The Non-uniform Random Seismic Response Analysis of Long-span Bridge Structure Based on Pseudo-excitation Method

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SUMMARY:
Most of the current studies on non-uniform earthquake response of long-span bridge structure are focused on deterministic analysis. In the paper, strong motion records with long-period information are firstly selected, whose response spectrum is taken as the target spectrum. Then the power spectrum density function is achieved with an iteration method. Afterwards, the relevant parameters are resolved by fitting the Clough-Penzien random vibration model. Finally, the non-uniform stochastic earthquake response of some long-span cable-stayed bridge is analyzed based on an improved pseudo-excitation of the direct displacement method with high efficiency. The results show that different apparent wave velocities have significant influences on the displacement response and the internal forces response of long-span bridge structure.

Keywords: Long-span cable-stayed bridge structure, Non-uniform random earthquake response, Fitting of power spectrum， Pseudo-excitation method， Numerical analysis

1. INTRODUCTION

The seismic input excitation is the premise and basis of the structural seismic response analysis. In engineering field, uniform excitation is the common method in process of structural seismic response analysis. The uniform excitation is suitable for dynamic analysis of the common high-rise structure and towering structure, but this method is not suitable for dynamic analysis of such long-span structures as bridge, stadium, nuclear power plant, dam etc. The reasons are that amplitudes and frequencies of the input earthquake wave are varying on different structural supporting points due to the influence of wave passage effect, partially coherent effect, site effect and wave attenuation. So, multiple-excitation considering spatial variability is the more reasonable input for dynamic analysis of long-span bridges.

At present, the random vibration method which takes full account of the seismic statistical probabilistic characteristics is commonly adopted. In order to avoid the disadvantage of expensive computation, the pseudo-excitation method is proposed by Lin J.H.. But in the traditional pseudo-excitation method, it is necessary not only to calculate the pseudo-static displacement and relative dynamic displacement respectively, but also to write the special program for extracting the pseudo-static matrix, which is a very onerous process. Two improved pseudo-excitation methods without expensive computation have been proposed in recent decades, one is direct displacement method and the other is large mass method. The authors have compared the two methods with the traditional random vibration method, and the result shows that the direct displacement method is more simple and accurate.

The seismic response of the cable-stayed bridge is different from that of other bridges, which contains obvious spatial coupling effect. Many researchers have studied the dynamic response of the cable-stayed bridge, Wu F.W. analyzed the seismic performance of Sutong cable-stayed bridge by the traditional random excitation method; Liu G.H. studied the random non-stationary seismic response of

Jiujang highway cable-stayed bridge on Yangtze River based on the fitted response spectrum. In this paper, 118 strong motion records on soft sites with reliable long-period information are firstly selected from seismic record libraries in different countries and regions. Then these records are fitted as target response spectrum which is then iteratively converted to power spectral density function. Afterwards, the long-period power spectrum is fitted based on Clough-Penzien stochastic seismic model with the least squares method. Finally, we adopt the fitted stationary earthquake power spectrum and the direct displacement method to analyze the random seismic wave passage effect under non-uniform excitations. Through the comparison between the above results and the ones under uniform excitation, the influences of wave passage effect to structural internal force and displacement are primary discussed.

2. SEISMIC RESPONSE ANALYSIS METHOD WITH MULTI-SUPPORT RANDOM EXCITATION

2.1. Equation of Motion under Multi-support Excitation

Structural motion equation with multi-support excitation is firstly proposed by Dibaj and Penzien in 1969, and its simplified version is:

$$M\ddot{Y}(t)+C\dot{Y}(t)+KY(t)=M\alpha E_s\ddot{x}_g \tag{2.1}$$

where *M*, *C* and *K* are the structural mass, damping matrix and stiffness matrix respectively; $Y(t)$ is the relative displacement vector; α is the pseudo-static matrix; E_s is the indicated vector of inertial force at non-bearing nodes; $\ddot{x}_g$ is the ground acceleration of structural bearings in the direction of seismic wave propagation.

