

# Experimental Investigation of Inertial Force Control for Substructure Shake Table Tests

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## **SUMMARY:**

This study investigates the use of inertial masses integrated into substructure shake table tests. The substructure shake table test method considered here, will allow for performance evaluation of large and complex structural systems subjected to ground motions by the decomposition of the structure into experimental and computational substructures. The interface forces between computational and experimental substructures are incorporated through the inertial force of a mass controlled by a hydraulic actuator. Following numerical simulations containing detailed models of the controlled mass system, an experimental investigation of inertial force control is performed to verify the feasibility of the method. This paper presents a formulation of the substructure shake table test method, preliminary numerical simulations, and a detailed discussion of the experimental implementation of inertial force control. Experimental results are then compared with results from numerical simulations.

*Keywords: Hybrid Simulation, Shake Table Tests, Substructure Method, Force Control, Hydraulic Actuator*

## **1. INTRODUCTION**

Modern structures are pushing the limits of traditional engineering concepts with their overall size, complex geometries and new construction techniques. It is imperative that the characteristics (static and dynamic) of these novel structures are fully understood before construction begins. Structural response is especially of great concern during potentially devastating events such as earthquakes, where not only structural integrity is at risk but residents and important non-structural components of the building are also in danger. Computer simulations and analyses are convenient for determining basic structural properties but do not fully capture the extreme structural responses such as local failures and collapse mechanisms. Full scale structural testing can be used to evaluate structural responses that numerical simulations do not capture. While large-scale environments have been made available (e.g., NEES and E-Defence facilities), full-scale tests including realistic conditions are still limited. Consequently, the exploration of additional techniques to compliment current structural testing strategies is needed to enhance the capabilities of earthquake engineering research.

One very popular structural testing method is shake table testing. Shake table tests tend to give the best results since the structural responses generated are the most consistent with actual structural responses that occur during an earthquake. However shake tables are limited in terms of the overall size of the attached payload and thus most shake table tests are scaled down. These scaled tests do not offer great insight since scaled results are not always representative of the full-scale response. One technique that can be used to increase the capabilities of shake table tests is substructuring. Substructure testing allows for somewhat large-scale tests since only a portion of the structure is tested experimentally, but ensuring the experimental boundary conditions are met during the test is a major challenge. Therefore substructure tests are typically conducted in a pseudo-dynamic manner and thus the rate dependant properties of the structural response are usually not captured. The results from shake table tests can be further improved by combining the fully dynamic properties of shake table tests with the

versatility and scale of substructure testing techniques.

The idea of combining the concepts of substructuring with shake table tests are not new to the field. Several implementations and discussions can be found in engineering literature (e.g.: Ji *et al.* (2009), Lee *et al.* (2007), Igarashi (2000) and Neild *et al.* (2005)). Shao *et al.* (2011) proposed a substructure shake table test method to experimentally reproduce the responses of the middle portion of a structure where the interface forces are applied by an external hydraulic actuator. Nakata and Stehman (2012) introduced a substructure shake table test method where a controlled mass system, consisting of an actuator and an auxiliary mass, is used to apply the interface forces to the experimental substructure. In that study the computational forces are first converted into equivalent displacements and the interface forces are transmitted to the experimental substructure through the inertial forces of the mass.

This paper continues the study first introduced by Nakata and Stehman (2012). The substructure shake table test method presented here can be used to experimentally test the lower or middle portions of a structure while the rest of the structure is computationally analyzed. This method introduces a new approach to impose the computational forces to the experimental substructure. An actuator with an auxiliary mass is placed on the top floor of the experimental substructure where the actuator cylinder is attached to the structure while the piston and mass are free to move. The motion of the mass will then create an inertial force that gets transmitted to the structure. The main benefit of this method is the versatility of the ‘‘Controlled Mass System’’ since it does not require any external reactions such as a strong wall. This paper presents the fundamental concepts of the substructure shake table test method with detailed discussion of the controlled mass system and controller. Numerical simulations are conducted and experimental results are presented as well.

