SUMMARY:
The collapse-resistant capacity of structures is closely related to the failure mechanism, and the strong-column weak-beam global failure mode is what is expected for structural design. Based on this demand, this paper focuses on the development of plastic design methodology using energy balance. Considering the different structural system having different hysteretic behaviors that affect the cumulative inelastic strain energy, the modified energy balance equation is proposed. Since periods are principal quantities governing the energy input of MDOF systems, the multiple vibration periods and modes of MDOF are taken into account for the determination of total energy input. Design results of a 7-storey RC frame using the proposed energy-based procedure show that the energy-based designed frame can achieve a uniform storey drift and reduce the local member damage while having the potential to form strong-column weak-beam plastic hinges distribution, which gives good evidence to the proposed energy-based seismic design method.

Keywords: plastic design; failure mode; energy balance; strong-column weak-beam mechanism; RC frame

1. INTRODUCTION

Evidence from recent severe earthquakes such as Wenchuan earthquake 2008, et al, indicates that the current earthquake-resistant design of structures could not guarantee the strong-column weak-beam failure mechanism. Current seismic design codes (e.g., Code for Seismic Design of Buildings, 2010) are generally based on the elastic vibration condition, or elastic method. Code-conforming design is carried out in accordance with force-based linear elastic analysis method when the design base shear on behalf of seismic activities and lateral force distributions have been known, which is an iterative process to satisfy the strength and drift requirements. As a consequence, the inelastic activity which may include severe yielding and buckling of structural members and connections, can be unevenly and widely distributed, which may result in rather unpredictable response and undesirable collapse failure modes induced by severe ground motions. Therefore, it is imperative to develop a plastic design approach using strong-column weak-beam mechanism (Leelataviwat and Geol, 2002; Liao, 2010). An obvious characteristic of structural excursions into plastic behaviours is the energy dissipation capacity, which has not been considered into the structural design, however. In an effort to develop plastic limit-state design of frame structures using energy-based approaches, this investigation mainly focus on the implementation of strong-column weak-beam failure mechanism with a proposed energy-based plastic design method. More considerations regarding the energy balance equation and the seismic energy input would be investigated.

2. FAILURE MECHANISM ANALYZED

Structural energy dissipation capacity is directly related to the location and number of plastic hinges, which form different failure modes in different sequential order. Frame structures under lateral earthquake forces have many failure modes and structural failure is mainly caused by unreasonable
seismic failure modes. To sum up, different failure modes can be summarized as three main failure types: local mechanism, soft-storey mechanism and global mechanism shown in Fig. 1. Local failure mechanism and soft-storey mechanism are required to avoid when conducting structural design due to the lower lateral drift, energy dissipation capacity and ductility level. Global failure mechanism is kinematically admissible mechanism and the embodiment of strong-column weak-beam design criterion. This mechanism require that the flexural plastic hinges form at the ends of all beams combined with base columns, which could provide greatest possible number of plastic hinges to absorb seismic energy and result in the nearly uniform storey drift to realize a admirable level of ductility. In this study, global mechanism is designated as the energy dissipation mode and developing an energy-based approach to control failure mode is the primary aim.

![Figure 1. Main collapse failure modes](image)

### 3. STRUCTURAL SEISMIC ENERGY BALANCE

#### 3.1. Modified energy balance equation

For structural systems subjected to earthquake ground motions, the energy balance could be formulated as a supply-demand problem. This is due to the fact that all the seismic demands must be estimated and be satisfied with adequate energy-dissipating capacity along with strength, stiffness and ductility to absorb energy supply induced by intense ground excitations. Hence, the energy balance equation can be derived as follows (H. Akiyama, 1985):

\[ E_c + E_p + E_\xi = E_I \]  \hspace{1cm} (3.1)

Where \( E_c \) is the elastic vibrational energy, \( E_p \) is the inelastic strain energy, \( E_\xi \) is the energy consumed by damping mechanism and \( E_I \) is the total energy input exerted by an earthquake. Since the \( E_\xi \) will dissipate part of the seismic energy input, the \( E_d \) defined as \( E_I - E_\xi \), will contribute to the damage of structures. Therefore:

\[ E_c + E_p = E_I - E_\xi = E_d = \lambda E_I \]  \hspace{1cm} (3.2)

