Application of Fractal Theory on Bridge Health Monitoring System

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SUMMARY:

For the special structural characteristic, bridges are prone to suffer from multiple hazards such as earthquake, wind, or floods. Recently, as most of the unexpected damage or collapse of bridges are caused by hydraulic issue, monitoring the scour depth of bridges has become as important topic. Exploiting the advantage of Fractal Theory, which utilizes the similarity at all levels of magnification in dealing with complicated problems, a novel bridge health monitoring system is proposed. Different than the existing approach of scour monitoring, which mainly focuses on installing sensors on the substructure of bridges, by analyzing the vibration data measured from the superstructure of bridge structure through two dominating parameters of Fractal Theory including the fractal dimension and the topothesy, the health condition of bridge structure can be rapidly evaluated. To demonstrate the performance of the system, a series of experiment was carried out. The function of the two parameters was first determined by data collected from single bridge column scour test. As the fractal dimension gradually dropped following the trend of scour depth, it is treated as an alternative of the fundamental frequency of the bridge structures in the system. Meanwhile, positive correlation was also investigated between the topothesy and the amplitude of vibration data. With the combination of these two parameters, a safety index to detect the critical scour condition is developed. Experimental result has demonstrated that the critical scour condition can be warned by the safety index proposed. The monitoring system developed has made a big progress in bridge scour health monitoring and offered an alternative choice than traditional scour monitoring technology.

Keywords: Fractal Theory, Bridge Scour, Structural Health Monitoring

1. INTRODUCTION

Structural Health Monitoring (SHM), an interdisciplinary concept originated from aerospace engineering, has been widely applied into different research fields. Due to natural disasters such as earthquake or flooding and the inevitable aging problem, structures are found to collapse without any warning. As the economy and society may be stricken seriously by this kind of catastrophe, SHM has become an important issue all around the world. Structural engineers not only need to provide sufficient structural strength for safety but should also equipped structures with proper SHM system for long-term maintenance.

Recently, the pattern recognition technique has been proposed and widely accepted to improve the reliability of SHM (C.K. Coelho et al., 2009). By comparing the information collected from the structure and the scenario database, the efficiency of the SHM system can be largely enhanced. For example, Support Vector Machine (SVM), a branch of bioinformatics, has been effectively applied on different subject such as the damage detection of the helicopter propeller blades (P.M. Pawar et al., 2008), the improvement of existed bridge SHM system (S.H. Park et al., 2006), the influence on modal parameters of bridge structure by temperature variation (Y.Q. Ni et al., 2006), and the damage detection of car electronic system. In addition, the slowly variation of stiffness decay or system parameters by internal damage of nonlinear structures can also be detected (L. Bornn et al., 2010). However, some limitations and difficulties are still faced when applying the SVM algorithm for SHM. For example, the lack of training samples, which should be mostly obtained from field experiments under different damage conditions of structures really hinder its practicability. To solve this bottleneck, alternative methods are considered.

The core of fractal theory (FT), similar to the concept of pattern recognition technique, is that self-similarity can be found from object with high irregularity when viewing in different scale. Euripides S. Mistakidis simulated the surface roughness of shear wall element and found the close correspondence of fractal geometry between structural ultimate strength and the structural surface (S. Euripides et al., 2002). N. Pirmoradian, etc, al. used FT as an alternative index to quantify the stability of aggregate of the cultivated soil (N. Pirmoradian et al., 2005). To predict the cohesion process of clay soil, the fractal dimensions was used as a variation for the yielding force of soil (M. Son et al., 2009). Moreover, the affect of the stiffness of brittle materials and the fatigue of metal materials caused by size effect was also analyzed by FT (A. Carpinteri et al., 2009). By verifying the test of arch bridge modal, the nonlinear behaviors of structural dynamic responses can be obtained (S.f. Jiang et al., 2001). Furthermore, Pi Zhong Qiao, etc proposed a FT-based method to identify the fissure locations and the numbers of cracks of beam-type structure (P.Z. Qiao et al., 2008).