2.2. Simulation on Random Earthquake Motion Field under Multi-support Excitation

In random vibration analysis, stochastic load is applied in the form of the power spectral density function. Spatial ground motion model in frequency domain is usually described as the following power spectral matrix:

$$S(\omega)=\begin{bmatrix} S_{11}(\omega) & C_{12}(\omega)+iQ_{12}(\omega) & C_{13}(\omega)+iQ_{13}(\omega) & \cdots \\ C_{12}(\omega)-iQ_{12}(\omega) & S_{22}(\omega) & C_{23}(\omega)+iQ_{23}(\omega) & \cdots \\ C_{13}(\omega)-iQ_{13}(\omega) & C_{23}(\omega)-iQ_{23}(\omega) & S_{33}(\omega) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}_{nn} \tag{2.2}$$

where $S_{ii}(\omega)$ is self-power spectrum; $C_{ij}(\omega)$ and $Q_{ij}(\omega)$ are the real and the imaginary parts of the mutual power spectrum respectively; $2\leq n\leq 10$. In the multi-support excitation, each excitation can be input according to the following relationship:

$$S_{ij}(\omega)=S_0(\omega)(e^{-i\omega d_{ij}}) \tag{2.3}$$

where $S_0(\omega)$ is the input power spectrum; $d_{ij}={\{D_{ij}\}\{v\}}/{v^2}$ is the consumed time during the seismic waves transmission from the i^{th} to the j^{th} excitation point; $\{v\}$ is the wave velocity; $\{D_{ij}\}=\{x_i\}-\{x_j\}$ is the distance between the i^{th} and the j^{th} excitation point.

2.3. Improved Pseudo-excitation Method

For the traditional pseudo-excitation method, it is necessary not only to calculate the pseudo-static displacement and the relative dynamic displacement respectively, but also to extract the pseudo-static matrix by writing special programs. At present, there are two improved pseudo-excitation methods to avoid such expensive computation. One is large mass method which forms the virtual force by adding additional large mass on bridge bearings. The other method is based on direct virtual displacement excitations which are derived from acceleration power spectrum. The authors have compared such two methods, and the result shows that direct displacement method is more simple and accurate which is adopted in our analysis. The basic principles of direct displacement method are briefly described as follows:

In the linear system under harmonic wave excitation, if the acceleration excitation is formulated as $\ddot{x} = -\omega^2 A\sin(\omega t)$, then the velocity and the displacement can be expressed as $\dot{x} = A\omega\cos(\omega t)$ and $x = A\sin(\omega t)$, where A is the amplitude of the excitation, ω is the frequency of excitation, and t is the duration time of excitation. Therefore, it can be drawn that $x = -\ddot{x}/\omega^2$ by the relationship between acceleration and displacement, then the acceleration excitation can be transformed into virtual displacement excitation. By exerting virtual displacement excitation on the bridge bearings, the power spectrums of response can be calculated as the squares of the response amplitudes.

3. STATIONARY STOCHASTIC VIBRATION MODEL AND THE PARAMETERS

3.1. Calculation of Target Response Spectrum

We firstly select 118 horizontal component records of typical long-period seismic acceleration from the libraries, whose data is obtained from Mexico M8.1 earthquake in 1985, Taiwan Chi-chi M7.6 earthquake in 1999 and Japan Tokachi-oki M8.0 earthquake in 2003. Then all selected seismic waves are dealt with by a uniform low-frequency error correction. Then the average amplification factor response spectrums are calculated. Finally, the fitting of average amplification factor response spectrum with least squares method is performed. The non-piecewise form is adopted to fit the long-period section:

$$\beta_i = \begin{cases} 1 + (\beta_{\max} - 1)\dfrac{T}{T_1} & 0 \le T < T_1 \\ \beta_{\max} & T_1 \le T < T_g \\ \left(\dfrac{T_g}{T}\right)^r \beta_{\max} & T \ge T_g \end{cases} \tag{3.1}$$

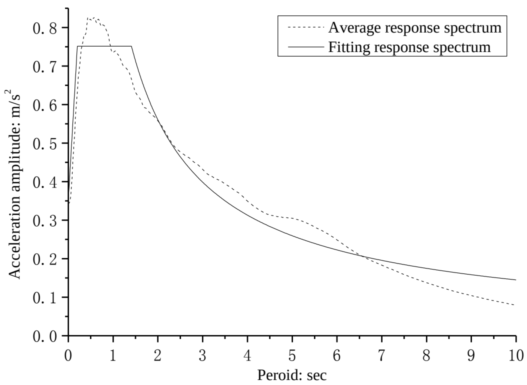


Figure 3.1. Average response spectrum and fitted response spectrum of soft soil site ($\xi = 0.05$)

For different damping ratios, the feature parameters of soft ground seismic response can be drawn. Due to the limited space, only the parameters for damping ratio $\xi = 0.05$ are listed here: $T_1 = 0.2$,

$r = 0.84$, $T_g = 1.41$, and $\beta_{\max} = 2.13$. Based on the given average amplification factors and the fitted amplification factors, and according to the provisions of seismic design code and maximal horizontal earthquake factors of the seismic fortification intensity of 7, the absolute acceleration response spectrum is calculated as the target response spectrum, see Fig. 3.1.