## 2. THE SUBSTRUCTURE SHAKE TABLE TEST METHOD

The substructure shake table method presented first by Nakata and Stehman (2012) is summarized here for convenience. In this method the computational forces are incorporated into the experimental substructure through the inertial force of a mass controlled by an actuator. This section presents the equations of motion for both entire and substructured systems. After defining the governing dynamics, constraints are discussed that will guarantee the substructured system has the equivalent characteristics as the entire structure.

### 2.1. Equations of Motion

Using Newton’s Second Law of motion, the equations of motion are derived for a structure subjected to ground motion. Fig. 1 presents a schematic diagram of both entire and substructured systems. The equations of motion for an arbitrary floor,  $j$ , are as follows:

*Entire system*

$$m_j \sum_{k=1}^j \ddot{x}_k + R_j(x_j, \dot{x}_j) - R_{j+1}(x_{j+1}, \dot{x}_{j+1}) = -m_j \ddot{x}_{g-e}, \quad (j = 1, \dots, n \text{ and } R_{n+1} = 0) \quad (2.1)$$

*Substructured system*

$$m_j \sum_{k=1}^j \ddot{x}_k + R_j(x_j, \dot{x}_j) - R_{j+1}(x_{j+1}, \dot{x}_{j+1}) = -m_j \ddot{x}_{g-e}, \quad (j = 1, \dots, i-1) \quad (2.2)$$

$$m_j \sum_{k=1}^j \ddot{x}_k + R_j(x_j, \dot{x}_j) - R_{j+1}(x_{j+1}, \dot{x}_{j+1}) = -m_j \ddot{x}_{g-e}, \quad (j = i) \quad (2.3)$$

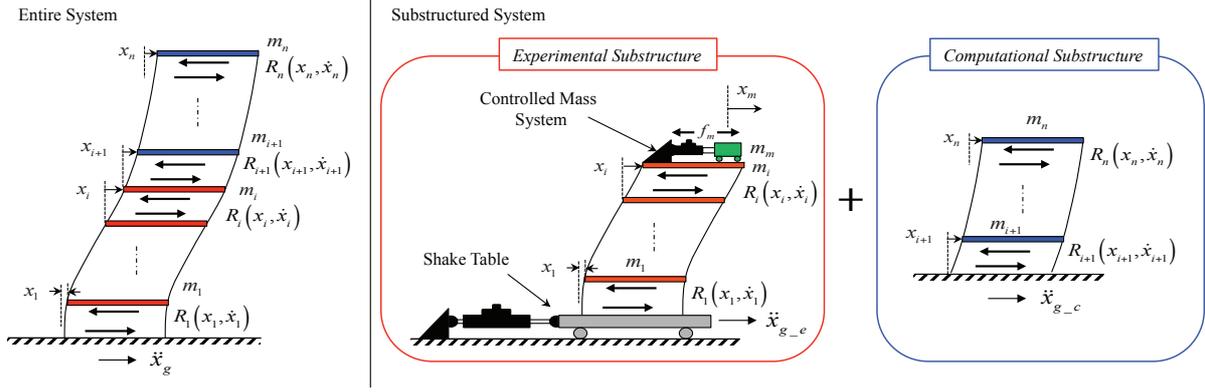


Figure 1. Graphical representation of substructure shake table test method

$$m_j \sum_{k=i+1}^j \ddot{x}_k + R_j(x_j, \dot{x}_j) - R_{j+1}(x_{j+1}, \dot{x}_{j+1}) = -m_j \ddot{x}_{g\_c}, \quad (j = i+1, \dots, n \text{ and } R_{n+1} = 0) \quad (2.4)$$

where  $\ddot{x}_g$  is the true ground acceleration;  $\ddot{x}_{g\_c}$  and  $\ddot{x}_{g\_e}$  are the “ground” accelerations for the computational and experimental substructures respectively;  $m_j$  is the mass of the  $j$ -th floor;  $x_j$ ,  $\dot{x}_j$  and  $\ddot{x}_j$  are the relative displacement, velocity and acceleration of the  $j$ -th floor with respect to the  $(j-1)$ -th floor;  $R_j(x_j, \dot{x}_j)$  is the restoring force from the columns connecting floors  $j$  and  $j-1$ ; and  $f_m$  is the force transmitted from the controlled mass system to top floor of the experimental substructure. The reader should note that Eqns. 2.2 and 2.3 are with respect to the experimental substructure and Eqn. 2.4 describes the dynamics of the computational substructure.