Due to the threat of multi hazard of bridges in Taiwan, real-time bridge health monitoring has become more and more important over the last decade. Among all the monitoring items, the monitoring of scour depth for the bridge foundation is the most difficult part. Traditionally, sensors are installed directly on the substructure to indicate the possible scour depth. However, for the complicated conditions faced during flooding or typhoon where the structural foundation could be caisson or group piles, the sensors may not work well as expected.

In order to improve the reliability of traditional bridge SHM system, an alternative solution is provided by introducing a FT-based algorithm with analyzing the dynamic response of superstructure only. The scour level of the bridge column can be estimated immediately to bring great contribution to the field of bridge SHM. Moreover, as close correlation between the scour condition and the frequency characteristic are investigated, a safety index is also proposed to assess the stability of the bridge. With the support of the safety index, real-time warning signal can be sent out to enhance the safety of human lives and properties.

2. FRACTAL THEORY

The concept of Fractal Geometry (FG) was first proposed by mathematician Mandelbrot to describe the complex nonlinear phenomena in nature such as the shape of mountain, turbulent flow, or length of coastline where the characteristic of self-similarity can be found between multi-scale structures (B. Mandelbrot, 1982). By analyzing the self-similarity based on FG in small scale where the identification can be easily achieved, the implicit regularity of the whole structure can be estimated with the help of partial structure. Although no regularity is observed from the original structure, the self-similarity characteristic started to appear by extracting a smaller part of the whole structure. Significant similarity which may not be detected from the original observation dimension is then found.

Fractal dimension and topothesy, commonly noted as D_s and G, are the two dominated parameters in FT. By using these parameters, the density variation and amplitude variation of vibration signal can be illustrated, respectively. The derivation of fractal dimension and topothesy is described briefly as follows:

2.1. W-M fractal function (weieratrass-mandelbrot fractal function)

In order to describe the non-differential characteristic and continuity of fractal curve, a "Riemann Function" $z(x)$ was first proposed by Riemann as (B. Mandelbrot, 1982)

$$z(x) = \sum_{n=1}^{\infty} n^2 \cos(n^2 x) \tag{1}$$

where n is continuous integer.

As some critical points on the fractal curve still couldn't be defined, the Rieman Function was modified into "Complex Weieratrass Equation" as

$$z(x) = (1 - w^2)^{-1/2} \sum_{n=0}^{\infty} w^n \exp(2\pi i b^n x) \tag{2}$$

where $w=b^{-H}$, and b is an amplified scale parameter.
The fractal dimension D_s in two dimensions can be derived by

$$D_s = H + 2 \quad (1 < D_s < 2) \tag{3}$$

The equation can be further simplified into the Weieratrass Equation to simulate signals with fractal characteristic where the non-differential characteristic in peak and continuity of fractal curve are satisfied for the self-similarity of fractal curve as

$$z(bx) = b^H z(x) \tag{4}$$

To consider the amplitude effect of signals, the Weieratrass Equation was rewritten into Weieratrass-Mandelbrot Fractal Function (W-M Fractal Function) as

$$z(x) = G^{(D_S-1)} \sum_{n=n_1}^{\infty} \frac{\cos(2\pi\gamma^n x + \phi_n)}{\gamma^{(2-D_S)n}}, 1 < D_S < 2, \ \gamma > 1 \tag{5}$$

where G is topothesy; D_s is fractal dimension, and γ^n represents the frequency mode with the random variable phase ϕ_n.
To extract D_s and G from the original signal z(x), a Scaling constant C_p, related to signal amplitude, is introduced.

