

# Pushover Analysis for Unreinforced Masonry Buildings subjected to Torsion – Comparison of Numerical and Analytical Calculations



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## SUMMARY:

In this paper the seismic safety of an existing unreinforced masonry residential building subjected to torsion is investigated. A building with rigid diaphragm was chosen and two different displacement based procedures were carried out: a software calculation with the Finite Macro Element Program 3muri and an analytical calculation according to the Documentation SIA D 0237 (2010). The results are then presented and compared. The differences in the obtained pushover-curves are shown and the main influencing parameters and modelling differences are discussed. A special focus is placed on the influence of torsional behaviour and its treatment in the analytical calculation. Thus, the paper gives practical engineers useful information regarding building modelling and a better understanding of influence of parameters depending on the procedure used.

*Keywords: unreinforced masonry, torsion, displacement based, pushover curve*

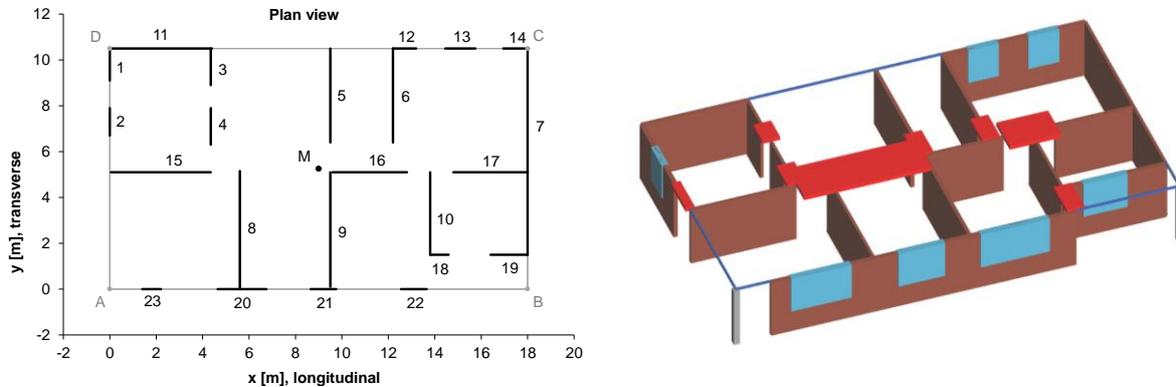
## 1. INTRODUCTION

Seismic verification of existing buildings is a challenging task because, especially in regions with moderate seismicity like Switzerland, where the majority of the building stock was built without seismic considerations. A large portion of existing residential buildings in Switzerland are unreinforced masonry with reinforced concrete slabs (rigid diaphragms). For retrofit purposes more and more of these buildings are checked for seismic safety. In many cases the assessment is done by practical structural engineers involved in the retrofit project, who usually possess limited knowledge and experience in earthquake engineering. Commonly used force based methods like the equivalent force and the response spectra methods are too conservative and would lead to high retrofitting costs. Therefore many engineers are using more sophisticated software tools. Unfortunately the verification of software results is difficult due to the multitude of calculation settings and options, the lack of useful intermediate results, and the general non transparent nature of the software. The Swiss Society of Engineers and Architects (SIA) has published a Documentation SIA D 0237 (2010) describing a displacement based procedure (originally developed at the ETH Zurich) to verify the seismic safety of these masonry buildings. One of the main advantages of this procedure is the possibility to carry out hand calculations.

## 2. DESCRIPTION OF THE EXAMINED BUILDING AND MODELLING ASSUMPTIONS

The purpose of the current study is to compare results obtained from an analytical calculation according to Documentation SIA D 0237 (2010) with the results of the numerical calculation using the commercial version no. 5.0.211 of Finite Macro Element Program 3muri. A typical Swiss unreinforced masonry building, built in 1968, was chosen for this study. The idealized plan view of the building and the used wall numbers are shown in Fig. 2.1. All four floors above ground level have identical plan view and therewith the bearing walls are vertically continuous. In order to simplify matters, the basement floor was not considered in this study. The geometry of the building is defined

by its 18 m length, 10.5 m width and a total high of 10.6 m with constant floor heights of 2.65 m. The length and thickness of the unreinforced masonry walls are given in Table 3.2. The parapets below the windows are 1.02 m high (including the slab thickness) and 0.10 m thick. The lintels above the doors are about 0.3 m high and of the same thickness as the adjacent walls. It is neglected in the following calculations as seen in the rendered view in Fig. 2.1. The reinforced concrete slabs are 16 cm thick.



**Figure 2.1.** Idealized plan view of the building, numbering of the walls and corners; rendered view of one floor

The masonry walls are made of hollow clay bricks, and resemble today's Swiss masonry type MB with following properties: design compressive strength  $f_{xd} = 3.50 \text{ N/mm}^2$  and  $f_{yd} = 1.58 \text{ N/mm}^2$ , characteristic Elastic modulus  $E_{xk} = 7'000 \text{ N/mm}^2$  and Shear modulus  $G_{xk} = 2'800 \text{ N/mm}^2$  (Poisson's ratio  $\nu = 0.25$ ). The concrete floor slabs are consistent with today's concrete type of C25/30. The total mass of the building is 660 t (the first three floors weigh 170 t, and the fourth floor weighs 150 t).

The normal forces for the analytical calculations acting on different walls are estimated using load areas. The normal forces  $N_{xd}$  acting on the 1<sup>st</sup> Floor are shown in Table 3.2. The normal forces are considered to be constant for analytical calculation, whereas the normal forces change depending on the actual deformation state while carrying out numerical calculation using the program 3muri.