3.2. Transformation from Target Response Spectrum to Stationary Stochastic Earthquake Power Density

At present, many studies on transformation from target response spectrum to stationary stochastic earthquake power density have been carried out. The scholars studying this subject include Vanmarcke E.H., Maharaj K., Pfaffinger D.D., Jiang J.R., Sun J.J., Zhao F.X. etc. On the basis of previous works, the transformation method of stationary earthquake power density based on target response spectrum is adopted in this paper, and the steps are detailed in the following sections:

1) First, the initial value of ground motion power density $G^0(\omega_i)$ $(i = 0,1,...,N)$ is selected, and the constant $G^0(\omega_i)$ is white noise. In order to reduce the iterations, the proposed Eqn. 3.2 by Kaul in 1978 is used to calculate the initial value of ground motion power density, which is expressed as:

$$G(\omega) = \frac{\frac{\xi}{\pi\omega}}{-\ln[-\frac{\pi}{\omega T}\ln(1-r)]} R^2(\omega,\xi) \tag{3.2}$$

where $R(\omega,\xi)$ is the target response spectrum; ω and ξ are the structural natural frequency and damping ratio; $G(\omega)$ is the corresponding power spectral density; r is the probability of the response exceeding the response spectrum *R*.

2) Given an elastic oscillator with single degree of freedom as the case, let its natural angular frequency be ω_i, its damping ratio be ξ, and the unilateral power spectrum density of the input stationary stochastic seismic acceleration be $G(\omega)$, and according to the random vibration theory, the average maximum absolute acceleration response can be calculated as:

$$A_m(\omega_i,\xi) = p\sigma_0(\omega_i,\xi) \tag{3.3}$$

where

$$p = \sqrt{2\ln(\nu\tau)} + 0.5772 / \sqrt{2\ln(\nu\tau)} \tag{3.4}$$

$$\sigma_0(\omega_i,\xi) = [\int_0^\infty G(\omega)\frac{1+4\xi^2(\omega/\omega_i)^2}{[1-(\omega/\omega_i)^2]^2 + 4\xi^2(\omega/\omega_i)^2} d\omega]^{1/2} \tag{3.5}$$

Considering the narrow-band characteristic of response, the variable $\nu\tau$ are calculated according to the empirical formula proposed by Der K. in 1980. The Eqn. 3.5 can be calculated with the proposed analytical solutions by Pfaffinger D.D. in order to enhance the efficiency.

3) The iterative error can be defined as follows:

$$E(\omega_i) = \frac{|R_a(\omega_i,\xi) - \bar{A}_m(\omega_i,\xi)|}{R_a(\omega_i,\xi)} \times 100\% \tag{3.6}$$

Power density function of stationary stochastic seismic motion can be calculated iteratively according to Eqn. 3.3 to Eqn. 3.5 until the iterative error is below 1%. If the result does not meet the accuracy requirements, $G^k(\omega_i)$ will be rectified according to Eqn. 3.7, and the step 3) is repeated.

$$G^{k+1}(\omega_i) = G^k(\omega_i) \times \frac{R_a^2(\omega_i, \xi)}{\bar{A}_m^2(\omega_i, \xi)} \qquad (3.7)$$

Through such three steps, the stationary ground motion power density based on the target response spectrum can be obtained, which is illustrated as the dashed curve in Fig. 3.2.

3.3. Fitting of Stationary Ground Motion Power and Spectral-parameter

The Clough-Penzien model is proposed in 1975, which introduces the parameters for reducing the low-frequency components and is quite suitable for random seismic response of long-period structure. The expression of this model can be formulated as follows:

$$G(\omega) = \frac{1 + 4\xi_g^2 \dfrac{\omega^2}{\omega_g^2}}{(1 - \dfrac{\omega^2}{\omega_g^2})^2 + 4\xi_g^2 \dfrac{\omega^2}{\omega_g^2}} \times \frac{\dfrac{\omega^4}{\omega_1^4}}{(1 - \dfrac{\omega^2}{\omega_1^2})^2 + 4\xi_1^2 \dfrac{\omega^2}{\omega_1^2}} \times G_0 \qquad (3.8)$$

where ω_g and ξ_g are the dominant frequency and the damping ratio of the site soil; ω_1 and ξ_1 are the frequency and the damping parameter for turning low-frequency components; G_0 is the power spectrum density of white noise.