2.2. Relationship between scaling constant C_p and fractal parameters G, D_s"

Based on the research conducted by Yan and Komvopoulos (W. Yan et al., 1998), tough surface can be simulated by introducing the structure function as

$$S(\tau) = 2^{2(4-D)} G^{2(D-2)} (\ln\alpha)(\tau)^{2(3-D)} \tag{6}$$

Where τ stands for the sampling interval of instrumentation; D is fractal dimension in three dimension; α expresses a dominant parameter of frequency density at the surface.

The structure function S can also be derived from the power spectrum function $P\,(f)$ as (M.V. Berry, 1979)

$$S(\tau) = \int_{-\infty}^{\infty} P(f)\left[\exp(if\tau) - 1\right]df = \frac{2C_P}{\eta - 1} \sin\left(\frac{\pi}{2}(2-\eta)\right)\Gamma(2-\eta)|\tau|^{\eta-1} \tag{7}$$

where f=1/ τ ; η is a constant, C_p is the Scaling constant from the spectrum function of

$$P(f) = \frac{C_P}{f^\eta} \tag{8}$$

The structure function S can be rewritten by calculating the exponential value τ under the condition of $\eta = 7 - 2D$ as

$$S(\tau) = \frac{C_P}{3-D} \sin\left(\frac{\pi}{2}(2D-5)\right)\Gamma(2D-5)\tau^{2(3-D)} \tag{9}$$

where Γ is Gamma function.

Combining equation (6) and equation (9), the Scaling constant C_p can be solved as

$$C_P = \frac{(3-D)2^{2(4-D)}G^{2(D-2)}\ln\alpha}{\sin\left(\frac{\pi(2D-5)}{2}\right)\Gamma(2D-5)} \tag{10}$$

The fractal topothesy can then be expressed as

$$G = \left\{ \frac{C_P \cdot \sin\left(\frac{\pi(2D-5)}{2}\right)\Gamma(2D-5)}{(3-D)2^{2(4-D)}\ln\alpha} \right\}^{1/(2D-4)} \tag{11}$$

As relationship between fractal dimensions in two dimensions and three dimensions was proven to be $D_s = D - 1$ (B. Mendelbrot et al., 1984), equation (10) and (11) are converted into two-dimension form to fit the requirement in this study as

$$C_P = \frac{(2-D_s)2^{2(3-D_s)}G^{2(D_s-1)}\ln\alpha}{\sin\left(\frac{\pi(2D_s-3)}{2}\right)\Gamma(2D_s-3)} \tag{12}$$

$$G = \left\{ \frac{C_p \sin\left[\frac{\pi(2D_S-3)}{2}\right]\Gamma(2D_S-3)}{(2-D_S)2^{2(3-D_s)}\ln\alpha} \right\}^{\frac{1}{(2D_S-2)}} \tag{13}$$

To estimate D_s and G correctly from measured signal, some algorithms such as Slit Island method, Box Counting method, Variation method, and Power Spectrum method were proposed (M. V. Berry, 1979). As studies of Dubuc et al (B. Dubuc et al., 1989) indicate that Variation method and Power Spectrum method are more suit for calculating fractal parameters when the fractal interval is located between one dimension to two dimension, the Variation method and the Power Spectrum method are used to get the D_s and G, respectively. By means of the calculated fractal parameters D_s and G, the density variation and amplitude variation can be evaluated more properly

Since preliminary study has shown that the physical characteristic of the bridge column are mainly caused by the change of scour depth while minor impact is caused by the force exerted by the current on the bridge column, an unique SHM methodology is proposed. The two dominant parameters D_s and G under different stage of scour depth are first obtained by applying the processing algorithms mentioned above on the vibration signal measured of the superstructure. The safety of the bridge can then be inferred based on the proposed method established from the experimental database.