Where \( \lambda \) is the modification factor of earthquake energy input due to damping, and the factor is a function of damping ratio: \( \lambda = \frac{1}{1 + 3\xi + 1.2\sqrt{\xi}} \) (H. Akiyama, 1985). For structural systems with reduced hysteretic behaviours, the energy dissipation capacity would decrease due to the reduction of areas enclosed by hysteretic loops with pinching and degradation. To consider the reduced energy dissipation capacity, a factor \( \eta \) is introduced to modify the inelastic strain energy \( E_p \):

\[ E_c + \eta E_p = \lambda E_I \]  \hspace{1cm} (3.3)
How to determine $\eta$ will be discussed with more detailing in the latter part. Eqn. 3.3 gives the energy balance equation which is applicable for all structural systems. To drive the energy-based design approach into practice, many efforts have been made by researchers (Housner 1956, Akiyama 1985, Uang 1988, Leelataviwat 1999, Teran-Gilmore 2003, et al). Leelataviwat et al (1999) proposed a performance-based plastic design framework in an innovative manner to eliminate the need for the drift check using yielding mechanism and target drift, the basic idea of which is deriving the ultimate design base shear and distributing the lateral design forces to structural members through energy balance concept shown in Fig. 2. The energy-based seismic design is primarily based on the principle that the work needed to push the structures to the target drift is equal to the energy dissipated by plastic hinges of all beams and base columns.

![Figure 2. Illustration of energy balance concept](image)

3.2. Seismic Energy Input

In the undamped elastic system, total energy input is produced highly selectively by a single component with the fundamental frequency, and in other words, the natural, or natural period and mass of vibration system, are to be one of principal entities which govern the total energy input of the system. However, for damped systems and reduced hysteretic systems, periods are principal quantities which govern the energy input of these systems (H. Akiyama, 1985). For multiple degree of freedom structural systems, all of the vibration frequencies or periods contribute to the energy input induced by earthquake excitations which are full of a variety of frequency components. The total seismic energy input can be expressed as (Bai, et al, 2012):

$$E_1 = \sum_{n=1}^{N} \Gamma_n^2 \cdot E_{SDOF,n}$$  

(3.4)

Where $E_{SDOF,n}$ is the seismic energy input of $n$th-mode SDF system subjected to $\ddot{\ddot{x}}_n(t)$ and $\Gamma_n$ is the modal participation factor which can be expressed as follows:

$$\Gamma_n = \frac{L_n}{M_n^*}, \quad L_n = \phi^\top M l, \quad M_n^* = \phi^\top M \phi$$  

(3.5)

Where $\phi$ is the natural mode of $n$th frequency; $M$ is the mass matrix of MDOF system; each element of the influence vector $l$ is equal to unity. Deriving from Eqn. 3.4, the seismic energy input of multistorey MDOF can be calculated through a superposition of every $n$th-mode SDOF energy input with the proportional factor of $\Gamma_n^2$. So to get the energy input of MDOF only needs to perform the mode analysis to get $\Gamma_n$ and calculate the energy input of corresponding $n$th-mode SDOF. Through Housner’s assumption (G. W. Housner, 1956), the seismic energy input of the above $n$th-mode SDF
system can be approximated as follows:

$$E_{SDOF, n} = \frac{M'_a V_{max,n}^2}{2} = \frac{M'_a S_{v,n}^2}{2}$$

(3.6)

In many design codes (e.g., Chinese Seismic Design Code), the velocity response spectra have not been defined. Ideally, it also can be developed using the same method and same ground motion data that the acceleration response spectra do. However, most code-based design velocity and displacement spectra are generated from the acceleration spectra assuming the peak response is governed by the equations of steady-state sinusoidal response (M.J.N. Priestley et al., 2007). Therefore, knowing one of the spectra, the other two can be obtained using their physical presences of relationships. As a consequence, $E_{SDOF, n}$ can be expressed as:

$$E_{SDOF, n} = \frac{M'_a S_{v,n}^2}{2} - \frac{1}{2} M'_a \left( \frac{S_{v,n}}{\omega_n} \right)^2 - \frac{1}{2} M'_a \left( \frac{T_{p,n}}{2\pi} S_{a,n} \right)^2$$