Therefore, such model is chosen to fit the stationary ground motion power spectral model, whose related parameters can be calculated by the least squares method. The fitted power spectral density is shown as the solid curve in Fig. 3.2. When the damping ratio is given as $\xi = 0.05$, other parameters are as follows: $\omega_g = 0.717\,rad/s$, $\xi_g = 0.395$, $\omega_1 = 3.471\,rad/s$, $\xi_1 = 0.770$, and $G_0 = 1.619\,m^2/s^3$.

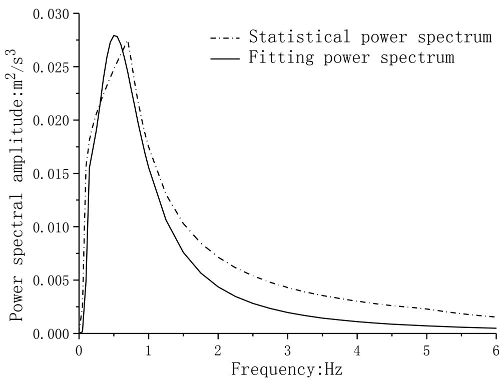


Figure 3.2. Statistical power spectrum and fitted power spectrum of soft soil site

4. ENGINEERING BACKGROUND AND THE FINITE ELEMENT MODEL

4.1. Engineering Background

The main bridge of some existing cable-stayed bridge adopts the half floating system with twin towers and double cable planes, which spans 44+136+336+136+44m. The main bridge girder is a combination section. The south tower and the north tower of the gate-shaped cable tower are 110.8m and 106.1m in height respectively. For simplicity, the cross beams, longitudinal girders, cable-pylons and transitional piers are simulated with the BEAM44 unit in ANSYS; the bridge deck is simulated with the SHELL63 unit; and the cables are simulated with the LINK10 unit.

The convergent points between the pylon beams and the bridge deck are free in the longitudinal and the vertical direction, and obey to master-slave constraints in the transversal directions; the convergent points between the bridge deck and the transitional piers are free in the longitudinal directions, and obey to master-slave constraints in the vertical and the transversal direction. The finite element model in this paper is illustrated in Fig. 4.1.

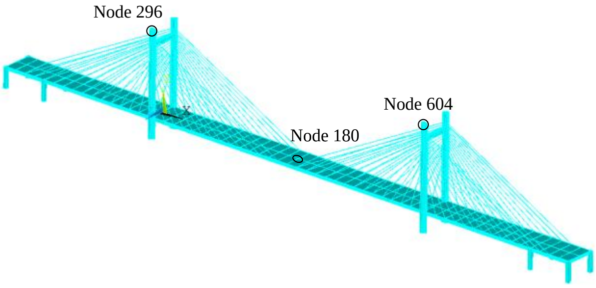
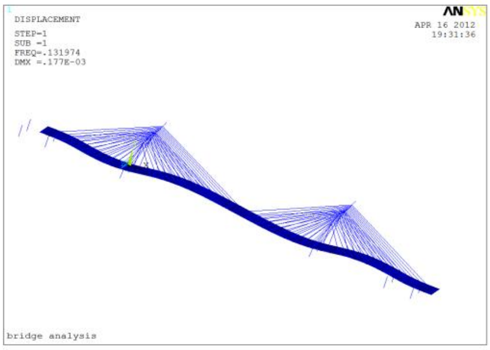


Figure 4.1. Finite element model of cable-stayed bridge

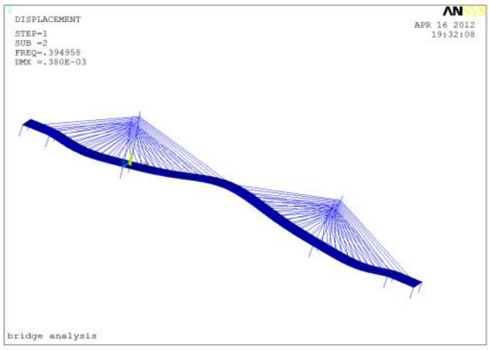
4.2. Analysis of Natural Vibration Characteristics