There are certain conditions that need to be met during the entire duration of the test to ensure that the substructured system performs equivalently to the reference entire system. The first is the experimental acceleration compatibility condition. This condition requires that the shake table can properly recreate the ground motions, namely:  $\ddot{x}_{g\_e} = \ddot{x}_g$ . The second is the computational acceleration compatibility condition. This condition states that the ground acceleration of the computational substructure needs to be the absolute acceleration of the separation floor (floor  $i$ ), namely:  $\ddot{x}_{g\_c} = \ddot{x}_{g\_e} + \sum_{k=1}^i \ddot{x}_k$ . The final requirement is interface force compatibility. Here the force applied by the controlled mass system must be equal to the base shear from the computational substructure (computational base shear), namely:  $f_m = R_{i+1}(x_{i+1}, \dot{x}_{i+1})$ .

## 2.2. Implementation Considerations

Since the performance of the substructure shake table test method relies heavily on the ability to meet the above compatibility conditions, said conditions should be considered when discussing the experimental setup. However, limited resources and technical challenges may make it difficult to achieve all of the needed requirements. This section discusses implementation strategies with regards to the compatibility conditions.

To meet the experimental compatibility requirement, acceleration tracking control of shake tables is needed. The topic of acceleration control of shake tables has been investigated by a number of researchers (e.g., Nakata (2010) and Conte *et al.* (2002)). These methods have shown to be promising and thus this study does not delve into the details of acceleration control for shaking tables. This paper does not discuss the implementation of acceleration control of shake tables and thus interested readers are encouraged to refer to the relevant literature.

For the computational acceleration requirement, the absolute acceleration of the separation floor must be measured and made available to the computational substructure analysis program. This process can

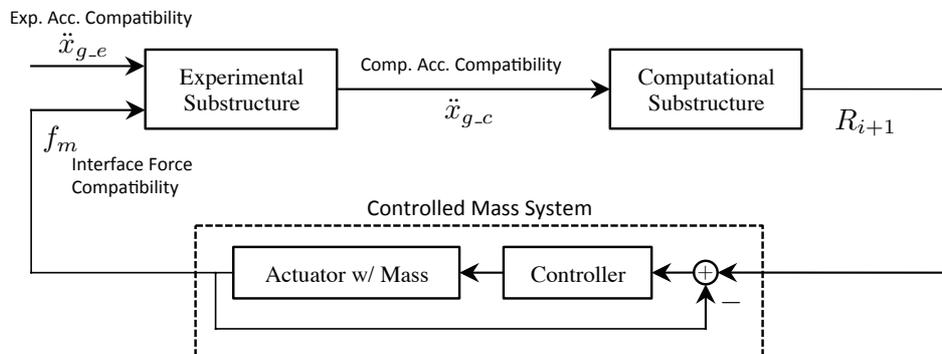
easily be achieved by utilizing any of the available real-time data acquisition systems (e.g. Labview RT and SCRAMNet). The use of real-time computations and data acquisition have been used in structural testing for quite some time now and thus this compatibility condition is not a major challenge in terms of hardware. Since this requirement can be satisfied with little effort it is not discussed in this study.

The interface force compatibility condition requires the computational base shear to be applied to the experimental substructure with minimal time lag. Thus the controlled mass system needs to be controlled with reference to the computational base shear. The control method chosen for this study is direct force control. The specifics of the controller and experimental setup will be discussed later. Since hydraulic actuators have an inherent time lag due to their dynamics properties it is expected that delay compensation methods may be needed in the experimental implementation. There are a many actuator delay compensation methods available that are based off of the pioneering work done by Horiuchi *et al.* (1999).