(3.7)

Where $T_{p,n}$ is the period of $n$th mode, and $S_{a,n}$ is the corresponding pseudo-acceleration response spectra. It is must be pointed out that the aforementioned presentation is based on the assumption that the response of a MDOF structural system can be expressed as superposition of the responses of each $n$th-mode SDOF system, which is appropriate in the liner elastic range. Of course, since the superposition principle does not apply in nonlinear system, the assumption seems to be arbitrary. However, such an assumption constitutes the basis on which the energy-based design method is built. Therefore, the classical structural dynamic theory is used to build the seismic energy input, and the $n$th mode frequency $\omega_n$ and modal vectors $\Phi_n$ are supposed to be constant.

The total seismic energy input $E_t$ is a function of two factors: natural periods and modal vectors which directly determine the values of generalized mass $M'_a$ and modal participation factor $\Gamma_n$. The two factors are influenced by member stiffness of structural system. The element stiffness is generally based on the gross section stiffness. It should be seen that the stiffness will degrade due to crushing of concrete, softening of longitudinal reinforcement and cumulative damage on plastic-hinge zones. Thus, it is required to have a reduction of element stiffness to perform the modal analysis to determine the natural periods and mode vectors. Some codes (ATC-32, 1996; Eurocode 8, 1998) specify 50% stiffness reduction of the initial elastic stiffness, while in New Zealand Concrete Design Code, the value is as low as 35% for beams. In Chinese Seismic Design Code, the recommended stiffness of concrete members is 85% of the gross section stiffness when calculating the elastic deformation level and the proportion of 85% is adopted in this study.

3.3. Elastic Vibrational Energy

For elastic vibrational energy $E_e$, H. Akiyama (1985) gave the following expression:

$$E_e = \frac{1}{2} V_y \Delta_y = \frac{1}{2} V_y \times \frac{V_y}{M \omega_n^2} = \frac{1}{2} M \left( \frac{T_e}{2\pi} \frac{V_y}{W} \right)^2$$

(3.8)

Where $W$ is the seismic weight of structure, $T_e$ is the fundamental elastic period, $g$ is the acceleration due to gravity, $V_y$ is the design base shear.

3.4. Inelastic Strain Energy

As mentioned above, the energy-based seismic design is based on the approximation that the work needed to push the structures to the target drift is equal to the energy dissipated by plastic hinges of all beams and base columns. This implies that the structural type and material are not included in the
design considerations since only unidirectional deformation of plastic hinges is used in the design process discussed in later part, i.e. the hysteretic behavior of structures can not be taken into consideration. It is required that the structural system have a stable hysteretic behaviors with no degradation and pinching which is case of typical ductile steel frame structures, shown in Fig. 3. For simplicity, the stable hysteresis curve can be equivalent to the bilinear model, where $A_F$ is the area of enclosed by full hysteresis loop and $ARPP$ is the area of rigid-perfectly-plastic loop which encompasses the hysteresis loop of area $A_F$ (H. M. Dwairi et al, 2007).

![Figure 3. Stable hysteresis loop and its model](image)

$\Delta_{AF} = \text{Area of enclosed by full hysteresis loop with no degradation and pinching}$

$\Delta_{max}$

$K_I$ R-P-P

$\Delta_{AP} = \text{Area of enclosed by hysteresis loop with degradation and pinching}$

However, it is important to note that actual hysteresis loops of structures are not perfect, i.e., they are reduced in areas or pinched. For such buildings, calculation of inelastic strain energy based on Fig. 3 will overestimate the energy dissipation capacity, resulting in the excessive damage and absence of pre-defined failure modes. Therefore, to make the energy-based method more appealing for design procedures of structural systems, the structural hysteretic behaviors which have direct influence on energy dissipation capacity, must be considered. For reduced hysteretic structural systems, such as reinforced concrete structures, the hysteretic behaviors have the characteristics of stiffness degradation, strength deterioration and pinching effects, which are shown in Fig. 4, where $A_P$ is the area of enclosed by reduced hysteresis loop. To quantify the energy dissipation capacity of reduced hysteretic structural systems reasonably, a correction factor is introduced to modify the energy balance equation, which is defined as follows:

$$\eta = \frac{A_P}{A_F} = \frac{A_P}{ARPP} \left( \frac{A_F}{ARPP} \right)^{-\frac{3}{2}}$$ (3.9)

