Development of Fragility Functions for Seismic Damage Assessment Using Kernel Smoothing Methods

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SUMMARY:
Efficient and reliable assessment of structural damage immediately following an earthquake is important to facilitate emergency response and mitigate losses from subsequent structural failures. With recent developments in structural health monitoring systems, more ground motion and structural response data are available. From these measured data, we can estimate seismic damage of the structure using the probabilistic relationships between structural responses and the corresponding damage state of a structure, which is defined as a fragility function in performance-based earthquake engineering. In this paper, we introduce a new framework for developing fragility functions using various kernel smoothing methods on the basis of acceleration responses. This framework is applied to simulated data using an analytical model of a four-story steel moment-resisting frame. As a result, we can obtain fragility functions that are smooth and densely represented by utilizing the data more efficiently.

Keywords: fragility function, kernel smoothing, seismic damage assessment, sparse data, non-homogeneous data

1. INTRODUCTION

Performance-based earthquake engineering (PBEE) has received increasing attention among structural engineering researchers and practitioners (e.g., Ghobarah, 2001; Krawinkler and Miranda, 2004) in order to predict the performance of a structure subjected to earthquakes in a probabilistic manner (SEAOC, 1995; Ghobarah, 2001; Porter et al., 2007). PBEE is a methodology in which the design criteria are expressed in terms of achieving performance objectives when the structure is subjected to various levels of seismic hazard (Ghobarah, 2001). In the conventional PBEE framework, conditional probabilities of the following four parameters are defined progressively to compute annual loss rate of a structure due to earthquakes: intensity measure (IM), which quantifies the intensity of an earthquake ground motion, engineering demand parameter (EDP), which represents the structural response to the earthquake, damage measure (DM), which describes the discrete physical damage state of the structure, and decision variable (DV), which relates to the actual loss, such as casualties, downtime, and monetary loss. Among these parameters, a fragility function is defined as the probability of a DM conditioned on an IM or an EDP to predict the probability of the structure being in a specific damage state given a certain intensity of an earthquake.

In this paper, we adopt the fragility function from PBEE to estimate seismic damage of a structure using acceleration measurements of the structural responses recorded during an earthquake. In PBEE, fragility functions often involve estimating the damage as a function of structural response, such as the story drift ratio (SDR) and the peak floor absolute acceleration. Structural displacements are, however, difficult to estimate accurately in practice. Therefore, we define fragility functions that relate acceleration responses of the structure to SDR, which in turn can be used for damage classification. Noh et al. (2011b) showed that their wavelet-based damage sensitive feature (DSF) extracted from structural acceleration responses is more highly correlated with SDR than other conventional measures, such as spectral acceleration and peak roof acceleration. In addition, the conventional data binning method may not be appropriate for constructing fragility functions using sparse or non-homogeneous data. To address this problem, kernel smoothing methods are applied. A kernel is a weighting function that is used to estimate a probability distribution function from noisy observations (Want, 1995). In other words, we can estimate a function output (e.g., damage state) at a given input value (e.g., structural response) as a weighted sum of output observations using kernels. More details are given in section 2.3.

The framework presented here consists of first obtaining the structural responses and the resulting damage states from an analytical model or an instrumented structure of interest. We then extract a DSF from each structural response and apply the kernel smoothing methods to define the probabilistic mapping between the DSF and the damage state The kernel method was first introduced in fragility analysis by Noh et al. (2011b). In addition to the kernel smoothing method of Noh et al. (2011b), two alternative methods for computing the probabilistic mapping that give different levels of information are introduced in this paper. For validation, the framework was applied to the simulated data obtained from the analytical model of the four-story steel special moment-resisting frame subjected to a set of scaled earthquake ground motions (Lignos and Krawinkler, 2009). The results show that the kernel smoothing methods can construct smooth and continuous fragility functions, unlike the data binning method which results in sparse and discrete functions.