The data flow diagram of the substructure method can be seen in Fig. 2. As shown in the diagram the substructure method consists of a feedback loop with both computational and experimental processes. Accuracy and stability of the method rely on the precision and time delay of each process. Fig. 2 also indicates how each of the compatibility conditions affects the overall performance of the method.

### 3. SIMULATION MODELS AND PARAMETERS

To evaluate the performance of the substructure shake table test method numerical simulations are conducted to replicate the overall experimental procedure. Results from the numerical simulations will be used to determine and investigate the influential parameters in the system. Also numerical simulations will serve as a stepping-stone for the experimental implementation by allowing for proper design of the experiment as well as to point out possible limitations. This section will present the models chosen for the numerical simulations presented.



**Figure 2.** Data flow diagram for the substructure shake table test method

#### 3.1. Structural Models

The structure to be analyzed is a 6-story shear building and is separated at the second floor, refer to Fig. 1 with  $n=6$  and  $i=2$ . The structural properties are selected so that physical construction is feasible in the SSHT lab at Johns Hopkins, namely the floor mass is 35kg and the damping ratio is 5%. The first natural frequency of the test structure is 2.5 Hz.

#### 3.2. Structural Analysis

Newmark's numerical integration method is used to solve the equations of motion for both

experimental and computational substructures. This is completed in a time stepping manner where the computational base shear is fed to the experimental substructure one-time step after it is calculated. The parameters for the simulations are selected as  $\gamma=1/2$  and  $\beta=1/6$  with a sampling rate of  $\Delta t = 0.001$  seconds.

### 3.3. Controlled Mass System

Since a hydraulic actuator is used to apply the computational base shear to the experimental substructure the simulations require an accurate model of the actuator to represent the interface force compatibility. The actuator model is based off of a 911D Shore Western Inc. hydraulic actuator with a 52.7 kg mass attached to the piston rod; the controlled mass system is shown in Fig. 3. This actuator has a stroke length capability of +/- 78mm and a dynamic force capacity of 24.5 kN. To model the system, Gaussian band limited white noise is fed to the actuator servo valve and the force is measured. Then using spectral methods of system identification, an empirical frequency response curve is generated and finally curve fitting techniques allow for an analytic expression of the experimental data. Eqn 3.1 shows the open loop transfer function from valve command to measured force:

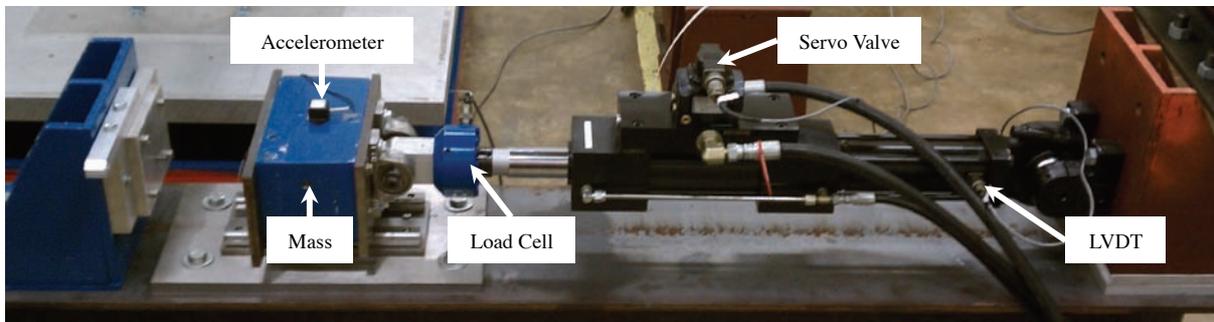
$$H_{fc} = \frac{-1.883 \cdot 10^{12} s^2 - 4.275 \cdot 10^{14} s - 4.087 \cdot 10^{15}}{s^6 + 775.3 s^5 + 5.794 \cdot 10^5 s^4 + 2.168 \cdot 10^8 s^3 + 5.427 \cdot 10^{10} s^2 + 8.812 \cdot 10^{12} s + 8.929 \cdot 10^{14}} \quad (3.1)$$

while the stabilizing force feedback controller was designed as:

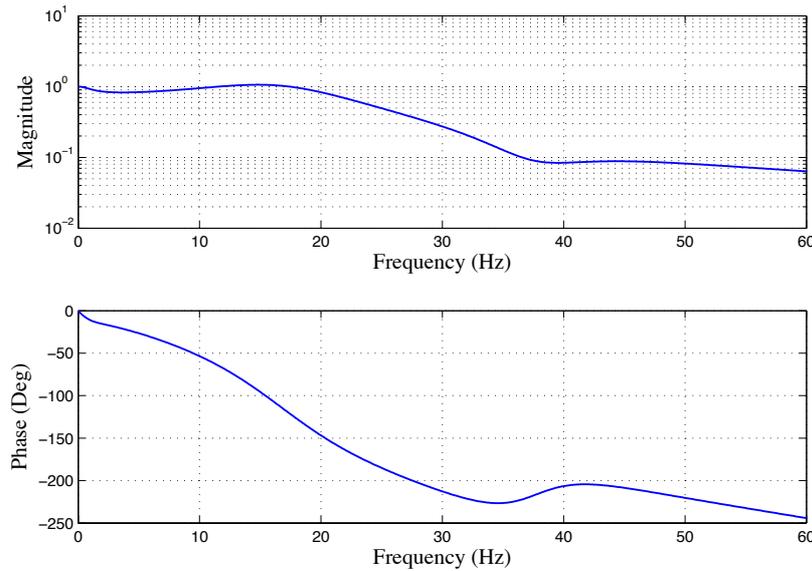
$$H_c = \frac{-0.4s^3 - 96.1s^2 - 2.635 \cdot 10^4 s - 4.281 \cdot 10^6}{s^4 + 3.440 \cdot 10^2 s^3 + 53394 \cdot 10^4 s^2 + 7.956 \cdot 10^5 s} \quad (3.2)$$

Fig. 4 shows the closed loop transfer function from reference force to measured force of the controlled mass system. The closed loop response of the controlled mass system has desirable attributes with an effective bandwidth of 20Hz and minimal phase lag up to 10 Hz. Since the lag between desired and applied forces is expected to be the governing factor in terms of stability of the substructure method, the closed loop system was designed to exhibit minimal phase lag between measured and reference forces.

The open loop transfer function, Eqn. 3.1, along with the transfer function of the stabilizing controller, Eqn. 3.2, are converted into their respective state space representations to allow for time domain simulation of their performance in conjunction with the structural analyses. These state space realizations are combined to build a closed loop model of the controlled mass system, which is then used in series between the computational and experimental substructure.