Table 4.1. Natural frequencies of the cable-stayed bridge

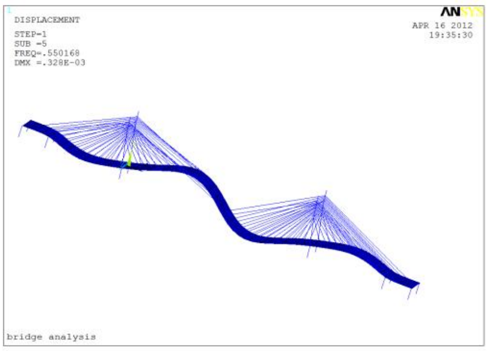
Order of vibration mode	Frequency/Hz	Order of vibration mode	Frequency/Hz
1	0.132	6	0.637
2	0.395	7	0.820
3	0.412	8	0.849
4	0.421	9	0.867
5	0.550	10	0.979



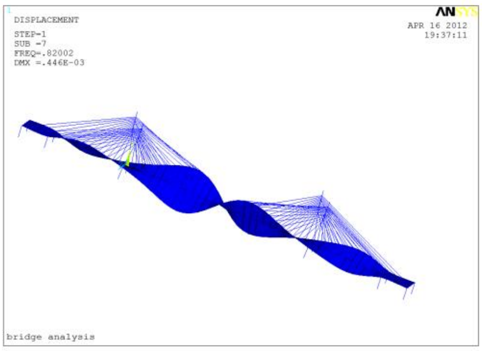
(a) The 1st vibration mode (Longitudinal drift)



(b) The 2nd vibration mode (Symmetric vertical bending)



(c) The 5th vibration mode (Antisymmetric vertical bending)



(d) The 7th vibration mode (Torsion)

Figure 4.2. Cable-stayed bridge vibration mode diagram

Dynamic characteristic analysis is the basis of seismic response analysis and seismic design. To better understand the seismic response characteristics of the cable-stayed bridge, the natural vibration characteristics are firstly analyzed, and then the first 50 modes are extracted and expanded with the accurate method of subspace iteration. The first 10 natural frequencies are listed in Tab. 4.1, and some of the corresponding modal pictures are shown in Fig. 4.2.

It can be seen from the table and the figures above that the basic natural period is 7.58 second, so this cable-stayed bridge is a typical long-period structure. In addition, it demonstrates the characteristics of 3D and coupling due to the intensive natural frequencies. The first mode is longitudinal drift, which makes a great contribution to the longitudinal seismic response of the bridge. The second mode is symmetric vertical bending, which have large affect to seismic response and wind-resistance stability of the bridge. The seventh mode is the torsional mode, which mainly affect the bridge flutter and wind-resistance stability. In brief, all the longitudinal drift mode, vertical bending mode and torsional mode have important significance on the seismic performance and wind-resistant performance of the cable-stayed bridge.

5. ANALYSIS OF EARTHQUAKE RESPONSE

5.1. Analysis of Wave Passage Effect

For studying the mechanism of wave passage effect to structural response, the authors adopt the direct displacement method to calculate the random seismic response under uniform excitation and non-uniform excitation respectively. Because there are six bridge piers connecting with ground, the non-uniform excitation contains six different excitation points. This study takes the assumption that the seismic waves spread from the left to the right with the constant velocity which equals to 300 m/sec, 600 m/sec and 1000 m/sec respectively.