3. EXPERIMENT VERIFICATION OF SINGLE BRIDGE COLUMN SCOUR

3.1 Single bridge column test

In order to verify the performance of the proposed bridge SHM concept, a series of single bridge column experiment was conducted. As shown in the left side of figure 1, a single-column specimen was embedded into the riverbed with sand of specific depth, and two high-resolution sensors were deployed on the top of the bridge column to capture the dynamic response of the specimen. The flow velocity was controlled in a constant speed to preclude any unnecessary interference from the environment. Totally 24 minutes of dynamic response data were recorded under a sampling rate of 200Hz through the whole experiment progress until the collapse of the bridge column. As vibration of the superstructure is mainly expressed on velocity term in the transversal and vertical directions during the scour experiment, velocity responses from both directions were considered and analyzed. The embedded depth of the specimen foundation was set as 15cm, and the total weight of the specimen was 25kg. The whole scour time history is shown in figure 2 where the scour phenomenon was observed from the first minute, and the bridge column collapsed at the 24th minute. As shown in the figure, the amplitude gradually exaggerated for the first five minutes and then turned stable while the scour depth slowly increased. The whole bridge column only vibrated significantly at the final stage of experiment, and the collapse was observed due to the instability of the structure.

Figure 1. Single bridge column experiment

In order to investigate the variation of fractal parameters during scour, the 24-min time history of horizontal vibration response was set into segments of one minute with 12,000 points each.

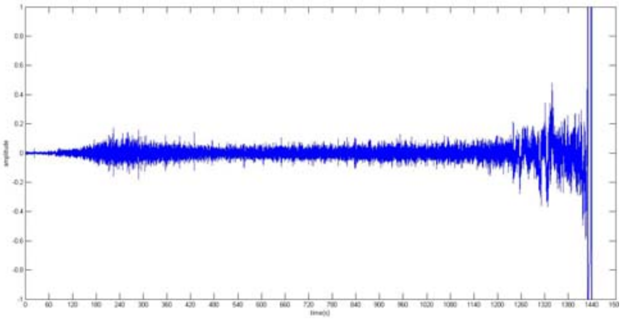

Figure 2. The scour time history of horizontal amplitude

By analyzing the data collected with the proposed algorithm, the 24 individual groups of fractal parameters D_s and G can be calculated and compared. As the topothesy G is treated as an index representing the vibration amplitude, the tendency of G was first examined. The variation of horizontal amplitude, which represents the mean absolute value of the signal amplitude in each minute is depicted in Figure 3 for comparison. The variation of the G values calculated is then illustrated in Figure 4.

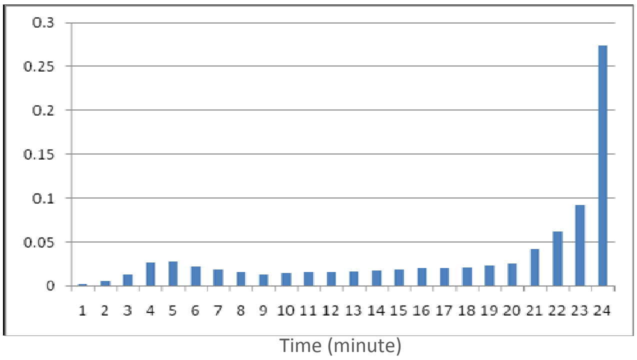

Figure 3. The time history variation of horizontal amplitude

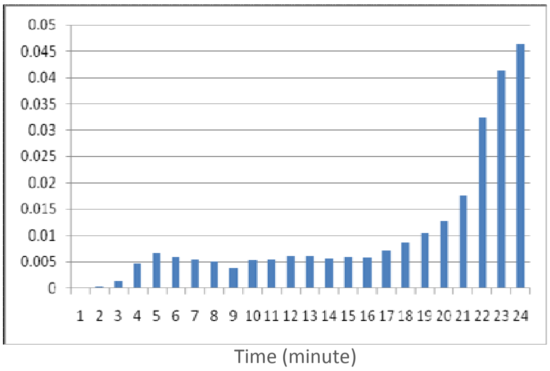

Figure 4. The time history variation of topothesy G

As both figures showed similar trend when the scour phenomenon progressed, the basic theory of FT for topothesy *G* to reflect the variation of the amplitude of measured signal is verified. To further investigate the relationship between the topothesy *G* and amplitude, Figures 3 and 4 are converted into figure 5 for comparison. Data captured in the last minute is removed due to the complicated bridge collapse condition. As shown in the figure, the relation can be regressed as linear with slight deviation where the advantage of applying FT can be reflected, and the reason of these differences is discussed.