### 3. ANALYTICAL CALCULATION USING DISPLACEMENT BASED METHOD

#### 3.1 Description of the method used

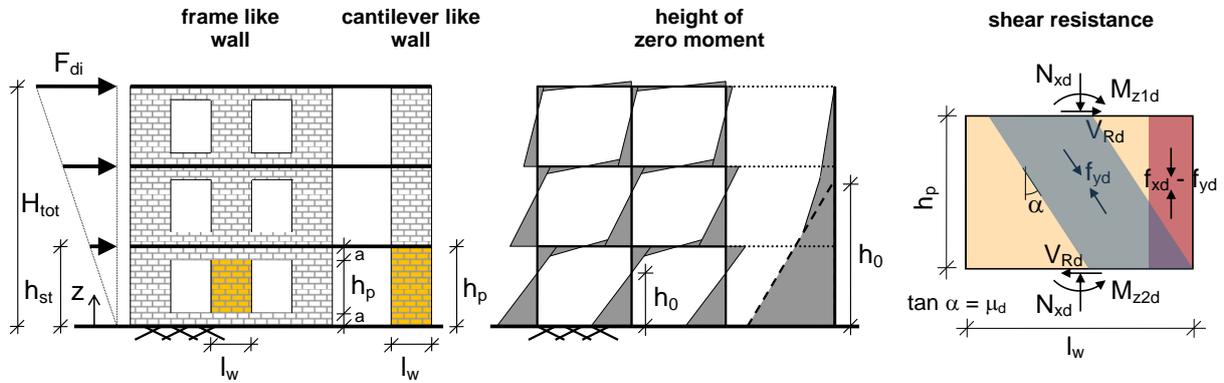
The principals of the displacement based method (DBM) applied here are defined by the “capacity spectrum method based on inelastic demand spectra” also called N2-Method developed by Fajfar P. (1999). The Swiss Society of Engineers and Architects (SIA) published the pre-standard SIA 2018 (2004) for calculating the earthquake safety of buildings based on the abovementioned method. A detailed design procedure for masonry buildings was developed by Lang (2002). It is described and modified in the documentation SIA D 0237 (2010) “Examination of masonry buildings with regards to earthquakes”. The Swiss “standards” describe the earthquake safety of a building by means of a compliance factor  $\alpha_{eff}$ . It is defined by the ratio between the seismic resistance (displacement capacity using the DBM) and the acting seismic load (displacement demand using the DBM). The seismic resistance is defined by the capacity curve or so called “pushover curve” in this study. The applicability of the DBM described in the SIA D 0237 is limited to the masonry buildings retaining the vertical continuity of the earthquake resisting masonry walls. The common Swiss masonry buildings usually fulfil this criterion.

#### 3.2 Developing the pushover curve of a single wall

The simplified linear elastic ideal plastic pushover curve of a multi-story masonry wall according to SIA D 0237 is defined by three values: the shear resistance  $V_{Rd}$  in the lowest floor (base shear), the yielding displacement  $\Delta_y$  and the ultimate displacement  $\Delta_u$ , both at the top of the wall.

Before a pushover curve of a multi-story masonry wall can be developed, two main influences (besides the material and geometrical properties) that define the distribution of forces and moments over the height of the multi-story wall have to be investigated. The first consideration is the influence of the height of zero moment  $h_0$  in the wall, which depends on if and how the different multi-story walls of the building are coupled by means of the rigid reinforced concrete slabs and/or masonry parapets/lintels connecting them. SIA D 0237 suggests choosing  $h_0$  for each wall of the building on the basis of engineering judgement and experience or calculating it using a FE-Program. SIA D 0237 recommends approximating the height of the first floor as  $h_0 = h_{1, \text{floor}} = h_{\text{st}}$ . The second main influence is the distribution of the inertia forces over the height of the building. Uniform or triangular distributions are commonly used. The triangular distribution is used in this study because it provides a more conservative approach. After defining  $h_0$  in the wall and choosing the force distribution the three values of the pushover curve can be calculated as follows (the nomenclature is shown in Fig. 3.1):

The shear resistance  $V_{Rd}$  is calculated in this study according to the Swiss Standard SIA 266 (2003) by superposition of a vertical and a diagonal stress field (see Fig. 3.1). It may be reasonable to assume the wall height  $h_p$  to be the height of the opening (e.g. window, door) in the case of a frame wall (walls connected with masonry parapets/lintels), as done in Lang (2002). The openings are arranged centrally with respect to the floor height in the analytical calculation as illustrated in Fig. 3.1.



**Figure 3.1.** Nomenclature and definitions used for multi-storey walls and calculation of shear resistance  $V_{Rd}$

The yielding displacement  $\Delta_y$  at the top of the wall according to SIA D 0237 can be calculated by using Eqn. 3.1 as an approximation, at which  $\Delta_y$  at the top of the wall is calculated by linear extrapolation of the yielding displacement of the first floor. Thereby bending and shear stiffness are considered. This approximation is valid for strongly coupled frame behaviour. Alternatively,  $\Delta_y$  can be calculated by using the normalized elastic deformed shape of the multi-story wall generated by a FE-Program.

$$\Delta_y = \left[ \frac{V_{Rd} \cdot h_{st}^3}{3EI_{eff}} + \frac{M_{z1d} \cdot h_{st}^2}{2EI_{eff}} + \frac{V_{Rd} \cdot h_{st}}{GA_{eff}} \right] \cdot \frac{H_{tot}}{h_{st}} \quad \text{with} \quad M_{z1d} = V_{Rd} \cdot (h_0 - h_{st}) \quad (3.1)$$

The following useful analytical solutions, suggested by the authors of this paper, take into consideration the elastic displacement figure of the multi-story wall: For an ideal frame situation with  $h_{0,f} = h_{st}/2$  Eqn. 3.2 can be used; Eqn. 3.3 can be suggested for an ideal cantilever situation with  $h_{0,c} \approx 2/3 H_{tot}$ ; and an interpolation based on Eqn. 3.4 can be used for  $h_{0,f} < h_0 < h_{0,c}$ . The value of the power  $x$  can be determined by a FE parametric study using frame models.  $EI_{eff}$  and  $GA_{eff}$  correspond to the effective bending and the shear stiffness of the wall at the first yield, respectively. SIA D 0237 suggests the use of the cracked stiffness of masonry walls as their effective stiffness, where  $E_{eff} = 0.3 \cdot E_{xk} = 0.3 \cdot 1000 \cdot f_{xk}$  and  $G_{eff} = 0.3 \cdot G_k = 0.3 \cdot 0.4 \cdot E_{xk}$ .