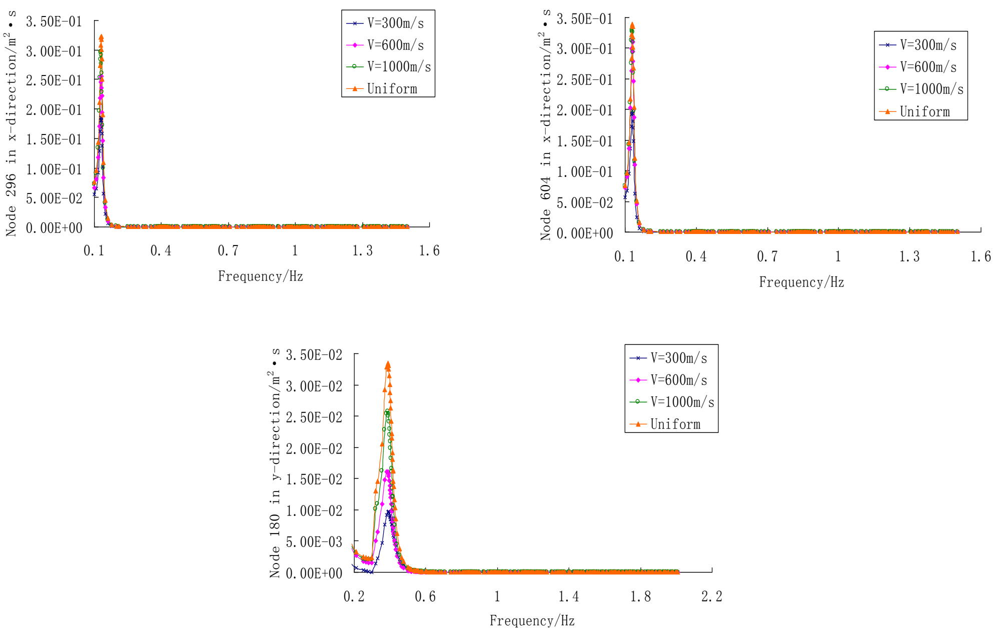


Figure 5.1. Comparative curves of displacement power spectrums of chosen nodes

In our study, 15 nodes from the bridge pylons and the bridge deck are chosen to analyze the wave passage effect. Owing to the limitation of space, Fig. 5.1 shows the absolute displacement power spectrums of only the left pylon vertex point (node 296), bridge deck middle point (node 180) and right pylon vertex point (node 604). From our analysis, we can see that the absolute displacement response power spectrum is quite small when the frequency is greater than 2Hz and is exceedingly large at $\omega = 0$. In addition, the commonly adopted power spectrum densities are trimmed for low frequency domain and is set to zero at $\omega = 0$. Thus, only the response power spectrum densities within the range of 0.1Hz~2.2Hz are provided.

By comparatively analyzing the results, when the apparent velocity is between 300m/s and 600m/s, the displacement power spectrums are much smaller than the corresponding results of uniform excitation, and the response of the right tower is 6% larger than that of the left tower at the same velocity. When the wave velocity equals to 1000 m/sec, the results of non-uniform excitations are almost consistent with the results of uniform excitation. That is because phase difference brought by wave passage effect becomes smaller gradually with the velocity increasing and the asynchronization of different excitation points tends to be consistent. It is also illustrated that the influence of wave passage effect to the bridge seismic response is closely related to wave velocity, and the influence will reach the maximum when the wave velocity falls into a specific range. The uniform excitation can be actually regarded as non-uniform excitation with infinite apparent velocity. The foundation motion of a bridge will have different phases under non-uniform excitation, which may leads to opposite effects such as additional uneven settlement of dynamic bearings and the inertia force offset of superstructure. So it is difficult to distinguish the advantage and the disadvantage of the wave passage effect to structural seismic by experience and the corresponding analysis is necessary for different structures.

5.2. Structural Peak Response Analysis

Based on the random vibration theory, various orders of response spectrum moments are calculated. Then the longitudinal bending moments and relative displacements of the left bridge pylon and the bridge deck are estimated for non-uniform excitations with different wave velocity. Details are illustrated in Fig. 5.2-Fig. 5.5.

The conclusions can be easily drawn that the peak bending moments of the pylon root have increased trend with the increasing wave velocity, and the peak bending moments at low wave velocity reduce as much as 20% compared with the results under uniform excitation. On the other hand, the peak moments of bridge deck reduce nearly 30% for the wave velocity between 300 m/sec and 600 m/sec. When the wave velocity is greater than 600 m/sec, it demonstrates increased tendency, although variation arises at some locations. When infinite wave velocities are applied, the results will tend to be consistent with the results of uniform excitation.

The wave passage effect makes the displacement peak of the pylon top about 32% lower, but the influence to the pylon bottom can be ignored. The displacement response of the left tower increases with the wave velocity increasing. The vertical peak relative displacements of main span are obviously 33% smaller than the results of uniform excitation. Also the response of the side spans has the same tendency but without too much distinctness, which means decreased influence of wave passage effect to the vertical displacement response of the main span. In conclusion, the wave passage effect is advantageous to the design of such cable-stayed bridge.