It can be seen that $A_F$ and $ARPP$ are constants for a specified hysteresis loop and as a result, $\eta$ is a function of the only parameter of $A_P$ which is dependent on structural type and material. By calculating the areas, $A_F/ARPP$ can be obtained as follows:

$$\frac{A_F}{ARPP} = \frac{(\mu - 1)(1 - r)}{\mu(1 + r - \mu - r)}$$ (3.10)

Where $r$ is the second slope stiffness ratio and $\mu$ is ductility factor ($\mu = \Delta_{max}/\Delta_y$). For a given $r$ and $\mu$, the value of $A_F/ARPP$ can be obtained. In order to apply energy-based seismic plastic design to a variety of structural types, it is imperative to develop a practical procedure to consider the discrepancy of energy absorption of hysteretic systems. Notably, different structural systems have different hysteretic behaviors, i.e. hysteretic models and hysteretic rules. Four hysteretic models were selected for this study: Elasto-Plastic model, small Takeda model, Large Takeda model and Ring-Spring model. Deriving from the determination of hysteretic damping, $A_F/ARPP$ can be expressed as follows:

$$\xi_H = 2\pi \frac{A_F}{ARPP} \Rightarrow \frac{A_F}{ARPP} = \frac{\pi}{2} \cdot \xi_H \Rightarrow \eta = \frac{\pi \mu(1 + r - \mu - r)}{2(\mu - 1)(1 - r)} \cdot \xi_H$$ (3.11)

Where $\xi_H$ is the hysteretic damping. Taking into account the effect of hysteretic model type and
effective period, H. M. Dwairi et al (2007) proposed new hysteretic damping equations which have obvious relationship to the theoretical area-based approach of Eqn. 3.11, and \( \xi_H \) is represented in the Eqn. 3.12, where \( C \) depends on the hysteretic rule and effective periods and the values are shown in Table 3.1.

\[
\xi_H = C \left( \frac{\mu - 1}{\pi \mu} \right) \Rightarrow \eta = C \cdot \frac{1 + r \mu - r}{2(1 - r)}
\]

(3.12)

**Table 3.1. Coefficient of \( C \) by H. M. Dwairi (2007)**

<table>
<thead>
<tr>
<th>Model</th>
<th>Ring-Spring</th>
<th>Large Takeda</th>
<th>Small Takeda</th>
<th>Elasto-Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{eff} &lt; 1s )</td>
<td>0.3+0.35(1-( T_{eff} ))</td>
<td>0.65+0.5(1-( T_{eff} ))</td>
<td>0.5+0.4(1-( T_{eff} ))</td>
<td>0.85+0.6(1-( T_{eff} ))</td>
</tr>
<tr>
<td>( T_{eff} \geq 1s )</td>
<td>0.3</td>
<td>0.65</td>
<td>0.5</td>
<td>0.85</td>
</tr>
</tbody>
</table>

For the design of any structural system, \( \eta \) can be obtained when having determined the hysteretic model appropriately and ductility level for a given performance objective. Substituting Eqn. 3.4 and Eqn. 3.8 into Eqn. 3.3 gives:

\[
E_p = \frac{\lambda E_i - E_e}{\eta} = \left( \frac{\lambda \cdot \sum_{n=1}^{N} I_n^2 \cdot E_{SDOF,n} - M}{2 \cdot \left( \frac{T_c}{2\pi} \cdot \frac{V}{W} \cdot g \right)^2} \right) \cdot \eta^{-1}
\]

(3.13)

### 3.5. Design Base Shear and Lateral Forces

![Formation of global failure mechanism](image)