2. FRAMEWORK FOR DEVELOPING FRAGILITY FUNCTIONS

A framework for developing fragility functions for structures subjected to earthquake ground motions has been developed using a wavelet-based DSF. The DSF is computed from each floor absolute acceleration response and is used as a indicator of structural damage. The framework consists of three steps: (1) collecting absolute acceleration response data and corresponding damage state from a structure subjected to various intensities of seismic loading; (2) extracting DSF values from these data using appropriate statistical pattern recognition methods; and (3) constructing fragility functions using kernel methods. It is assumed that, as with PBEE, a reliable analytical model of a structure or information from an instrumented building, which is sufficient for developing such a model, is available. The three steps of the framework are summarized in Figure 2.1, and more details of the procedure are described in the following sections.

2.1. Data Collection

In the first step of the framework (Figure 2.1), an analytical model of a structural system is subjected to a set of ground motions using incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2002). The IDA involves a set of ground motions, which is scaled to various intensities and applied to a structure to evaluate its seismic performance under various intensities of loading. The set of ground motions can be selected by numerous methods (Katsanos et al., 2009), and each ground motion is scaled by a set of scale factors. Since the statistical techniques are used in the framework as discussed in the following sections, we need to have an adequate amount of unbiased structural response data for various damage states. Vamvatsikos and Cornell (2002) use 40 records to compute statistics for different engineering demand parameters (EDPs) of a structural system. In this paper, the wavelet-based DSF, which is introduced in Noh et al. (2011a;b), is used as a measure of seismic performance of a structure; thus, the absolute acceleration response of each floor of the structure, from which the wavelet-based DSF is extracted (see section 2.2), is collected during each ground motion excitation. The corresponding maximum story drift ratio (SDR) for each story is also obtained from the same model in order to determine the damage states of the structure (see section 2.3). The SDR at each story is computed as the maximum drift difference between two consecutive floors normalized by the height of the story.