**Figure 3.** Experimental setup of the controlled mass system



**Figure 4.** Closed loop transfer function of the controlled mass system

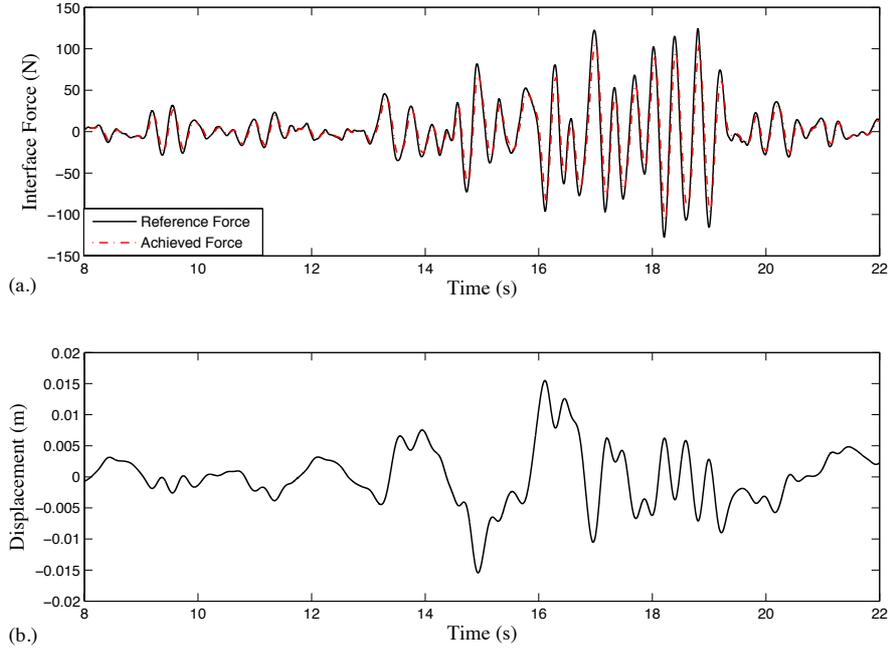
#### 4. NUMERICAL SIMULATIONS

As mentioned previously numerical simulations were performed to examine the performance of the substructure shake table test method using inertial force control. Results from the simulations will give insight into the expected performance of an experimental implementation. The results presented here make use of the models discussed in Section 3. The numerical simulations are conducted in a sequential time domain fashion using Matlab (2011).

The simulation results presented here are for the structure described in Section 3.1 when subjected to the 1999 Chi-Chi (Taiwan) ground motion with a scaled PGA of 0.5g. The results from the simulation are presented in the same order that they are generated during the simulation so as to give readers an idea of the simulation procedure. Firstly, the ground motion excites the experimental substructure and the absolute acceleration of the separation floor is then fed to the computational substructure. Next, the computational base shear is calculated from the response of the computational substructure and is used as the reference force for the controlled mass system. The time history of the controlled mass system response for this simulation is shown in Fig. 5. Fig. 5a shows the force tracking results of the controlled mass system while Fig. 5b shows the displacement time history of the actuator piston and it is noted that the actuator remained within its capabilities.

From Fig. 5 it is seen that the controlled mass system is doing an acceptable job of reproducing the reference computational base shear. The results from the simulation show that the closed loop controlled mass system is indeed exhibiting minimal phase lag between reference and measured forces and thus the phase lag can be considered negligible. Note that the achieved force is what is actually fed back into the experimental substructure in place of the numerically calculated computational base shear. This procedure of data flow is repeated throughout the duration of the simulation. The response of the substructured system is compared to the response of the entire system to check the performance of the substructure method.

Fig. 6 shows a comparison of the top floor responses for both substructured and entire systems during the Chi-Chi ground motion. In this simulation the substructured system performs comparatively well with the entire structure. The major characteristics of the structures response are captured with a 9.59% RMS error for displacement, 10.08% RMS error for velocity and 9.64% RMS error for acceleration. Where the RMS error is defined by Eqn 4.1 as:



**Figure 5.** Response of the controlled mass system: a.) reference and achieved force comparison; b.) displacement

$$E_{RMS} = \frac{\sqrt{\frac{1}{N} \sum_{k=1}^N (\varphi_E[k] - \varphi_S[k])^2}}{\max\{\varphi_E[t]\}} \quad (4.1)$$

where  $\varphi_E$  and  $\varphi_S$  are the structural response time history vectors for entire and substructured systems respectively;  $N$  is the number of time steps and  $k$  is the time step index. The RMS errors presented here are calculated using the top (6-*th*) floor structural responses.