**Figure 5. Formation of global failure mechanism**

The formation of the global failure mechanism due to lateral seismic forces and gravity load is shown in Fig. 5. Assuming the lateral forces distribution in the following form (Shih-Ho Chao, 2007):

\[
F_i = \begin{cases} 
V_y \left( W_i h_i / \sum_{j=1}^{N} W_j h_j \right)^{0.75n+1} ; i = N \\
V_y \left( \sum_{j=1}^{N} W_j h_j / \sum_{j=1}^{N} W_j h_j \right)^{0.75n+1} - \left( \sum_{j=1}^{N} W_j h_j / \sum_{j=1}^{N} W_j h_j \right)^{0.75n+1} ; i = 1: n-1 
\end{cases}
\]

(3.14)

Where \( W_j \) is seismic weight of \( j \)th floor and \( h_j \) is the height of \( j \)th floor from ground. The plastic strain energy is derived by the lateral forces with a uniform plastic rotation \( \theta_p \). Therefore, the following expression can be established:

\[
E_p = \sum_{i=1}^{N} F_i h_i \theta_p = \sum_{i=1}^{N} \mu_i V_y h_i \theta_p = V_y \left( \sum_{i=1}^{N} \mu_i h_i \right) \theta_p
\]

(3.15)

Where \( \mu_i = F_i / V_y \), and \( \mu_1 + \mu_2 + \cdots + \mu_N = 1 \). Equating Eqn. 3.13 with Eqn. 3.15 gives:
It is must noted that the $P-\Delta$ effects is not included in the Eqn. 3.16. In the process of lateral forces pushing the structure monotonously to the target drift, the gravity will contribute to the damage of structure since the structure “drop slightly” due to the lateral drift. The design lateral forces can be expressed as: $F_{i}^{u} = F_{i} + \Delta F_{i} = \mu V_{y} + W/\theta_{u}$, where $\Delta F_{i}$ is the extra lateral force due to $P-\Delta$ effects.

4. STRUCTURAL PLASTIC DESIGN

Current well-established Capacity Design Method (T. Paulay and M. J. N. Priestley, 1992) is used widely to ensure the strong-column weak-beam failure mechanism. However, as mention above, this design method could not guarantee the elimination of column yielding which has been confirmed by researchers and post-earthquake surveys (Kuntz and Browning, 2003; Jinping Ou et al., 2008). This is due to the fact that the structural design for a given performance objective is based on the elastic analysis method which is not suitable when structure members experience inelastic excursions into nonlinear behaviors, and a single column-to-beam strength ratio is adopted to ensure the global mechanism. To ensure the formation of the global failure mechanism, Sutat Leelataviwat et al (1999) proposed a practical design procedure of moment-resisting steel frame structures which is based on the conventional plastic design concept with some modifications. Wen-Cheng Liao (2010) extended this plastic procedure into the design of reinforced concrete frame structures.

In the proposed plastic design procedures, the moment of column-base can be determined based on design criteria that soft-storey failure mechanism would not occur in the first storey for amplified design lateral forces with an overstrength factor to account for the overstrength due to strain hardening. For the design of beams, it can be determined by plastic design approach based on the philosophy that external work done by design lateral forces equals to internal work absorbed by all beam end plastic hinges and column-base end plastic hinges, for a kinematic rotation of the whole structure into the global mechanism. Specially, a distribution factor of beam strength which is based on the design storey shear is introduced to proportionate the beam moments for all storey levels. The distribution of moment in the columns can be figured out by subjecting the column to the updated design forces whose magnitude can be obtained by equating the overturning moment to the moments generated by the fully strain-hardened and material-overstrength floor beams. When all member internal forces have been found, the section detail design can be carried out under the specified design provisions, such as ACI 318 and Chinese Code for Design of Concrete Structures. It is must be noted that the whole design considerations are based on the assumption that joint and shear failure would not occur provided that enough transverse reinforcement and efficient detailing are confirmed.