$$\Delta_{y,f} = \left[ \frac{V_{Rd} \cdot h_{st}^3}{3EI_{eff}} + \frac{M_{z1d} \cdot h_{st}^2}{2EI_{eff}} + \frac{V_{Rd} \cdot h_{st}}{GA_{eff}} \right] \cdot \frac{H_{tot}}{h_{st}} \quad \text{with} \quad M_{z1d} = V_{Rd} \cdot \left( \frac{h_{st}}{2} - h_{st} \right) \quad (3.2)$$

$$\Delta_{y,c} = \left[ \sum \left( \frac{H_{tot}}{2} - \frac{z_i}{6} \right) \cdot F_{di} \cdot z_i^2 \right] \cdot \frac{1}{EI_{eff}} + \left[ \sum F_{di} \cdot z_i \right] \cdot \frac{1}{GA_{eff}} \quad \text{for} \quad \sum F_{di} = V_{Rd} \quad (3.3)$$

$$\Delta_y = \frac{\Delta_{y,c} - \Delta_{y,f}}{(h_{0,c} - h_{0,f})^x} \cdot (h_0 - h_{0,f})^x + \Delta_{y,f} \quad \text{with} \quad x \approx 1.17; \quad h_{0,f} = \frac{h_{st}}{2}; \quad h_{0,c} \approx \frac{2 \cdot H_{tot}}{3} \quad (3.4)$$

The ultimate displacement  $\Delta_u$  at the top of the wall can be calculated according to SIA D 0237 using Eqn. 3.5 and Eqn. 3.6. It is assumed that the wall failure occurs in the lowest floor due to the high concentration of forces, which is justified for most walls with the exception of the ones subjected to low normal force (e.g. top floor). SIA D 0237 outlines an additional sub-method to account for wall failure at the top floor. This may predict a lower shear resistance and is not used in the calculated example.  $\Delta_{pl}$  represents the failure displacement at the top of the first floor, which is estimated by the ultimate limit drift ratio  $\delta_{ud}$  of the wall in the first floor multiplied with the floor height  $h_{st}$ .

$$\Delta_u = \left( 1 - \frac{h_{st}}{H_{tot}} \right) \cdot \Delta_y + \Delta_{pl} \quad \text{with} \quad \Delta_{pl} = \delta_{ud} \cdot h_{st} \quad (3.5)$$

$$\delta_{ud} = \delta_0 \cdot \left( 1 - \frac{\sigma_n}{f_{xd}} \right) \cdot r \quad \text{with} \quad \delta_0 = 0.8 \%; \quad \sigma_n = \frac{N_{xd}}{l_w \cdot t_w} \quad (3.6)$$

Eqn. 3.6 represents an empirical solution based on interpretation of results of different experiments carried out on masonry walls and it provides a design value for cantilever like walls.  $\sigma_n$  and  $t_w$  in Eqn. 3.6 are the normal stress of the wall, and the wall thickness, respectively. The value of  $\delta_{ud}$  should be decreased using a reduction factor  $r$  of 0.5 for frame like walls with a fixed condition at the top according to SIA D 0237. The authors of this paper suggest the reduction factor  $r$  to be determined by a linear interpolation between  $r = 0.5$  for  $h_0 = h_{st}/2$  and  $r = 1.0$  for  $h_0 \geq h_{st}$ .

The value of  $\Delta_u$  calculated according to the Eqn. 3.5 is an approximation, which should carefully be used. Suggestions were made to use it only for strong coupled frame behaviour. A more realistic determination of  $\Delta_u$  can be achieved through Eqn. 3.7, which also accounts for the normalized elastic deformed shape of the multi-story wall.  $\phi_1$  is the fraction of the normalized elastic deformed shape at the first floor and can be calculated using Eqn. 3.8. The precision of the Eqn. 3.7 can be improved by setting the height of the plastic zone  $h_p$  equal to the height of the opening (e.g. window, door, room height between the slabs), instead of the total height of the first floor  $h_{st}$ .

$$\Delta_u = \left( 1 - \phi_1 \cdot \frac{h_p}{h_{st}} \right) \cdot \Delta_y + \Delta_{pl} \quad \text{with} \quad \Delta_{pl} = \delta_{ud} \cdot h_p \quad (3.7)$$

$$\phi_1 = \frac{\phi_{1,c} - \phi_{1,f}}{(h_{0,c} - h_{0,f})^y} \cdot (h_0 - h_{0,f})^y + \phi_{1,f} \quad \text{with} \quad y \approx 0.35; \quad \phi_{1,f} = h_{st}/H_{tot} \quad (3.8)$$

In Eqn. 3.8 the first floor fraction of the normalized elastic deformed shape of an assumed ideal cantilever  $\phi_{1,c}$  and of an assumed ideal frame  $\phi_{1,f}$  are used. The suggested value of the power  $y$  can be determined by a FE parametric study using different frame models and situations.

**Table 3.1.** Three different possibilities to calculate the pushover curve using the principal method of SIA D 0237

	<b>C1</b> SIA D 0237 Approximation	<b>C2</b> Adjusted SIA D 0237	<b>C3</b> Using FE-Program
$h_0$	$h_0 \approx h_{st}$ (generally)	based on engineering judgement	calculated with FE-Program
Inertia force	not defined	triangular	triangular or uniform
$V_{Rd}$	SIA 266 or EC 8, $h_p = h_{st}$	SIA 266 or EC 8, $h_p \leq h_{st}$	SIA 266 or EC 8, $h_p \leq h_{st}$
$\Delta_y$	(3.1) $E_{eff} = 0.3 \cdot E_{xk}$ , $G_{eff} = 0.12 \cdot E_{xk}$	(3.4) using (3.2) and (3.3) $E_{eff} = 0.3 \cdot E_{xk}$ , $G_{eff} = 0.12 \cdot E_{xk}$	calculated with FE-Program $E_{eff} = 0.3 \cdot E_{xk}$ , $G_{eff} = 0.12 \cdot E_{xk}$
$\delta_{ud}$	(3.6) no reduction $r = 1$	(3.6) reduction factor $r$	(3.6) reduction factor $r$
$\Delta_u$	(3.5)	(3.7) using (3.8)	(3.7) using $\phi_1$ from FE-Calc.
Note	valid for strong coupling (frame behaviour)	uses elastic deformed shape by interpolation with $h_0$	depends on FE modelling assumptions and quality