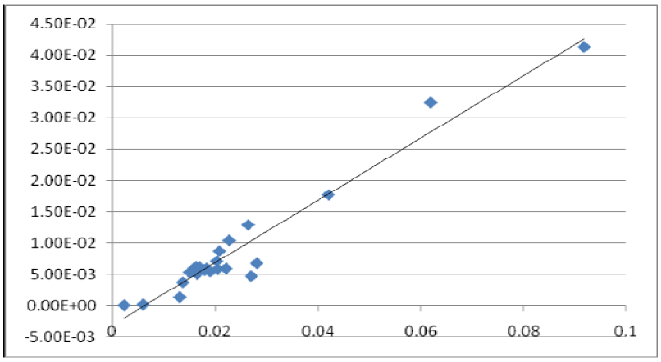

Figure 5. The relationship of amplitude and time history

According to the concept of FT, the characteristic of self-similarity for the vibration signal can be described clearly, especially under events with high nonlinearity such as the complicated scour problem. This characteristic was observed significantly on the topothesy *G*. As shown in the first ten minutes, the bridge column was under the progress of initial scour, and serious variation on vibration amplitude was found with the water level increase. However, as self-similarity should exist since the whole bridge column was still under a stable condition, flocculation on the topothesy *G* was comparatively minor and kept in almost the same level near 0.005. Moreover, similar phenomenon was also investigated for the last five minutes of the scour process. The amplitude was exaggerated dramatically only on the last stage with the sudden collapse of the bridge column. Oppositely, the topothesy *G* gradually amplified from the 20th minute and gave a warning signal on the 22nd minute, three steps before the catastrophe. The precious early warning would be of great help to save properties and lives when applied to practical bridges.

3.2. The relationship between fractal dimensions D_s and signal frequency

The fractal dimension D_s was calculated from the 24-minute scour time history of 12,000 points each. In order to improve the sensitivity and reliability of the D_s value calculated, a special strategy was introduced where the total 12,000 data points were first divided into 12 sections of 1,000 points in each section. The mean value of the 12 D_s values obtained was then calculated as shown in figure 6 to show the minute variation of D_s related to time history. Meanwhile, as mentioned in section 2, D_s can be used to determine the density variation of the signal measured. As a result, relationship between D_s and the dominant frequency of the signal in each section was studied. The Short Time Fourier

Transform (STFT) technique was used to rapidly analyze the dominating frequency transversal direction of the single bridge column as shown in figure 7 for comparison. The figure was expressed in the form of contour where the longitude coordinate is the elapsed time in second; the latitude coordinate is shown in frequency, and the power density is expressed by colors. Comparison between figure 6 and figure 7 has demonstrated the basic concept that the density variation, which is equivalent to dominating frequency of structures, can be illustrated by the fractal dimension D_s.