5. DESIGN EXAMPLES

To implement the energy-based design approach, a seven-storey RC planar frame is built (see Fig. 6). For comparison, the code-conforming design results of the 7-storey building can be obtained as well as material behaviors (Bai jilin, et al, 2011). The first three vibration modes and periods of the building for 15% reduction of the gross section stiffness are shown in Fig. 7. The vibration periods for the first three modes are 1.28, 0.41 and 0.23 s, respectively. The fundamental period for linearly elastic vibration is 1.18 s. The first three periods are used to determine the seismic energy input, which could achieve a good approximation for the total energy input. The yielding drift can be developed from the yielding curvature expression (M. J. N. Priestley, 2007): $\theta_{y} = 0.5\varepsilon_{y}L_{b}/h_{b}$, where $\varepsilon_{y}$ is the yielding strain of beam reinforcement, $L_{b}$ is the beam span and $h_{b}$ is the beam depth. Therefore, the ductility factor $\mu = \theta_{u}/\theta_{y}$ and $\theta_{p} = \theta_{y} - \theta_{u}$. For the 7-storey frame, $\theta_{y} = 0.0075$, $\theta_{u} = 0.02$, and as a result, $\mu = 2.67$, $\theta_{p} = 0.0125$. 

\[
V_{y} = -\frac{4\eta W\pi^{2} \left(\sum_{n=1}^{N} \mu_{n} h_{n}\right) \Theta_{\theta}}{T_{\gamma} g} + \frac{4\eta W\pi^{2} \left(\sum_{n=1}^{N} \mu_{n} h_{n}\right) \Theta_{\theta}}{T_{\gamma} g} + \frac{8\lambda W\pi^{2} \left(\sum_{n=1}^{N} F_{n}^{2} \cdot E_{ADOF,n}\right)}{T_{\gamma} g}
\]
The small Takeda model \((r=0.1)\) is used and the correction factor of inelastic strain energy \(\eta\) can be obtained with the value of 0.32. OpenSees software (McKenna et al, 2004) is used to implement the nonlinear analysis. The beams and columns are modeled using nonlinear beam-column elements with fiber section which can simulate the interaction of moment and axial load. Three ground motions of 1940 El Centro (Imperial Valley, USA), 1992 Landers (Yermo Fire, USA) and 1989 Loma Prieta (Capitola, USA) were selected as the ground motion excitations which were scaled to a peak ground acceleration (PGA) of 400 cm/s\(^2\) to represent rare earthquakes.

Fig. 8 shows the storey drift ratios of RC frame designed by Code-conforming provisions and energy-based method. It can be seen that the structure designed by energy-based method has a uniform storey drift with obvious reduction of peak interstorey drift, which demonstrates the structure can prevent damage concentrating on no specific storey to form soft-storey mechanism. To investigate the local member damage, the distributions of plastic hinges along with the plastic rotations of element ends are illustrated in Fig. 9. For convenience of demonstration, the plastic rotations of structure subjected to each ground motion fall into four categories. As mentioned above, the current code-conforming design could not guarantee the absence column yielding, which is validated by the case study. However, it is well represented in Fig. 9 that the energy-based design method can fulfill the design philosophy of strong-column weak-beam, indicating the implementation of the global failure mechanism. Also, the energy-based designed structure has more plastic hinges to dissipate the earthquake-induced energy, and as a result, the member damage (plastic rotations) can be reduced comparable to that of the code-conforming structure. It is of great importance that excessive damage of local members will have direct influence on the collapse-resistant capacity, so from this point of view, the energy-based design approach has a potential to enhance the earthquake-resistant capacity.
6. CONCLUSIONS

To implement the design procedure of global failure mechanism (strong-column weak-beam), this investigation developed an energy-based seismic design with a more reasonable manner to consider the energy balance equation. Considering the effects of reduced hysteretic behaviors of structural systems on the energy dissipation capacity, a modified energy balance equation is derived with a unified form. This modification mainly depends on the areas of hysteresis loops for different reduced hysteretic systems and the modification value is a function of hysteretic model and levels of ductility. The multiple vibration periods and modes are taken into account for the determination of total energy input since periods are principal quantities governing the energy input of MDOF hysteretic systems. For practical design, the first few periods and modes can be used to calculate the total energy input with an admirable accuracy. The structural design procedure using energy-based approach can be derived from the mechanism of global failure. Design example of a 7-storey RC frame structure is conducted using the proposed energy-based procedure and code-conforming provisions. The results of nonlinear time-history analysis show that the energy-based designed frame can achieve a uniform storey drift and reduce the local member damage comparable to that designed by code-conforming while having the potential to form strong-column weak-beam plastic hinges distribution, which gives good evidence to the proposed energy-based seismic design method.

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