Integrated Design of Damper Placement and Parameters for Passive Control of Randomly Base-Excited Structures

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SUMMARY:
An integrated design scheme for simultaneous optimization of damper placement and parameters of passively controlled structures is proposed in the present paper. A sequential procedure with probabilistic criteria of a minimum storey controllability index gradient and an optimum energy trade-off, both in function of exceedance probability of structural response and control action, is defined to efficiently schedule the optimal topology and optimal parameters of dampers. The resolution procedure of the integrated scheme is detailed, and a numerical example is investigated involving a randomly base-excited ten-storey shear frame equipped with viscoelastic dampers. Numerical results reveal that the proposed integrated scheme can gain the maximum control effectiveness with the minimum control cost. The controlled system response is significantly reduced and becomes smoother along the height of the structure, which results in a more desirable structural performance. Using the design criterion, meanwhile, of the minimum storey controllability index gradient, the structural stochastic optimal control achieves the objective performance more efficiently than that of the maximum storey controllability index currently in use.

Keywords: Damper placement; parameter optimization; storey controllability index gradient; energy trade-off; exceedance probability

1. INTRODUCTIONS

In recent years, structural control has demonstrated its value for mitigating natural hazards and enhancing the safety and serviceability of structural systems (Housner et al, 1997). It includes, typically, four kinds of control modalities, i.e. passive control, active control, semi-active control and hybrid control. Passively controlled modality is widely used in the engineering application due to its feasible and stable behavior upon structural systems compared to other three controlled modalities. The passively controlled modality, generally, operates through attaching isolation devices or energy-dissipation components to the structure towards strengthening structural damping, structural stiffness and structural intensity. The control effort provided by this modality just relies upon the response of structures subjected to the external dynamic loading, rather than rely upon the power as required in other three controlled modalities. The efficiency and sustainability of the passively controlled modality have been verified by its applications to the control of engineering structures since 1990s. Development of the energy-dissipation components, up to date, is still one of the challenging issues in the field of structural control (Gajan & Saravanathiiban, 2011).

Control algorithms relevant to the passive control are mainly involved in the optimization of parameters and placement of passive energy-dissipation devices. Criteria for optimizing controller parameters have been proposed that include the minimization of structural displacements using the capacity spectrum method (Kim et al, 2003), the minimization of cost over the life-cycle of structures using the genetic algorithm (Park et al, 2004). While criteria for optimizing control placement include minimization of a modal control index (Chang and Soong, 1980), minimization of an energy control index
(Chen et al, 1991). Pioneering work has also contributed to the controller placement in connection with multistory buildings using the concept of degree of controllability (Laskin, 1982). Although a lot of investigations were devoted to the optimization of parameters and placement of passive energy-dissipation devices, the uncertainties existing about the dynamics of the structural control system or its operational environment has not been received sufficient attention. One might not ensure the structural safety using control devices with deterministic control policies. Recently, a physical approach to structural stochastic optimal controls on the basis of the generalized density evolution equation has been proposed, which is applicable to practical random excitations, such as earthquake ground motions, strong winds and sea waves (Li et al, 2010). This approach allows a complete probabilistic design of controller parameters and placement towards structural performance, as gaged by probability density function of system responses of controlled structures.

In the present paper, an integrated scheme with routine of two-step optimization is proposed that serves to schedule the optimal topology and optimal parameters of viscous dampers mounted on passively controlled structures. Performance function in function of exceedance probability of structural response and control action is defined as the kernel of probabilistic criteria of the two-step optimization. A control criterion with minimum storey controllability index gradient is proposed to efficiently search for the optimal topology of viscous dampers. For illustrative purposes, a ten-storey shear frame structure controlled by viscous dampers is investigated. The concluding remarks are included in the final section.