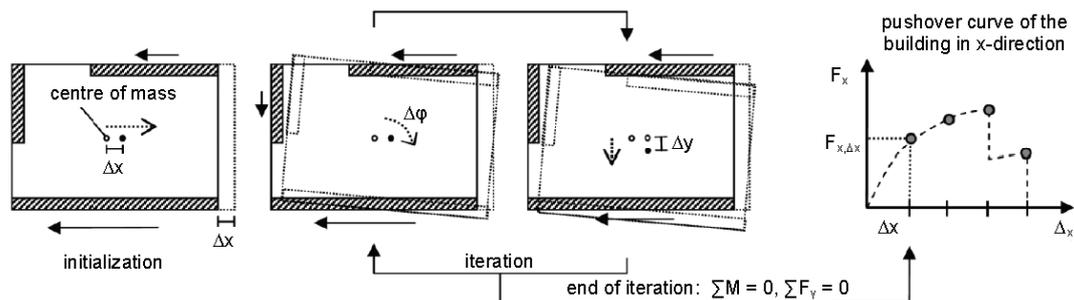
Three different possibilities (referred to as procedure C1, C2 and C3 only in this paper) are used to define the pushover curve based on the principal method of SIA D 0237, (see Table 3.1). Table 3.2 illustrates the considered  $h_p$  and the estimated  $h_0$  for the examined building using the adjusted SIA D 0237 procedure C2. These two values can be determined by setting:  $h_0 = h_{st}$  and  $h_p = h_{st}$  applying the approximate procedure C1. Fig. 3.3 shows the pushover curves of the individual walls using the different calculation procedures C1 (approximation) and C2 (adjusted by interpolation). The former leads to a smaller yield displacement and a longer yielding phase due to similar displacement capacity.

**Table 3.2.** Wall properties and parameters used with calculation procedure C2

Wall number	1	2	3	4	5	6	7	8	9	10			
Length $l_w$ [m]	1.4	1.2	1.6	1.6	4.1	4.1	9.0	5.1	5.1	3.6			
Thickness [cm]	18	18	12	12	15	15	12	12	15	12			
$N_{xd}$ 1 <sup>st</sup> floor [kN]	80	161	188	342	513	513	427	644	518	400			
$h_p$ (1 <sup>st</sup> floor) [m]	1.63	1.63	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65			
$h_0$ [m]	2.3	2.3	2.6	2.6	4.0	4.0	5.3	4.6	4.6	4.0			
Base shear $V_{Rd}$ [kN]	28	41	39	42	179	179	247	217	203	122			
Drift limit $\delta_{ud}$ [%]	0.63	0.55	0.57	0.39	0.61	0.61	0.71	0.56	0.65	0.59			
Wall number	11	12	13	14	15	16	17	18	19	20	21	22	23
Length $l_w$ [m]	4.4	1.0	1.3	1.1	4.3	3.2	3.2	0.8	1.6	2.1	1.1	1.1	0.8
Thickness [cm]	18	18	18	18	15	15	15	18	18	18	18	18	18
$N_{xd}$ 1 <sup>st</sup> floor [kN]	150	51	202	51	342	284	270	72	144	137	137	68	68
$h_p$ (1 <sup>st</sup> floor) [m]	2.65	1.63	1.63	1.63	2.65	2.65	2.65	1.63	1.63	1.63	1.63	1.63	1.63
$h_0$ [m]	4.0	2.0	2.0	2.0	4.0	3.3	3.3	2.0	2.6	2.6	2.0	2.3	2.0
Base shear $V_{Rd}$ [kN]	73	15	59	17	140	105	101	16	45	57	38	19	15
Drift limit $\delta_{ud}$ [%]	0.76	0.55	0.45	0.56	0.68	0.66	0.67	0.52	0.67	0.70	0.48	0.63	0.52

### 3.3 Developing the pushover curve of the building and comparison of procedures C1 and C2

The pushover curve of the building is determined by mathematical addition of the different pushover curves of the single walls. This approximation is valid for buildings subjected to no or very little torsion (the centre of mass is near the centre of stiffness). For buildings subjected to relevant torsion SIA D 0237 is referencing different methods to take torsion into account. In this paper the principals of Mistler and Butenweg (2005) are used as global approximation by forcing the compatibility of displacement and the equilibrium of force at the top of the building in each step, see Fig. 3.2.