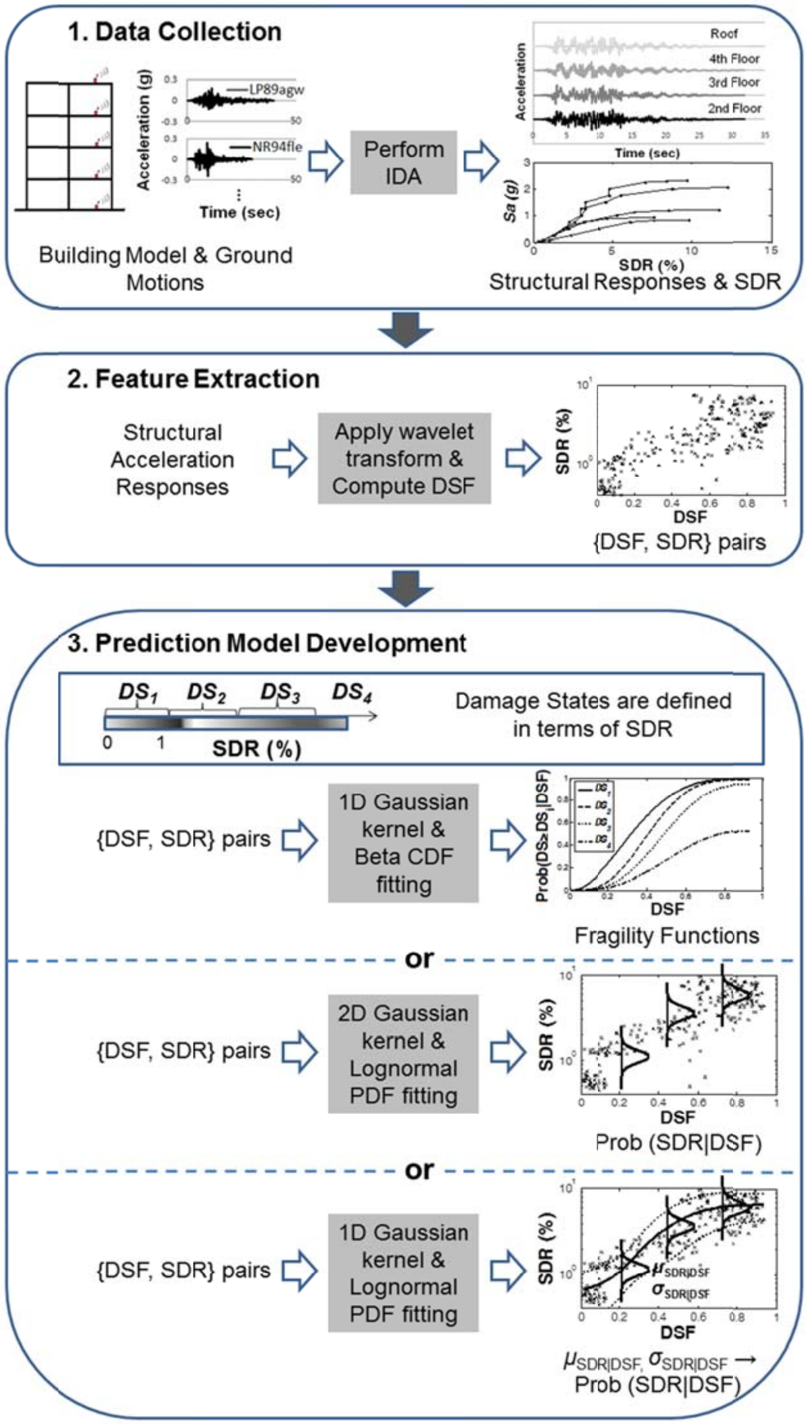


Figure 2.1. Summary of the proposed framework

2.2. Feature Extraction Using Wavelet Analysis

In the second step depicted in Figure 2.1, the wavelet-based DSF, described in Noh et al. (2011b), is extracted from the individual absolute acceleration histories for each floor of a structural system. This DSF represents approximate energy loss at the natural frequency of the structure due to damage. The wavelet-based DSF is used because the structural response to earthquake strong motions has time-varing characteristics. Wavelet analysis is suitable for studying time-varying characteristics of non-stationary signals such as earthquake responses because it represents the signal as a sum of dilated and time-shifted wavelets that are localized in time. Nair and Kiremidjian (2007) and Noh et al. (2011a) show the relationship between structural parameters and various wavelet-based DSFs including the one used in this paper. Before the wavelet transform is applied, each acceleration response is standardized by subtracting the mean of the response to offset different initial conditions of the measurements. Note that the DSF value varies between 0 (when there is no damage) and 1 (when the structure is severely damaged).

2.3. Prediction Model Development Using Kernel Smoothing

The final step of the framework (see Figure 2.1) is to construct fragility functions based on the wavelet-based DSF using kernel smoothing methods. A fragility function is defined as the conditional probability of being or exceeding a damage state given a DSF value. The fragility functions are empirically computed using kernel smoothing methods using SDR and DSF pairs collected and computed in the previous steps. A kernel is a symmetric weighting function used for non-parametric estimation, and the kernel smoothing methods make non-parametric estimations of functions from noisy observation based on their weighted sum. Then, a conventional cumulative distribution function (CDF) is fitted to the empirical fragility functions. In addition to the method that uses one-dimensional kernel for the DSF values to compute conditional probabilities, which Noh et al. (2011b) introduced, two methods are presented in this section to provide different types of information about the conditional probability of the SDR given the DSF. First method uses two-dimensional kernel for the DSF and SDR to directly estimate each conditional probability. Another one uses one-dimensional kernel to estimate the mean and the variance of the conditional density function, and then a probability density function (PDF) is fitted to the empirical conditional probability distribution. These three methods are summarized in Table 2.1.

Table 2.1. Summary of three methods for probabilistic mapping between the DSF and the SDR

Methods	**Outcome**	**Advantages**
One-dimensional Gaussian kernel for the DSF and the beta CDF fitting (Noh et al., 2011b)	Fragility function $(\mathrm{Prob}(\mathrm{DS}\geq DS_i \mid \mathrm{DSF}=dsf))$	Beneficial when the damage states are clearly defined in terms of the SDR
Two-dimensional Gaussian kernel for the DSF and the SDR and the lognormal PDF fitting	Conditional probability of the SDR given the DSF $(\mathrm{Prob}(\mathrm{SDR}=sdr \mid \mathrm{DSF}=dsf))$	Beneficial when the damages states are not clearly defined. The conditional probability can be computed for any SDR value.
One-dimensional Gaussian kernel for the DSF and the lognormal PDF fitting	Conditional mean and standard deviation ($\mu_{\mathrm{SDR}\mid\mathrm{DSF}}$, $\sigma_{\mathrm{SDR}\mid\mathrm{DSF}}$) Conditional probability of the SDR given the DSF $(\mathrm{Prob}(\mathrm{SDR}=sdr \mid \mathrm{DSF}=dsf))$	