2. INTEGRATED SCHEME FOR DAMPER DESIGN

Consider an $n$-degree-of-freedom linearly damped structural system subjected to a finite mean-square random excitation and governed by the following equation of motion:

$$M \ddot{X}(t) + C \dot{X}(t) + K X(t) = B_s U(t) + D_s F(\varpi, t)$$

where
$$X^i(t) = \{X_i(t)\}_{i=1}^n$$
refers to an $n$–dimensional vector, $X_i(t)$ is the inter-story drift between stories $i$ and $(i-1)$; $F^i(t) = \{F_{i}(\varpi)\}_{i=1}^p$ represents a $p$–dimensional random excitation vector, in which $\varpi$ represents a point in the set of basic random events charactering the external excitation. $M$, $C$ and $K$ are $(n \times n)$ mass, damping and stiffness matrices, respectively; $D_s$ is a $(n \times p)$matrix denoting the location of excitations; $B_s$ is a $(n \times r)$ matrix denoting the location of viscous dampers; $U^T(t) = \{U_i(t)\}_{i=1}^r$ denotes an $r$-dimensional control force vector relevant to viscous dampers.

The control force is a nonlinear mapping from the state vector involving the inter-story drift, inter-story velocity and storey acceleration to control force vector using any number of technologies and control logic. As a specific case that the control action linear relies on the state, the control force has the following general expression:

$$U(\Theta, t) = f(\tilde{M}, \tilde{C}, \tilde{K}) \begin{bmatrix} \ddot{X}(\Theta, t) \\ \dot{X}(\Theta, t) \\ X(\Theta, t) \end{bmatrix}^T$$

where $f(\tilde{M}, \tilde{C}, \tilde{K})$is the matrix function of the optimal control policy; $\tilde{M}$, $\tilde{C}$ and $\tilde{K}$ are the generalized mass, generalized damping, and generalized stiffness, respectively, associated with the control action; $\Theta = \Theta(\varpi)$ is a random vector defined on the set of basic events with the joint PDF $p_{\Theta}(\Theta)$. The vector of random parameters models the basic uncertainty in the systems and is used to implicitly parameterize the state and control force vectors.
An unified scheme for the optimal control policy can be developed by introducing a matrix function \( f(T, L) \) describing the dependence of the linear system dynamics on system parameters and control layout. The control force provided by viscous dampers is obtained as matrix \( \mathbf{f} \) operates on the state vector, as indicated in equation (2). Here, \( T = [T_m, T_c, T_k] \) is the optimal parameter vector denoting the generalized mass, generalized damping and generalized stiffness; \( L' = [L_x', L_y', L_z'] \) is optimal topology matrix denoting the dampers distributed in structural space with respect to dimensions \( x, y \) and \( z \). The lattices enclosed by beams and columns, especially for a frame, are set as the elements. In the topology matrix, zero denotes no dampers in the lattice, and the non-zero indicates a reference number of damper in the lattice and the sequence of placement of the damper. It is noted that the matrix function \( \mathbf{f}(\cdot) \) is a deterministic functional matrix even when used in conjunction with stochastic system control.

Its solution can be gained with benefit of the following two-step optimization routine; see Figure 1. The probabilistic criteria, i.e. Optimum energy trade-off criterion and Minimum storey controllability index gradient criterion, involved in the routine would be detailed in the next section.

### 3. PROBABILISTIC CRITERIA OF INTEGRATED SCHEME

#### 3.1 Criterion of optimal damper placement

In order to identify the optimal topology matrix, a controllability index related to the exceedance probability of quantities of interest is defined as

\[
\rho_i = \frac{1}{2} \Pr [\tilde{Z}_i - \tilde{Z}_{int} > 0] \Pr [\tilde{\tilde{U}}_i - \tilde{U}_{int} > 0] + \Pr [\tilde{\tilde{U}}_i - \tilde{U}_{int} < 0] \Pr [\tilde{\tilde{U}}_i - \tilde{U}_{int} < 0], \quad i = 1, 2, \ldots, n
\]

where \( \tilde{Z}_i, \tilde{\tilde{U}}_i \) are the extreme state and control force vectors of the \( i \)th element in the interval \([t_0, t_f]\), respectively, and where \( \Pr(\cdot) \) operates componentwise on its vector argument; \( \tilde{Z}_{int}, \tilde{\tilde{U}}_{int} \) are the threshold vectors corresponding to \( Z_i, U_i \).