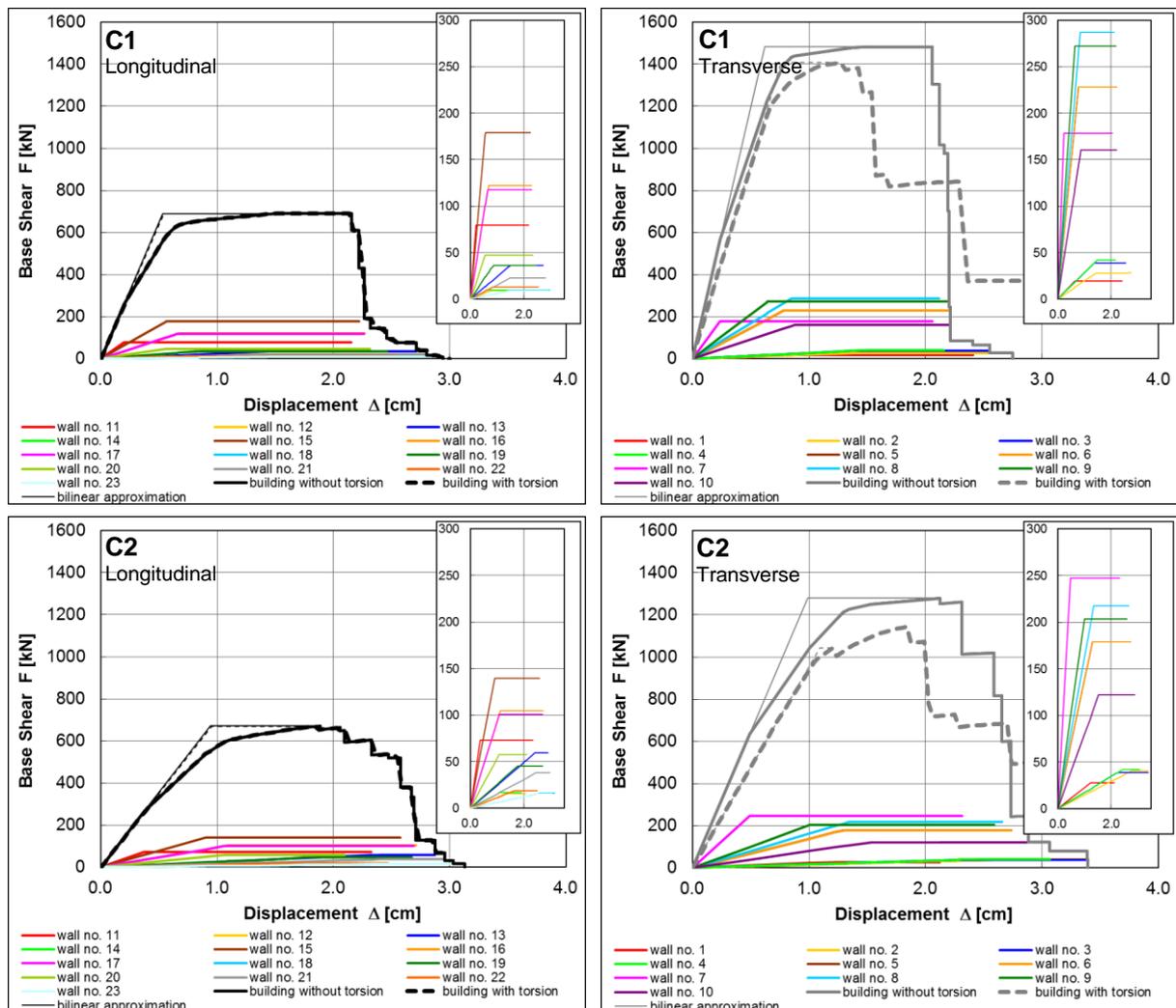
**Figure 3.2.** Consideration of torsion (based on Mistler and Butenweg 2005)

Thereby the pushover curves of the single walls are used as input. This approach is an approximation of the real behaviour, which doesn't meet the compatibility of the wall displacements in each floor. Thinking of the various uncertainties and model assumptions for the earthquake behaviour of a masonry building this approach is adequate for practical purposes for low to middle rise buildings up to about 6 to 10 floors. For high rise buildings this method isn't valid anyway because of the neglect of higher eigenmodes. The pushover curves for the example building are calculated using only unidirectional input in longitudinal or transverse direction. Therewith the comparison with the numerical calculation using 3muri is possible. A direction combination like in Eurocode 8 (e.g. 100%, 30%) could be easily implemented in the analytical calculation.

Fig. 3.3 shows the pushover curves of the example building using the different calculation procedures C1 and C2 and taking into account torsion. The comparison of the curves shows, that

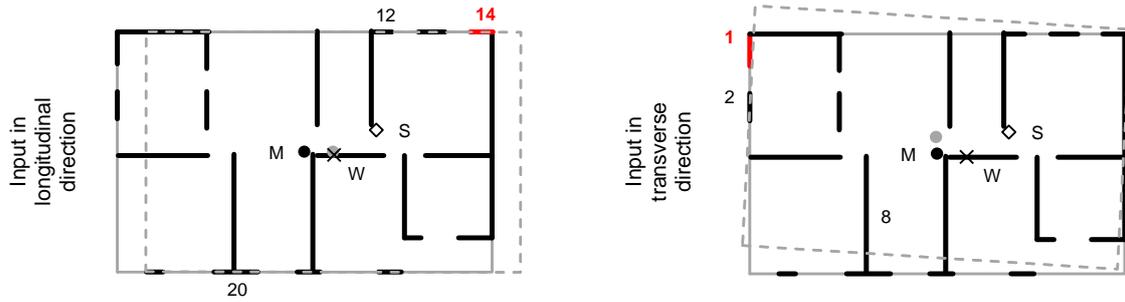
- the total longitudinal base shear (resistance) is much smaller than the one in transverse direction. This is due to the much longer and more loaded walls ( $N_{xd}$ ) in transverse direction, see Table 3.2.
- the total base shear (resistance) from calculation procedure C1 is bigger than the one of procedure C2 especially in transverse direction. The reason for that is the lower height of zero moment  $h_0$  in the heavily loaded walls in calculation procedure C1.
- procedure C1 results in a much lower yielding displacement (higher initial stiffness and frequency) than procedure C2 but in an equal global displacement capacity (failure), see Table 5.1 and Table 5.2. This result is explainable by the differences in the equations for  $\Delta_y$  and in the use of  $h_{st}$  instead of  $h_p$  in the equation for  $\Delta_u$  in case of the significant frame like walls.
- taking into account torsion results in lower frequency, lower total base shear (resistance) and lower displacement capacity, as well as horizontal loading on the walls orthogonal to the input direction.

The differences in the procedures C1 and C2 results in a much lower compliance factor  $\alpha_{eff}$  for procedure C2. For the calculated example the difference is of factor 2, see Table 5.1 and Table 5.2.



**Figure 3.3.** Pushover curves of the individual walls and the building using the calculation procedures C1 and C2

Fig. 3.4 shows the displacement figures in ground view for the state of first wall failure (procedure C2). The first walls to fail are the frame like walls no. 14, 12 and 20 for input in longitudinal direction and walls no. 1, 2 and 8 for input in transverse direction. The latter are walls subjected to torsion.