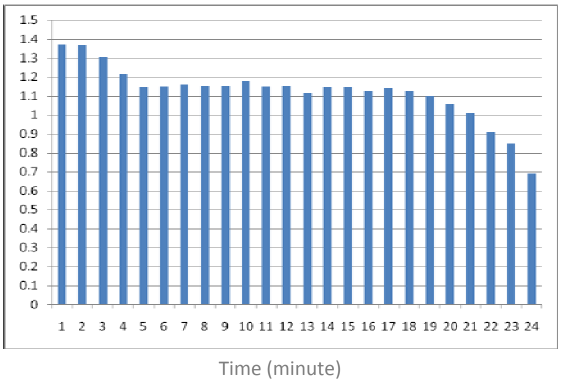

Figure 6. The time history variation of Ds

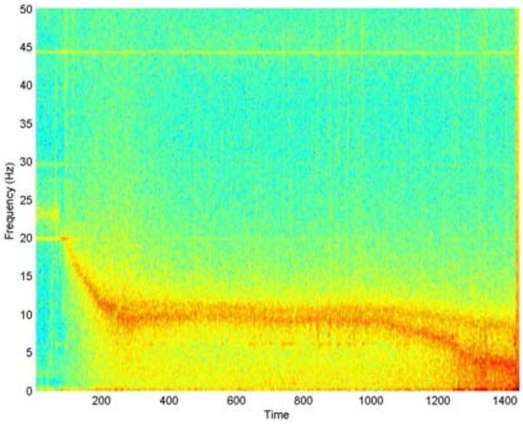

Figure 7. The STFT diagram from experiment

Meanwhile, variation of fractal dimension D_s is also studied. As shown in figure 6. Initially, the D_s was 1.37 and started to decrease as the river began to scouring the single-column specimen. D_s was kept on 1.37 for the first two minutes which reflects the condition of bridge column with only embedded sand. A transition period between non-scoured and initially-scoured due to the sudden change of foundation condition was then observed in the first four minutes, and D_s gradually changed to the value of 1.15. Similar to the plateau phenomenon found on the modal frequency of the STFT diagram, the D_s from the vibration signal measured fluctuated slightly between 1.1 and 1.2 from minute 5 to minute 20 as the scour depth continued changing, following the self-similarity criterion defined by equation 3 that the fractal dimension should fall between 1 and 2. It is inferred that though the boundary condition of the bridge column changed; however, the characteristics of self similarity existed to keep the fractal dimension constant.

The advantage fractal dimension was demonstrated again at the final stage of the scour experiment. As indicated in figure 6, the D_s value decreased below 1 at the 21st minute, which was three steps before the final collapse. Similar trend was also observed for topothesy G in the last section. Furthermore, D_s then dropped rapidly to 0.7 before the critical collapse of bridge foundation. By properly utilizing the trait of fractal dimension, the stability of bridge can be effectively evaluated.

As shown in figure 7, the natural frequency of the single bridge column dropped significantly from the point of 20 Hz and spread into a wide range with an approximate central frequency of 10 Hz from minute 5 to minute 20. The frequencies then descended to the area of 5 Hz in central before the sudden collapse of the bridge column. Based on structural dynamics, the first few modal frequencies can be roughly estimated by the STFT diagram. Although an apparent descending trend was observed for the dominating frequencies; however, the damage or scour condition cannot be quantified to offer early warning signal to bridge users. Moreover, huge computing resource is required if real-time extraction of modal frequency from STFT processing is desired, and practical application of STFT on bridge monitoring by comparing of the exact modal frequencies is hindered

Preliminary investigation has proven that the influence of scour depth variation on bridge dynamic response can be effectively reflected by D_s, and the variation of amplitude of the measured signal can be reflects by topothesy G. With the help of these characteristics, a safety factor composed of these two parameters is proposed to guarantee the safety of the bridge column.

3.3. Safety factor for bridge column scour

Nowadays, most of the failure conditions happened during bridge column collapse is classified as failure of bearing capacity (J. Nogues et al., 1992), and the boundary conditions can be further considered in three cases as

$$\begin{aligned}&\text{Case1: } \frac{h_{left}}{D} \le 0.3\\ &\text{Case2: } 0.3 \le \frac{h_{left}}{D} \le 1.3\\ &\text{Case3: } \frac{h_{left}}{D} \ge 1.3\end{aligned} \tag{14}$$

where h_{left} is the embedded depth of bridge column; D is the diameter of bridge column.

By selecting the proper case for evaluation, the safety factor under bearing capacity failure mode can be expressed as

$$(S.F.)_B = \frac{q_u}{q} \tag{15}$$

where q_u is the vertical ultimate bearing capacity of soil, and q is the vertical resultant force of soil.