Equation (3) indicates that the exceedance probability based controllability index characterizes system safety (the controlled inter-story drift), system serviceability (the controlled inter-story velocity), system comfortability (the controlled storey acceleration), controller workability (the constrained control force) and their trade-off, which is more comprehensive than the controllability index of single controlled quantity (Zhang and Soong, 1992).

Moreover, a controllability index gradient is defined as

\[
\Delta \rho_j^i = \frac{\rho_{j+1}^i - \rho_j^i}{\rho_{j+1}^i}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, r
\]
where $\rho_i^j$ represents the controllability index of the uncontrolled structure. It is noted that the structural topology is always updated with the newly added proper damper $j$ by introducing the criterion of minimum storey controllability index gradient (MinSCIG). The optimal topology design of each sequence involves the following optimization problem:

$$
(x^*, y^*, z^*) = \arg \min_{x, y, z} \{ \Delta \rho_i^j(x, y, z) \}, \quad i = 1, 2, \ldots, r; \quad j = 1, 2, \ldots, r
$$

Therefore, the next damper will be placed at $i_{opt}$, of which $\Delta \rho_i^j$ possesses the minimum of the gradient vector $(\Delta \rho_i^j, \Delta \rho_j^2, \ldots, \Delta \rho_r^j)$, and the corresponding location vector $(x^*, y^*, z^*)$ is ready to prompt an update structural topology until the predetermined objective performance is achieved. The optimal location of the first damper is fixed according to the maximum of argument $\rho_i^{opt}$ since no controllability index gradient is at current state. It is worth noting that the design criterion for the optimal location of the next damper, referred to as the principle of maximum storey controllability index (MaxSCI) (Zhang and Soong, 1992), achieves the objective performance more slowly than that of the minimum storey controllability index gradient, which will be illustrated in details in the following numerical example.

### 3.2 Criterion of optimal damper parameters

As was noted previously, the optimal damper placement relies upon the controllability index gradient. The optimal parameters of dampers, however, must resort to another optimization program. Therefore, a performance function with energy trade-off can be constructed in the exceedance probability of quantities of interest, namely,

$$
J(\bar{Z}, \bar{U}) = \frac{1}{2} \left[ P_{\bar{Z}}(\bar{Z} - \bar{Z}_{thd} > 0) P_{\bar{U}}(\bar{U} - \bar{U}_{thd} > 0) + P_{\bar{U}}(\bar{U} - \bar{U}_{thd} > 0) P_{\bar{Z}}(\bar{Z} - \bar{Z}_{thd} > 0) \right]
$$

where $\bar{Z} = \max(\max(X(t), t_i), \max(X(t), t_i))$, $\max(\max(X(t), t_i), \max(X(t), t_i))$ are equivalent extreme-value vectors of the state and control force in the interval $[t_i, t_j]$, respectively; $\bar{Z}_{thd}, \bar{U}_{thd}$ are the threshold vectors corresponding to $\bar{Z}, \bar{U}$. It is clear that the performance function with energy trade-off has the same physical meaning as the controllability index.

The optimal parameters of dampers, thereby, are obtained by minimizing the performance function with energy trade-off, and the damper configuration is determined as the criterion of the MinSCIG at each stage. It is noted that the exceedance probability included in the controllability index and performance function can be readily solved, since the state vector $\bar{Z}$ and the control force vector $\bar{U}$ are governed by the generalized density evolution equations (GDEEs) (Li et al., 2010).