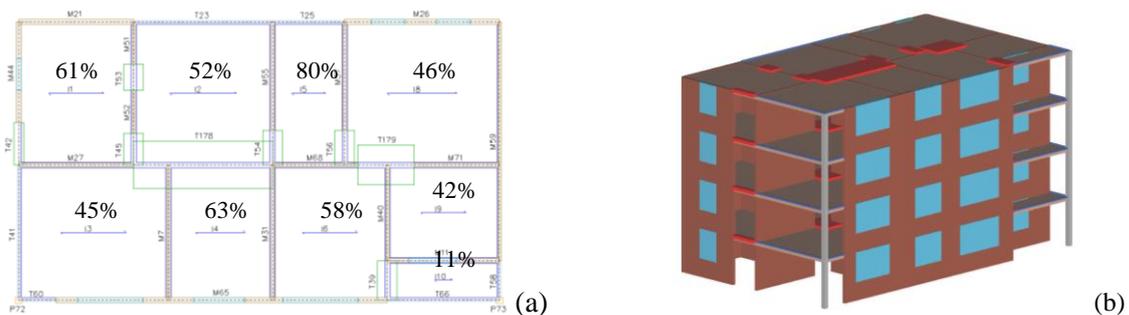
**Figure 3.4.** Displacement figure in ground view for the state of first wall failure (method SIA C2), Red: first wall failure and corresponding wall no., additionally stated wall no.: subsequent wall failures, M: centre of mass, W: centre of resistance, S: centre of stiffness

#### 4. NUMERICAL CALCULATION USING THE MACRO ELEMENT PROGRAM 3MURI

For the numerical calculation the Macro Element Program 3muri was used. Four different studies, named 3muri M1, M2, M3 and M4 were carried out with the building corresponding to the analytical study. Several material and model parameters that were used are shown in Table 4.1. The diaphragm was modelled as concrete with uncracked stiffness ( $E_{ck}$  and  $G$ ). Using the 3muri program, masonry walls in a building have to be connected with coupling beams. This can be done by using either fixed beams or beams with hinges. The former shows more realistic results for concrete floor slabs as the frame effect between concrete slabs and masonry walls can be considered. Thus, fixed beams were modelled (with the same thickness as the existing concrete slab and participating width according to SIA) for walls that were affected by the frame effect. They are depicted in Fig. 4.1 (a) as green rectangle or in Fig 4.1 (b) as red areas, respectively. For all other walls beams with hinges were modelled. Only for the fourth study (3muri M4) coupling beams with hinges were used for all walls. Another difference between the four models is the stiffness setting whereas the study M1, M3 and M4 were carried out using directly the Elastic modulus and M2 the reduced stiffness at cracking. All four calculations used a masonry compression strength of  $f_m = f_{cd} = 3.5 \text{ N/mm}^2$  and shear strength of  $\tau = f_{vd0} = 0.20 \text{ N/mm}^2$  according to SIA D 0237. The use of higher strength values than the design values would result in significant higher shear resistance (base shear) and compliance factor.

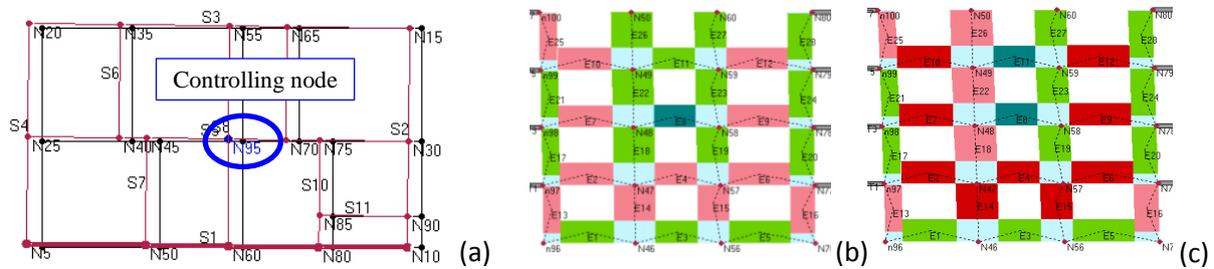
**Table 4.1.** Input Parameter of the four different calculations in 3muri

	<b>3muri M1</b> 3muri suggestion	<b>3muri M2</b> 3muri (initial crack)	<b>3muri M3</b> Authors suggestion	<b>3muri M4</b> No coupling beam
Elastic modulus (masonry / concrete)	$E_{xk} = 1000 f_{xk}$ $E_{ck}$	$E_{xk} = 1000 f_{xk}$ $E_{ck}$	$E_{eff} = 0.3 E_{xk}$ $E_{c,eff} = 0.3 E_{ck}$	$E_{eff} = 0.3 E_{xk}$ $E_{c,eff} = 0.3 E_{ck}$
Shear modulus (masonry / concrete)	$G_{xk} = 0.4 E_{xk}$ $G$	$G_{xk} = 0.4 E_{xk}$ $G$	$G_{eff} = 0.12 E_{xk}$ $G_{c,eff} = 0.3 G$	$G_{eff} = 0.12 E_{xk}$ $G_{c,eff} = 0.3 G$
Modelling of coupling beams	Fixed (frame effect due to floor slab)	Fixed (frame effect due to floor slab)	Fixed (frame effect due to floor slab)	Hinges
Stiffness at cracking	No checkmark	Checkmark	No checkmark	No checkmark