The fragility functions can be computed for each story of the structure using the DSF computed from individual story responses, or can be computed for the entire structure using the DSF from the roof absolute acceleration responses and the maximum SDR among all the stories. The fragility functions for each story can be used for more detailed diagnosis of damage at a specific story of a structure. The global fragility functions can be used after an earthquake to quickly assess the overall damage of a structure. The overall assessment of a structure would be particularly useful when multiple structures have to be assessed in a timely manner. In this section, the procedure is described for computing fragility functions for each story separately, but similar procedure can be applied to compute the fragility functions for the overall damage.

Damage states (DS) are discrete variables most often defined as ‘no damage,’ ‘slight damage,’ ‘moderate damage,’ and ‘severe damage.’ In this study, each damage state (DS_i) covers a range of SDR values. Using this definition of damage states, the fragility function can be defined as follows:

$$G_i(dsf) = \text{Prob}\{\text{SDR} \geq SDR_i | \text{DSF} = dsf\} = \frac{\text{Prob}\{\text{SDR} \geq SDR_i, \text{DSF} = dsf\}}{\text{Prob}\{\text{DSF} = dsf\}} \tag{2.1}$$

where $G_i(dsf)$ is the fragility function for being or exceeding damage state i given a DSF value, dsf, and SDR_is are monotonically increasing threshold values for increasing damage states, DS_i s.

Typically an empirical fragility function for each damage state described above is computed using data binning. From the numerical simulation and the structural damage diagnosis algorithm, pairs of DSF and SDR values, $\{ dsf_i, sdr_i \}$, are computed for acceleration responses at each floor. Data binning is then used to segregate DSF values into each bin and count the number of pairs whose SDR values belong to each set of DS_i within the bin (Porter et al., 2007). Alternatively, we can apply the kernel smoothing method using the following equation:

$$\text{Prob}(X = x) \cong \frac{\sum_{i=1}^{n} \frac{1}{h} K(\frac{x - x_i}{h})}{n} \quad . \tag{2.2}$$

where x_is are n realizations of the random variable X, K is a kernel, and h is a smoothing parameter or the bandwidth of the kernel K. Substituting the Equation (2.2) into the Equation (2.1), we can estimate the conditional probability of the SDR given the DSF using a two-dimensional kernel as follows:

$$\begin{aligned} \Pr\text{ob}(\text{SDR} = sdr | \text{DSF} = dsf) &\cong \frac{\frac{1}{N}\sum_{p=1}^{N} \frac{1}{h_1 h_2} K(\frac{dsf - dsf_p}{h_1}, \frac{sdr - sdr_p}{h_2})}{\frac{1}{N}\sum_{q=1}^{N} \frac{1}{h_1} K(\frac{dsf - dsf_q}{h_1})} \\ &= \frac{\frac{1}{h_2}\sum_{p=1}^{N} K(\frac{dsf - dsf_p}{h_1}, \frac{sdr - sdr_p}{h_2})}{\sum_{q=1}^{N} K(\frac{dsf - dsf_q}{h_1})} \end{aligned} \tag{2.3}$$

where $K(x, y)$ is a two-dimensional kernel centered at (x, y). This equation follows directly from the definition of the conditional probability and the kernel density estimation in Equation (2.2). If the two-dimensional kernel can be factorized into $K(dsf)$ and $K(sdr)$, then the Equation (2.3) can be rewritten as

$$\Pr\text{ob}(\text{SDR} = sdr | \text{DSF} = dsf) \cong \frac{\sum_{p=1}^{N} \frac{1}{h_1} K(\frac{dsf - dsf_p}{h_1}) \times \frac{1}{h_2} K(\frac{sdr - sdr_p}{h_2})}{\sum_{q=1}^{N} \frac{1}{h_1} K(\frac{dsf - dsf_q}{h_1})} \tag{2.4}$$