As the procedure for solving q is very tedious, and no dynamic effect is considered in the formula to reflect the practical condition for bridge scour, a formula based on FT is attempted for rapid evaluation on the safety factor to improve the practicability of traditional methods.

As shown from the previous sections that unique sensitivity to collapse can be reflected by both D_s and G in advance, the two parameters are used, and a formula to indicate the safety factor of bridge scour is proposed as

$$\text{FS} = (D_s - 1)(G + 1) + 0.3 \tag{16}$$

where the first term $(D_s - 1)$ is used to shift the D_s value between 0 and 1; the second term $(G + 1)$ is applied for enhancing the contribution of G in stability identification.

Performance of the safety factor is verified on both the horizontal and vertical direction of the single bridge column experiment, and the result is shown in figure 8 and figure 9. To fit the requirement of real-time bridge SHM, the safety factor should be kept within a stable range when the structure undergoes stable condition, and a warning signal should be given when the structure is suffering critical condition. As expected, the FS was moving around 1.1 for the first 19 minutes for both vertical and horizontal directions as shown in figure 9 and figure 10. The FS then started to decline at minute 19, as the scour situation became critical. Finally, at minute 24 the FS dropped below 1, where the bridge column was observed as collapsed, and the performance of the proposed FS has been demonstrated.

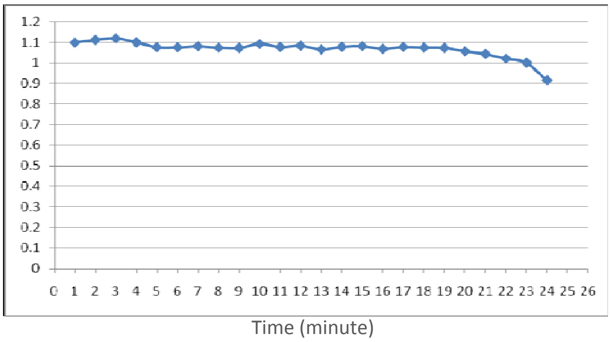

Figure 8. The bridge scour FS (horizontal)

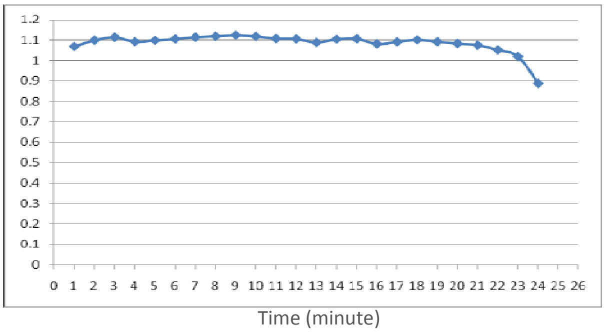

Figure 9. The bridge scour FS (vertical)

4. SUMMARY AND CONCLUSION

A novel bridge health monitoring system is proposed by exploiting the advantage of FT for checking the similarity at all levels of magnification in complicated problems. By analyzing the vibration data measured from the superstructure of bridge structure through two dominating parameters including the fractal dimension and the topothesy, the health condition of bridge structure can be rapidly evaluated. To demonstrate the performance of the system, a series of experiment was carried out.

The result of single bridge scouring test has shown that the variation of fractal dimension D_s can reflect the variation of the embedded depth of soil, and the variation of vibration signal amplitude can be reflected the topothesy G. Meanwhile, STFT Analysis of the vibration signal with was also conducted to establish reference in the frequency domain. It is found that D_s is highly related to the dominating frequency, and G is also closely related to the vibration amplitude

With the combination of these two parameters, a safety index to perform the possible trend for critical scour condition is developed. Experimental result has demonstrated that the scour trend of the scaled down specimen can be reliably estimated by the scour depth prediction formula. The critical scour condition can also be warned by the safety index proposed. The monitoring system developed has made a big progress in bridge scour health monitoring and offered an alternative choice than traditional scour monitoring technology.

