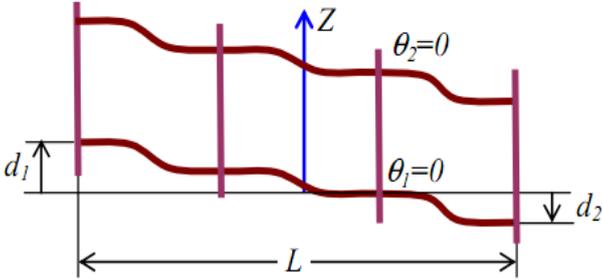


Figure 4.4. Deformation induced by node vertical displacement in frame structure

In wall structure, through plane-section assumption of the wall bending, the node vertical displacement seems to be relevant to the inter-story displacement (see Fig. 4.5). However, in this case the node rotation can also represent the effect of the vertical displacement on the inter-story displacement.

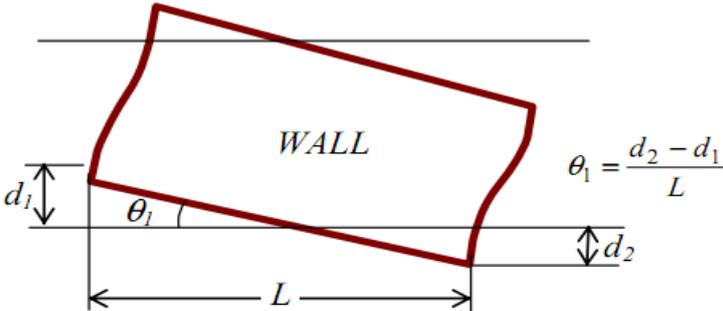


Figure 4.5. Deformation induced by node vertical displacement in wall structure

## 5. CONCLUSION

This paper clarifies the contents of inter-story drift, and the definition of the harmful and harmless inter-story drift. Four current methods for calculating harmful inter-story drift are presented. Their advantages and shortcomings are compared.

The secant method shows the idea that the harmless inter-story drift should be subtracted from total inter-story drift, but it may make some errors. By comparison, the improved secant method provides a relatively simple expression. However, they are based on the same principles and assumptions. To avoid the errors introduced by secant angle, the tangent method seems to be much reasonable, but the relationship between the global inter-story drift and local vertical member deformation needs to be studied. The fourth approach refers to fixing floor method, can directly calculate the harmful inter-story drift, but it is difficult for engineering application and impossible for dynamic analysis.

All these methods focus on the structure types predominated by flexural deformation. Thus, a new parameter defined as generalized shear deformation of region is introduced, especially for structure predominated by flexural-shear deformation. Finally, the advantages and unreasonableness are evaluated. It demonstrates that the definition of region rotation is not comprehensive when calculating the region harmful displacement.

## ACKNOWLEDGEMENTS

The authors acknowledge the financial support by Hunan Provincial Innovation Foundation for Postgraduate (CX2010B270), State Key Lab of Subtropical Building Science, South China University of Technology (2010KB13).

## REFERENCES

- Deng, M.K., Liang, X.W., Wang, Q.L. and Cai, D.Y. (2008). Research on Calculating Methods of Story Drift for Reinforced Concrete Shear Wall Structures. *Journal of Earthquake Engineering and Engineering Vibration* **28:3**, 95-103.
- GB50011. (2010). Code for Seismic Design of Buildings, China Architecture & Building Press, Beijing, China.
- JGJ3. (2005). Additional regulations of technical specification for concrete structures of tall building, China Architecture & Building Press, Beijing, China.
- Li, K.N. (2010). Three-dimensional nonlinear static/dynamic structural analysis computer program-user manual and data-input manual of Canny, Vancouver. <http://members.shaw.ca/CannyNAS/>.
- Wei, L. and Wang, S. (2006). Discussion on Inter-story Displacement Angle of High-rise Building. *Building Structure* **36:S1**, 49-55.
- Xin, D.K. et al. (2000). Limit Value of Interstory Drift of Reinforced Concrete super Highrise Buildings. *Journal of Earthquake Engineering and Engineering Vibration* **21:3**, 10-15.
- Xu, P.F. (2005). Design of Complex High-Rise Building Structures, China Architecture & Building Press, Beijing, China.
- Zhang, H., Yang, L.P. and Zhou, W.X. (1999). Discussion on Story Drift Limit of Super High-rise Reinforced Concrete Buildings. *Journal of Building Structures* **20:3**, 8-14.
- Zheng, J.D. and Xie, C. (2010). Harmful Displacement Calculation and Analysis of a High-rise Building with High Height-width Ratio. *Building Structure* **40:12**, 84-85.