The kernel assigns a different weight for each pair of DSF and SDR values. We use the kernel $K(x)$ whose weight is higher for the x values near 0. Using a rectangular kernel with height 1 is equivalent to the conventional data binning methods. Equation (2.4) estimates the conditional probability of SDR when the value of the DSF is dsf by using all the pairs of DSF and SDR values. Therefore, this method is more appropriate than discrete data binning method when the data are sparse or non-homogeneous. In addition, the use of all the pairs leads to a continuous and smooth representation of the fragility functions when compared with the data binning method.

A conventional CDF is then fitted to the empirically computed fragility functions. The advantages of fitting a conventional CDF are as follows: (1) the function is completely described by a few parameters; (2) the function is continuous, thus defined for all possible DSF values (no interpolation is necessary); and (3) the function increases monotonically. The lognormal CDF is used in conventional fragility functions, but other functions, such as the beta CDF and the truncated normal CDF, can also be used depending on the data. In general, the CDF that minimizes the fitting error, such as a root-mean-square error (RMSE), is selected. Several CDFs of interest are fitted to the data using a nonlinear least-square method, and the CDF that has the smallest RMSE is chosen. The lognormal distribution is appropriate for this conditional probability because the SDR values are bounded by zero on the lower side.

The advantage of this approach to the approach presented in Noh et al. (2011b) is that we do not need to discretize the range of the SDR into specific damage states. Instead, we can directly compute the conditional probability of the SDR given the DSF without computing the cumulative conditional distribution. In addition, this method considers the uncertainty in both the DSF and the SDR measurements unlike the previous method that considers the uncertainty of only the DSF by using the one-dimensional kernel.

The second method is to estimate the mean and the variance of the SDR given the DSF ($\mu_{\mathrm{SDR|DSF}}$, and $\sigma^2_{\mathrm{SDR|DSF}}$, respectively) and then fit a PDF. In other words, we can obtain the conditional probability distribution of the SDR given the DSF. This method is particularly useful when damage states are not clearly defined by the SDR or when the conditional density function of the SDR needs to be convolved with other conditional density function for further risk analysis. The estimates of the conditional mean, $\hat{\mu}_{\mathrm{SDR|DSF}}$, and the conditional variance, $\hat{\sigma}^2_{\mathrm{SDR|DSF}}$, for the DSF value of *dsf* can be computed using a kernel as follows:

$$\hat{\mu}_{\mathrm{SDR|DSF}=dsf} = \frac{\sum_m sdr_m \times K(\frac{dsf - dsf_m}{h})}{\sum_n K(\frac{dsf - dsf_n}{h})} \tag{2.5}$$

$$\hat{\sigma}^2_{\mathrm{SDR|DSF}=dsf} = \frac{\sum_m \left(sdr_m - \hat{\mu}_{SDR|DSF=dsf_m}\right)^2 \times K(\frac{dsf - dsf_m}{h})}{\sum_n K(\frac{dsf - dsf_n}{h})} \tag{2.6}$$

Once the mean and the variance are computed, the lognormal distribution function is used to fit the conditional distribution of the SDR given the DSF by the method of moments.

3. APPLICATION TO SIMULATED DATA

The framework for building fragility functions using kernel smoothing methods was validated using a set of numerically simulated data from a four-story two-bay steel special moment-resisting frame (SMRF). This frame is a perimeter lateral resisting system of an office building designed in Los Angeles based on current seismic provisions such as the IBC and the AISC. An analytical model of this frame has been developed in DRAIN-2DX analysis program and validated experimentally up to collapse (Lignos and Krawinkler, 2009).