REFERENCES

Coelho, C.K., Das, S., Chattopadhyay, A. (2009). A hierarchical classification scheme for computationally efficient damage classification. *Journal of Aerospace Engineering*. **223:5**,497-505.

Pawar, P.M. and Nam, J.S. (2008). Support vector machine based online composite helicopter rotor blade damage detection system. *Journal of Intelligent Material Systems and Structures*. **19:10**,1217-1228.

Park, S.H., Yun, C.B., Roh, Y.R., and Lee, J.J. (2006). PZT-based active damage detection techniques for steel bridge components. *Smart Materials and Structures* **15:4**,957-966.

Ni, Y.Q., Hua, X.G., Fan, K.Q. and Ko, J.M. (2006). Correlating modal properties with temperature using long-term monitoring data and support vector machine technique. *Engineering Structures* **27:12**,1762-1773.

Bornn, L., Farrar, C.R. and Park, G.H. (2010). Damage detection in initially nonlinear systems. *International Journal of Engineering Science* **48:10**,909-920.

Euripides, S. Mistakidis, E.S., Olympia, K. and Panagouli, O.K. (2002). Strength evaluation of retrofit shear wall elements with interfaces of fractal geometry, *Engineering Structures*. **24: 5**,649-659.

Pirmoradian, N., Sepaskhah, A.R. and Hajabbasi, M.A. (2005). Application of fractal theory to quantify soil aggregate stability as influenced by tillage treatments. *Biosystems Engineering*. **90:2**,227-234.

Son, M. and Hsu, T.J. (2009). The effect of variable yield strength and variable fractal dimension on flocculation of cohesive sediment. *Water Research.* **43:14**,3582-3592.

Carpinteri, A. and Puzzi, S. (2009). The fractal-statistical approach to the size-scale effects on material strength and toughness. *Probabilistic Engineering Mechanics*. **24:1**,75-83.

Carpinteri, A., Spagnoli, A. and Vantadori, S. (2002). An approach to size effect in fatigue of metals using fractal theories. *Fatigue and Fracture of Engineering Materials and Structures.***25:7**,619-627.

Jiang, S.f., Xu, F. and Fu, C. (2001). Intelligent damage identification model of an arch bridge based on box-counting dimension and probabilistic neural network. *Journal of Computational Information Systems*.**6: 4**,1185-1192.

Qiao, P.Z. and Cao, M.S. (2008). Waveform fractal dimension for mode shape-based damage identification of beam-type structures. *International Journal of Solids and Structures.* **45:22-23**,5946-5961.

Mandelbrot, B. (1982). The Fractal Geometry of Nature, W.H. Freeman, New York, U.S.A.

Yan, W. and Komvopoulos, K. (1998). Contact Analysis of Elastic-Plastic Fractal Surface. *Journal of Applied Physics.* **84**,3617-3624.

Berry, M.V. (1979). Diffractals. *J. Phys. A : Math Gen.* **12:6**,781-797.

Mandelbrot, B., Passoja, D. and Paullay, A. (1984). Fractal Character of Fracture Surface of Metals. *Nature*. **308**,721

Chester, S., Wen, H.Y., Lundin, M. and Kasper, G. (1989). Fractal-Based Characterization of Surface Texture. *Applied Surface Science*. **Vol IVX:** 185-192.

Dubuc, B., Quiniou, J.F., Roques, C.C., Tricot, C. and Zucker, S.W. (1989). Evaluating the Fractal Dimension of Profiles. *Physics Review*. **39:3**,1500-1512.

Nogues, J., Costa, J.L. and Rao, K.V. (1992). Fractal Dimension of Thin Film Surfaces of Gold Sputter deposited on mica: A Scanning Tunneling Microscopic Study. *Physica A*, **182**,532-541