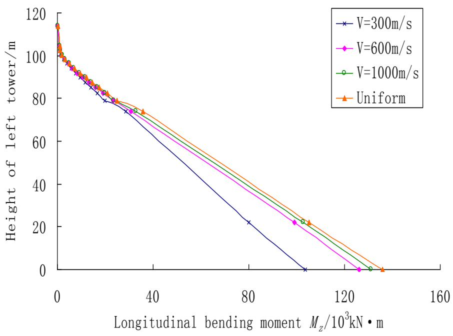


Figure 5.2. The peak value of vertical bending moment of left tower

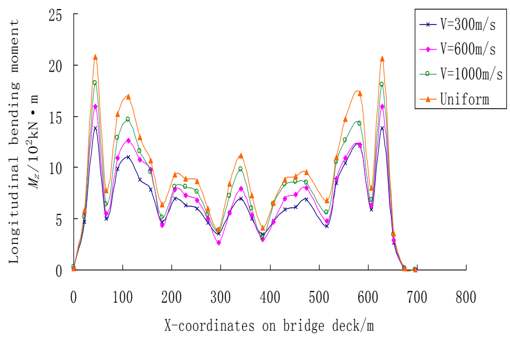


Figure 5.3. The peak value of vertical bending moment of bridge deck

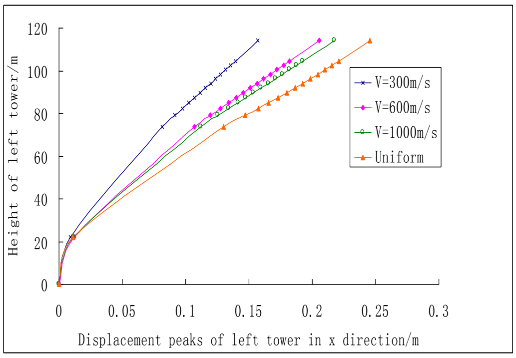


Figure 5.4. The peak value of relative displacement of left tower in x direction

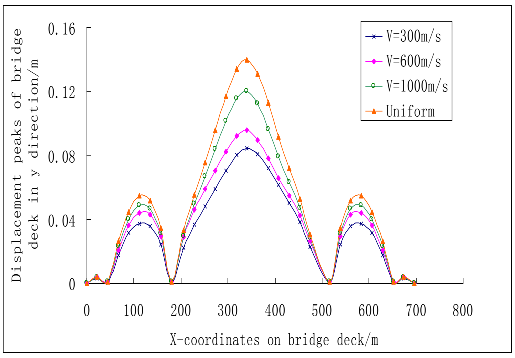


Figure 5.5. The peak value of relative displacement of bridge deck in y direction

6. CONCLUSION

In the paper, strong motion records with reliable long-period information are firstly selected, and the fitting process with least squares method is performed based on Clough-Penzien random vibration model. Then based on the input of the fitted stationary ground motion power spectrum and the direct displacement method, stochastic seismic wave passage effect of some long-span cable-stayed bridge under non-uniform excitation is carried out, and conclusions are drawn as follows:

1) The natural period of the cable-stayed bridge is as long as 7.58s, so it is a long-period structure. Its natural vibration has such characteristics of long-period, dense frequency distribution and uniform variation. In addition, the modes appear apparently three dimensional and mutual coupling.

2) The lower the wave velocity is, the more obvious the wave passage effect is, especially when the wave velocity is between 300m/s and 600m/s. With the wave velocity increasing (greater than 600m/s), structural seismic response has a trend to be consistent with the results of uniform excitation. The wave traveling effect to the structural seismic response has close relationship with the wave velocity of the input excitation and reaches maximum within a special range.

3) As shown in the absolute displacement power spectrum graph of selected nodes, fundamental frequencies have great influence on structural seismic response. The displacement spectrum in low frequency domain is much larger than that in high frequency domain. And the response of the right tower is 6% larger than that of the left tower at the same velocity

4) Due to the wave passage effect, bending moments power spectrum peaks of the tower bottoms and main spans of the bridge deck reduce as large as 20% and 30% respectively, and the variation of the tower tops and the side spans can be negligible; displacement power spectrum peaks of the tower tops and main spans of the bridge deck reduce 32% more or less, and only little variation emerges at the tower bottoms and the side spans. The influence of travelling wave effect to structural response is closely related with structural dynamic characteristics, component positions, apparent waves as well as response types (internal force or displacement).

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