The analytical model of the structure was subjected to a set of 40 ground motions scaled to various intensities, and absolute acceleration time-histories at each floor were obtained. The unscaled ground motions in this set have large magnitude (6.5 < M < 7.0) and distances from the rupture zone of 13 km

< R < 40 km (Medina and Krawinkler, 2003). The median of the acceleration spectrum of the unscaled motions matches the design level acceleration spectrum for the area in which the office building is designed. Hence, the ground motion set is a suitable representative one for the location of the structure. The response of the SMRF was evaluated up to collapse using IDA, where the spectral acceleration at the first mode period ($Sa(T_1$, 2%)) was used as an intensity measure of the ground motion. The absolute acceleration responses were collected for each level of intensity, and the wavelet-based DSF was extracted. A pair of DSF and SDR, { dsf_i, sdr_i }, was then computed for the individual floor absolute acceleration response for each ground motion excitation. Figure 3.1 shows the distribution of { dsf_i, sdr_i } pairs from all the ground motion excitations for each story. This figure shows that DSF and SDR are well correlated. The correlation coefficients (ρ) of the pairs for stories 1 to 4 are also shown in the figure. Based on these data, fragility functions are computed for damage assessment of the four-story SMRF.

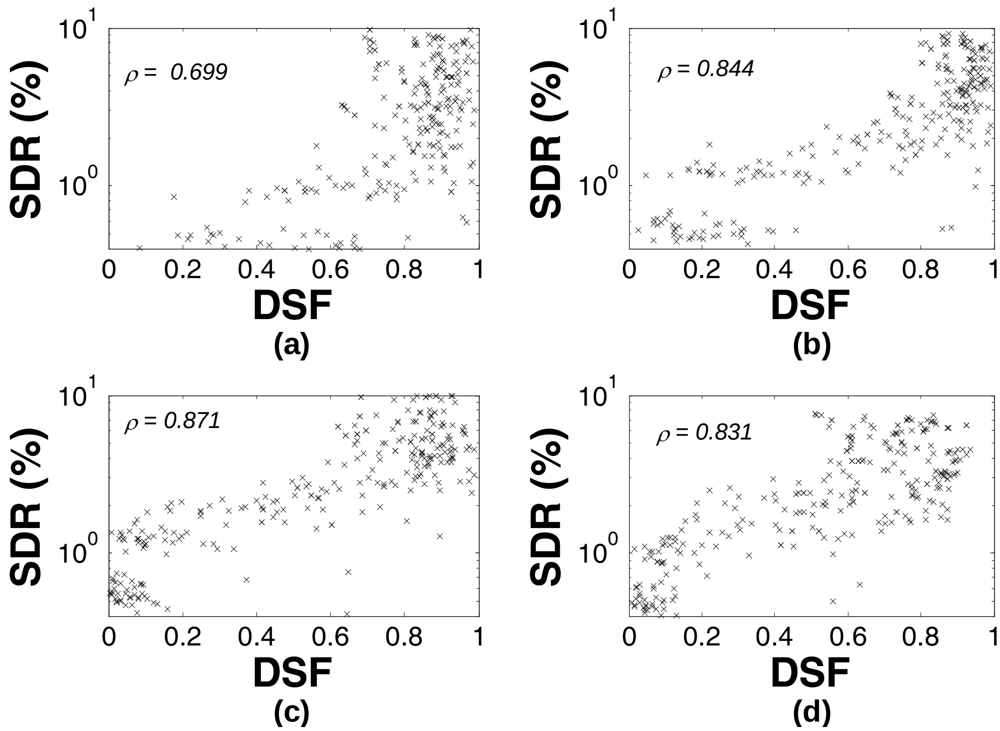


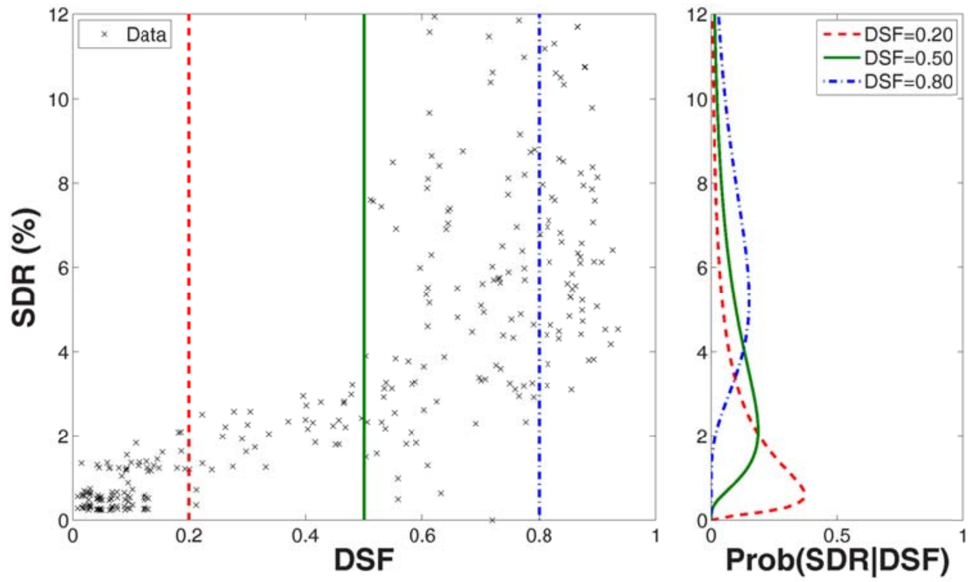
Figure 3.1. Scatter plot of DSF versus SDR: (a) story 1; (b) story 2; (c) story 3; (d) story 4

For the two-dimensional kernels, the standard Gaussian function is defined as K in Equation (2.3), and the smoothing parameter (or bandwidth) of 0.1 for the DSF and 0.0106 for the SDR, which are Silverman's optimum bandwidth (h) for the Gaussian kernel (Silverman, 1986). It is given as

$$h = 1.06\hat{\sigma}n^{-\frac{1}{5}} \tag{3.1}$$

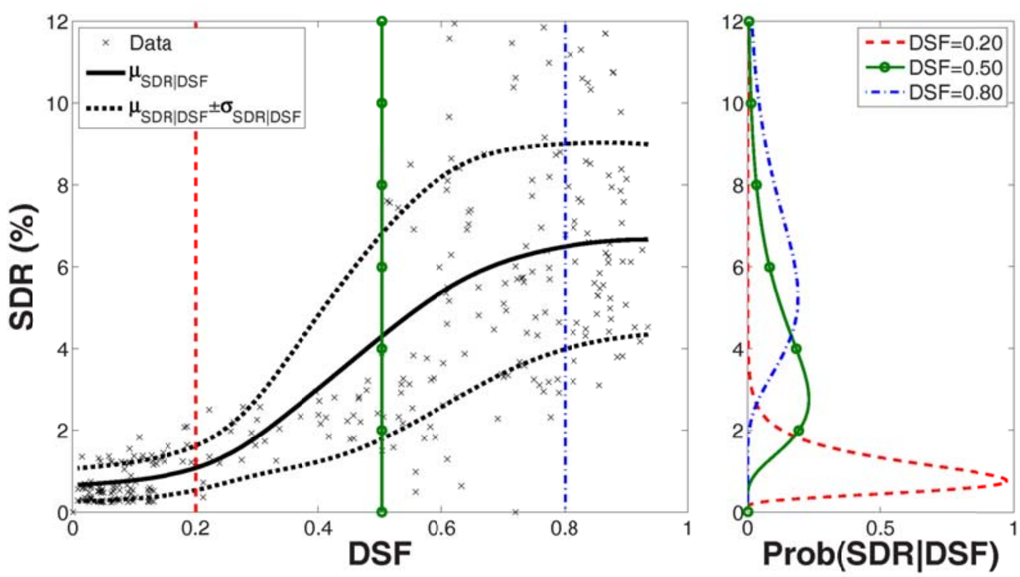
where $\hat{\sigma}$ is the sample standard deviation, and n is the number of samples. The Gaussian kernel is a powerful kernel widely used in pattern recognition (Evalgelista, 2007). Figure 3.2 shows the scatter plot of the DSF from the roof acceleration responses and the maximum SDR among all the stories and the conditional density functions fitted to the lognormal PDF for several DSF values.