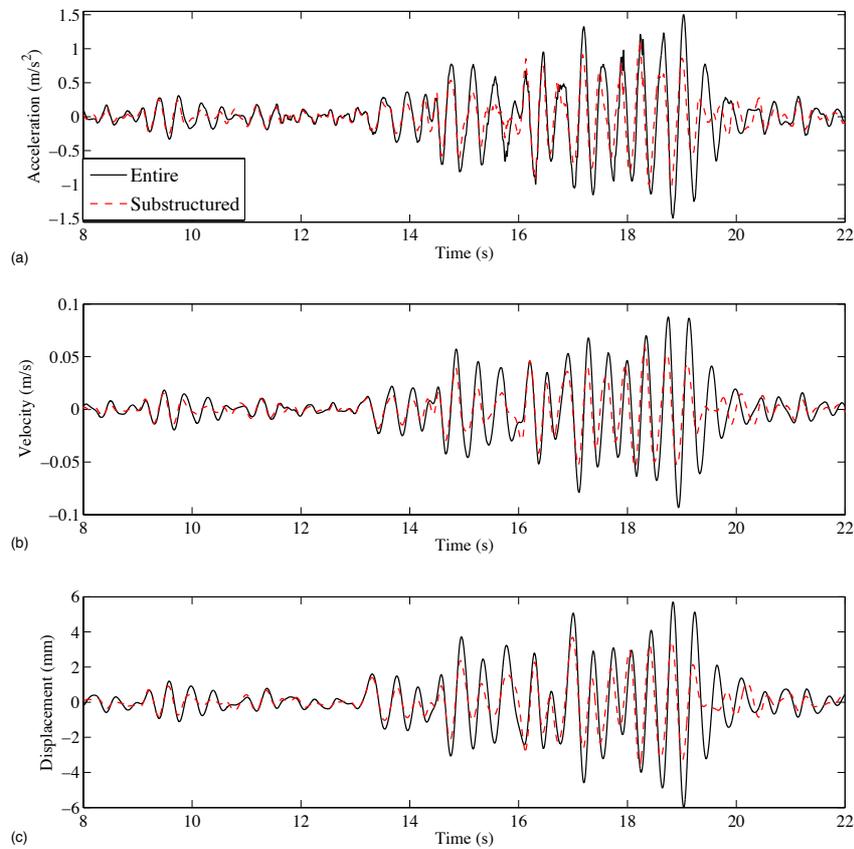
This numerical case study suggests that the proposed substructure shake table test method has promise to serve as an alternative technique to full-scale shake table tests of structures. The substructured system was able to capture the major vibratory response characteristics of the 6-story structure of interest. Further studies will be conducted to investigate the limitations and feasibility of the proposed method, mainly consisting of experimental investigations.

## 5. EXPERIMENTAL INVESTIGATION OF INERTIAL FORCE CONTROL

The previous discussion showed promising numerical results for the substructure shake table test method but before feasibility of the method is ensured, experimental validation is needed. This section discusses the experimental implementation and investigation of inertial force control that is needed for the proposed substructure shake table test method. Preliminary experimental investigations are conducted to evaluate the performance and to show the limitations and challenges of experimental inertial force control.

The experimental implementation can be seen in Fig. 3. Using the system models determined from the system identification process discussed in section 3.3 the stabilizing loop shaping controller, Eqn 3.2, was implemented in the SSHT lab at the Johns Hopkins University. Labview Real Time, Labview (2011), was used as the real time software interface to the controlled mass system as well as for data acquisition. The aforementioned continuous time controller is first discretized and then implemented

on a computer running Labview RT. The real-time operating system and computer are used at a rate of 4KHz, allowing seamless communication between the user and controlled mass system.