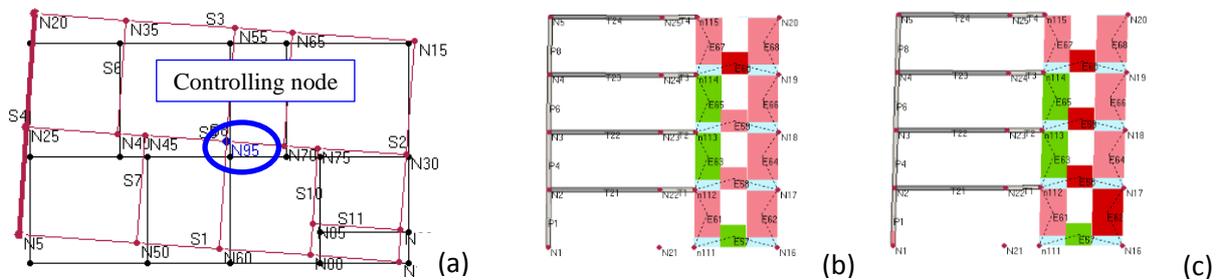


**Figure 4.1.** Load transfer in principal directions (longitudinal) of the slabs (a); overview of the 3muri model (b)

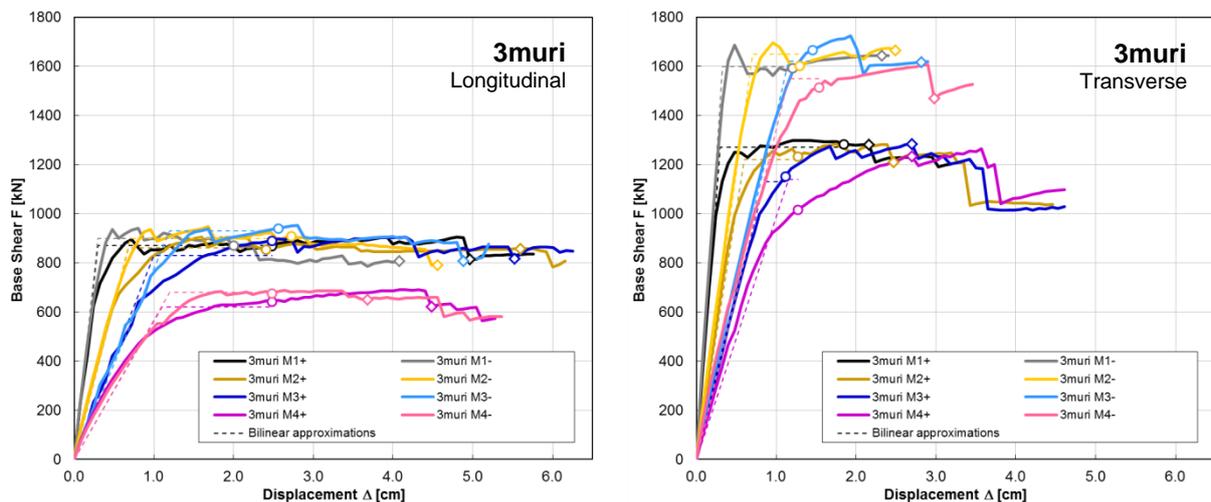
Carrying out the calculation, different program settings were used for all four studies like “existing building”, “Swisscode SIA”, the assumption of the mass distribution called "1°Modus" (with no additional eccentricity) and the calculation of the part of vertical load in principle direction of each floor slab. These settings should guarantee a good comparability with the analytical calculation. Fig. 4.1 (a) shows the load transfer in longitudinal direction of the floor slabs and in Fig. 4.1 (b) the 3D model is depicted. The red areas on the slabs defined the above mentioned fixed beams. The input parameters of the four different studies are summarized in Table 4.1.



**Figure 4.2.** "3muri M3" in negative longitudinal direction: displacement figure (a), failure of first parapet between wall no. 20 and wall no. 21 (b) and failure of first vertical elements; wall no. 20 and wall no. 21 (c)



**Figure 4.3.** "3muri M3" in positive transverse direction: displacement figure (a), failure of first parapet between wall no. 1 and wall no. 2 (b) and failure of first vertical element; wall no. 1 (c)



**Figure 4.4.** Pushover curves of the four 3muri models, for each input direction in positive and negative direction. The circle marks first parapet failure and the square first vertical wall failure.

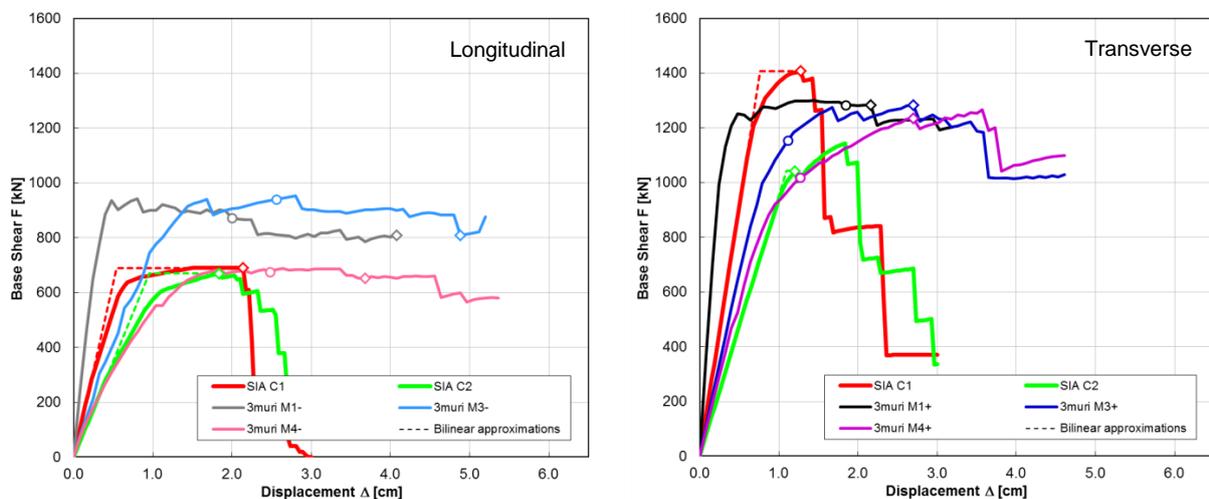
After carrying out the four studies the outcome was compared to the results obtained by the analytical calculation. In Fig. 4.2 and 4.3 the displacement figure and the significant failure mechanism for the model "3muri M3" are depicted whereas blue and dark red equals to the failure of the element. Fig. 4.4 show the different pushover curves, each taken at the controlling node (top slab). Tables 5.1 and Table 5.2 also summarize the results of the four 3muri models. As main influencing parameters the elastic

modulus (initial stiffness) as well as the modelling and the characteristics of the coupling beams were identified. The former can strongly influence the eigenfrequency and therewith the displacement demand and the yielding displacement, respectively. The latter can strongly influence the base shear by coupling the walls and normal force redistribution. This can be seen in Fig. 4.4 comparing the 3muri models M1, M2, M3 to model M4. The big differences in the base shear between the + and – direction for transverse input is the result of normal force redistribution at wall corners.