Alternatively, the conditional mean and the standard deviation of the SDR given the DSF were first computed using the Gaussian kernel, and then the lognormal distribution was fitted to the data using the method of moments. Figure 3.3 (a) shows the scatter plot of the DSF from the roof acceleration responses and the maximum SDR among all the stories and the conditional mean and the standard deviation of the SDR given the DSF, represented by the solid and the dotted lines, respectively. To compute $\hat{\mu}_{\mathrm{SDR|DSF}}$ and $\hat{\sigma}_{\mathrm{SDR|DSF}}$, we used the Gaussian kernels with the bandwidth of 0.1 for the DSF as before. Figure 3.3 (b) shows the conditional density functions of the SDR given the DSF values of 0.2, 0.5, and 0.8 using the lognormal distribution. Both the mean and the variance of the SDR increase as the DSF increases. This indicates that the DSF is positively correlated to the SDR, and the DSF can estimate lower levels of damage more confidently than more severe levels. Figure 3.4 shows $\hat{\mu}_{\mathrm{SDR|DSF}}$ and $\hat{\sigma}_{\mathrm{SDR|DSF}}$ for various DSF values in more detail.



(a) (b)

Figure 3.2. (a) Scatter plot of DSF versus maximum SDR; (b) Conditional probability density function of SDR given DSF using the two-dimensional kernel for the four-story steel SMRF



(a) (b)

Figure 3.3. (a) Scatter plot of DSF versus maximum SDR and the conditional mean and standard deviation; (b) Conditional probability density function of SDR given DSF for the four-story steel SMRF

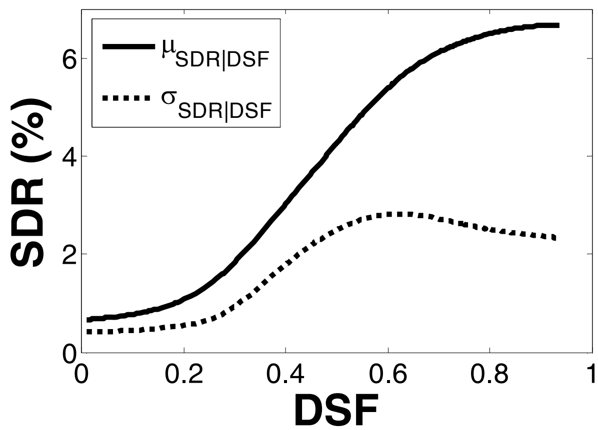


Figure 3.4. Conditional mean and standard deviation of SDR given DSF for the four-story steel SMRF

4. CONCLUSIONS

This paper presents a new framework to compute fragility functions using kernel smoothing methods for probabilistic seismic damage diagnosis. In this framework, the fragility function is used to define the probabilistic relationship between structural acceleration responses recorded during an earthquake, which is summarized using a wavelet-based damage sensitive feature (DSF), and maximum story drift ratio (SDR), which is closely correlated with structural damage. The analytical formulations that relate the DSF to structural parameters is given in Nair and Kiremidjian (2007) and Noh et al. (2011a), which provide the theoretical foundation for using the wavelet-based parameters. The relationship between the DSF and SDR is computed using two different kernel smoothing methods that non-parametrically estimate the conditional probability of SDR given the DSF value. These kernel-based methods provide smooth and continuous representation of fragility functions, unlike the data binning method, and are particularly beneficial when the data are sparse and/or non-homogeneous. The proposed framework is based on information retrieved from an extensive set of structural responses extracted from an analytical model of a structure subjected to a set of ground motions utilizing incremental dynamic analysis. The wavelet-based DSF is then computed based on the floor absolute acceleration time-histories, and related to SDR that gives DSF an engineering meaning. Finally, the conditional probability of SDR given the value of the DSF and their conditional mean and standard deviation are computed. These fragility functions can be computed for each story separately or for the entire structure to assess the overall damage state. The framework is validated using a set of numerically simulated data from a four-story steel special moment-resisting frame subjected to various intensities of 40 different ground motions.

The fragility functions are computed using a particular wavelet-based DSF in the study; however, the framework can potentially be used with any valid DSF that can reliably estimate the damage state of a structure. Further verification and testing of its damage assessment capabilities need to be performed as additional data for different types of structures become available, and the general form of the fragility functions for a group of similar types of structures can be explored. It is also necessary to investigate the feasibility of implementing this damage classification method using the DSF-based fragility functions on a wireless structural health monitoring system.

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