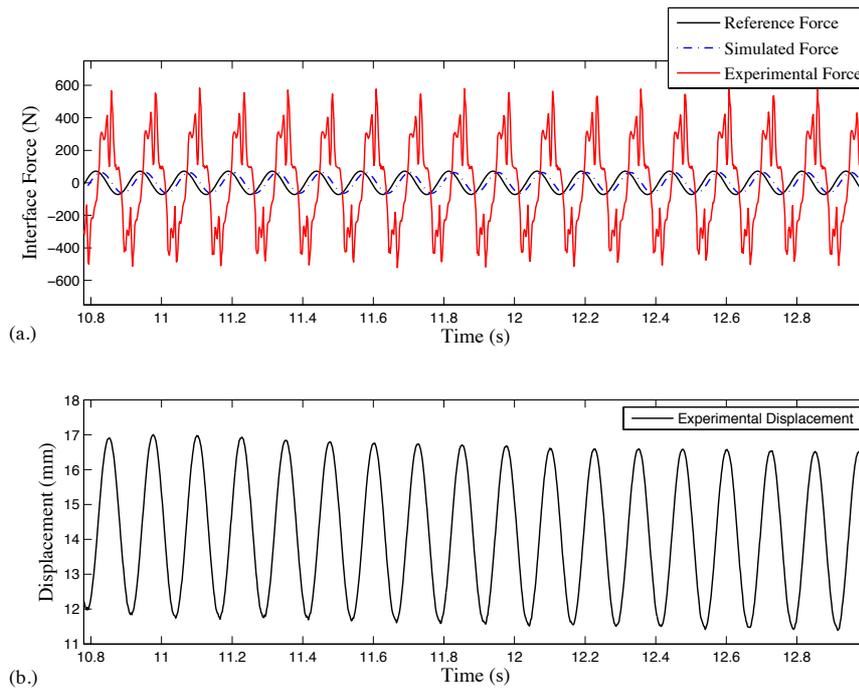


**Figure 6.** Comparison of top floor structural responses for entire and substructured systems: a.) accelerations; b.) velocities; c.) displacements

To experientially test the performance of the controlled mass system, a sinusoidal force signal with amplitude 89N and frequency 6Hz is used as the reference force signal to the controlled mass system. The results from this test are shown in Fig. 7. Fig. 7a shows a comparison of the reference and measured forces produced by the controlled mass system generated both experimentally and from numerical simulations of the controlled mass system. Fig. 7b displays only the displacement of the experimental controlled mass system during this test.

It is seen that the actual controlled mass system is not performing as well as its numerical model did in the simulation. The experimental setup exhibits extra vibrations in the force reading and therefore the experimental implementation of inertial force control needs to be further explored before it can be used in substructure shake table tests. It is concluded that inertial force control, in its current state, is not yet ready for use in structural testing applications and that other control methods may need to be considered.

Other control techniques include both state space controller design and nonlinear controller design. However, both design methods are not as versatile as the frequency domain design techniques used in this study, since each controller is designed based on a priori knowledge of the reference signal. In most real-time substructure testing the reference force is not known a priori and thus other controller design techniques may not be the optimal choice in terms of actual implementation.



**Figure 7.** Experimental performance of the controlled mass system: a.) reference and achieved force comparisons ; b.) experimental displacement

Apart from exploring different design methods there are other strategies that could lead to more accurate results from the controlled mass system, namely, the addition of physical mechanisms to linearize the system such as springs. Nakata (2012) showed that a hydraulic actuator directly connected to a mass spring system is conducive to frequency domain control techniques. Thus other directions will be explored to investigate the performance and control of the controlled mass system discussed here so that inertial force control can be utilized in future substructure shake table tests.

## 6. CONCLUSIONS

This paper discussed the overall concepts of a new substructure shake table test method that uses a controlled mass system to impose the interface forces from the computational substructure back to the experimental substructure. Compatibility conditions were formulated and an implementation strategy (inertial force control) was proposed to account for the interface forces between the two substructures.

Numerical simulations were performed to evaluate the performance of the substructure shake table test method. The numerical implementation included models of the controlled mass system to simulate how the interface forces are applied to the experimental substructure. The controlled mass system was able to track the reference force with very little phase lag and only slight variations in magnitude. The substructured system was able to capture the major dynamic properties of the entire structure resulting in a fair comparison of substructured and entire responses. Following numerical simulations an experimental investigation of the controlled mass system was performed. The controlled mass system did not perform as well as its numerical counterpart but other approaches will be considered to further investigate the future of inertial force control.

Additional numerical simulations are needed to investigate the influential parameters in the numerical simulations of the substructure shake table test method. The experimental investigation of the controlled mass system and inertial force control is on going at the SSHT lab at the Johns Hopkins University.

## ACKNOWLEDGEMENT

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