To determinate the displacement capacity of the building, the responsible engineer has to decide what element failure causes a failure of the building. In Table 5.1 and Table 5.2 the first failure (for this model always failure of parapets) as well as the failure of the first vertical wall element are documented. This gives a possible range of the compliance factor  $\alpha_{eff}$ . It varies strongly between the different 3muri models. It has to be mentioned that the partial safety factor used in the 3muri is not known, because the model parameters behind the 3muri setting "Swisscode (SIA)" are not explained in the program guidelines. Therefore, there is no information about what failure criteria (drift limits) are used with this setting. The authors assume that the drift limits of Eurocode 8 are used and not Eqn. 3.6.

## 5. COMPARISON OF ANALYTICAL AND NUMERICAL CALCULATION

Fig. 5.1 shows the comparison of the pushover curves for analytical calculation procedures C1 and C2 with the 3muri models. All calculations used  $E_{eff} = 0.3 \cdot E_{xk}$  except for calculation 3muri M1 that used  $E_{eff} = E_{xk}$ . The latter results in the stiffest pushover curve as expected. The main influencing parameter to the compliance factor  $\alpha_{eff}$  of the building is the Elastic modulus (initial stiffness) used respectively the resulting yielding displacement  $\Delta_y$ . This applies for both methods SIA and 3muri and it is because the displacement capacity is not strongly affected by the stiffness. In 3muri this fact is even more important due to the difference in the displacement capacity of parapet and wall failure. If the parapet is considered in the 3muri model it is taken into account for the stiffness.



**Figure 5.1.** Comparison of pushover curves of three 3muri models and of the analytical calculations

**Table 5.1.** Comparison of results for longitudinal direction (only controlling direction for 3muri calculations)

Longitudinal direction	SIA	SIA	3muri M1-		3muri M3-		3muri M4-	
	C1	C2	parapet	wall	parapet	wall	parapet	wall
Frequency [Hz]	3.1	2.3	4.3		2.3		2.0	
Yielding displacement [cm]	0.5	0.9	0.3		1.2		1.2	
First failure [wall-no.]	14	14	13 – 14	20&21	20 – 21	20&21	20 – 21	3
Displacement capacity [cm]	2.1	1.8	2.0	4.1	2.6	4.9	2.5	3.7
Performance point [cm]	0.78	1.36	0.46		1.43		1.86	
Compliance factor $\alpha_{eff}$ [-] <sup>1)</sup>	2.7	1.3	4.3	8.9	1.8	3.4	1.3	2.0

<sup>1)</sup> See Table 5.2

**Table 5.2.** Comparison of results for transverse direction (only controlling direction for 3muri calculations)

Transverse direction	SIA C1	SIA C2	3muri M1+ (-)		3muri M3+		3muri M4+	
			parapet	wall	parapet	wall	parapet	wall
Frequency [Hz]	3.8	2.7	5.3 (5.6)		2.9		2.5	
Yielding displacement [cm]	0.8	1.1	0.3		0.9		1.2	
First failure [wall-no.]	1	1	(1 – 2)	1	1 – 2	1	1 – 2	1
Displacement capacity [cm]	1.3	1.2	(1.2)	2.2	1.1	2.7	1.3	2.7
Performance point [cm]	0.52	1.00	0.23 (0.21)		0.76		1.01	
Compliance factor $\alpha_{\text{eff}}$ [-] <sup>1)</sup>	2.5	1.2	(5.7)	9.4	1.5	3.6	1.3	2.7

<sup>1)</sup> Earthquake input according to Code SIA 261 (2003): Zone 1 ( $a_{\text{gd}} = 0.6 \text{ m/s}^2$ ), soil class E ( $S = 1.40$ ), building class I ( $\gamma_f = 1.0$ ) and 5% damping

## 6. CONCLUSIONS

The characteristics of the examined building are relatively simple and regular. Despite this fact, the analytical and numerical calculations using DBM as well as the parametric studies show that the results are varying strongly depending on the way of modelling and on the input settings. Thus the earthquake engineering knowledge, the experience and the modelling decisions of the responsible engineer are very important.

Regarding the main influences the authors draw the following conclusions:

- As the Elastic modulus of the masonry and the procedure of calculating the initial stiffness (yielding displacement) are most influencing the results, the Elastic modulus should be determined taking initial cracking into account as suggested in the Documentation SIA D 0237 (e.g.  $E_{\text{eff}} = 0.3 \cdot E_{\text{sk}}$ ). This should be done not only in analytical but also in numerical calculations with 3muri.
- The approximation of the yielding displacement in analytical calculation according to SIA D 0237 should be used only carefully. More conservative results can be achieved by an adjusted procedure presented by the authors. Thereby the height of zero moment is the important parameter.
- The modelling of the coupling beams with fixed ends has to be done carefully in 3muri, because of their big influence on initial stiffness and shear resistance due to the frame effect.
- In accordance to D 0237 all calculations were carried out with design values for compression and shear strength of the masonry. This is also suggested by the authors for calculations with 3muri.
- The principal of the method of Mistler and Butenweg is suited to consider torsional behaviour especially in the case of analytical calculations.

More studies have to be conducted to show if these suggestions can guarantee a conservative approach in most cases and for buildings with different characteristics. Comparison with test results from large scale shake table testing of masonry buildings would be necessary additionally.

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