### 4. RESOLUTION PROCEDURE FOR INTEGRATED SCHEME

The resolution procedure of the functional matrix of generalized optimal control policy $f(\Gamma, L')$ involves optimization programs, including the sequential procedure to identify the optimal damper placement and the performance function minimized to lay down the optimal parameters of dampers. The following steps thus are used:

**Step 1:** Computation of controllability index of uncontrolled system. The numerical procedure involves: (i) Probability-assigned space partition to determine the representative point set $\sigma_{\text{opt}} = \{0, \theta_1, \theta_2, \ldots, \theta_{n_{\text{opt}}}\}$ and the corresponding assigned probabilities $p_i$ (Li and Chen, 2007). (ii) Deterministic dynamic simulation of the controlled system at the representative points to obtain the
state \(Z(\theta, t)\) and its derivative process \(\dot{Z}(\theta, t)\), the control force \(U(\theta, t)\) and its derivative process \(\dot{U}(\theta, t)\). (iii) Using the finite difference method to solve the GDEEs and get the numerical solutions of \(p_{zq}(z, \theta, t), p_{uq}(u, \theta, t)\), where the modified Lax-Wendroff difference scheme with TVD nature is usually preferred (Li and Chen, 2004). (iv) Repeating (ii) and (iii), running over \(q = 1, 2, \ldots, q_{\text{int}}\), and summing the results to obtain the desirable probability density by

\[
p_{z}(z; t) = \sum_{q=1}^{q_{\text{int}}} p_{zq}(z, \theta, t)S_{\theta}^{q}, \quad \text{and} \quad p_{u}(u; t) = \sum_{q=1}^{q_{\text{int}}} p_{uq}(u, \theta, t)S_{\theta}^{q}
\]

where \(S_{\theta}^{q}\) is the area measure of representative sub-domains, which is related with the partition strategy of probability-assigned space. (v) Computing the controllability index (gradient) by equations (3) and (4).

Step 2: Adding a new optimal damper. (i) Deploying the damper as the criterion of maximum (minimum) controllability index (gradient). (ii) Initial values chosen of optimized parameters based on their physical contexts, to the passively controlled system, for example, optimized parameters \(C_{d}, K_{s}\). (iii) Seeking for the optimal parameters by minimizing the performance function, which involves an iterative process of performing (ii) (iii) (iv) of Step 1.

Step 3: Computation of controllability index of controlled system. Running (ii) (iii) (iv) (v) of Step 1 for the controlled system with the newly added damper, the optimal location of the next damper is determined.

Steps 2 and 3 are repeated until the objective performance is achieved. The flow chart of these steps is shown in Figure 2.

5. NUMERICAL EXAMPLE

A ten-storey shear frame is controlled by \(r(1 \leq r \leq 10)\) viscous dampers. The storey mass and storey stiffness of the uncontrolled structure are shown in Table 1. With the application of Raleigh damping, the damping ratios for the first two vibrational modes of the entire building are assumed to be 0.02. The computed natural frequencies are 4.53, 11.92, 19.19, 25.92, 31.94, 38.82, 43.44, 47.40, 50.08 and 50.92 rad/sec. The threshold values, appearing in equations (3) and (6), of structural inter-storey drifts, inter-storey velocities, storey accelerations and control forces are 10 mm, 100 mm/sec, 3000 mm/sec², 200 kN, respectively. A physically-motivated stochastic ground motion model, specifically, a Kanai-Tajimi model with random site parameters is employed to simulate the ground motions, which is given by (Li and Ai, 2006)

\[
\ddot{X}(\Theta, \omega) = H(\Theta_{\omega}, \Theta_{\zeta}, \omega) \cdot \dot{U}(\Theta_{b}, \omega)
\]

\[
H(\Theta_{\omega}, \Theta_{\zeta}, \omega) = \frac{\theta_{\omega}^{2} + 2i \Theta_{\zeta} \omega}{\theta_{\omega}^{2} - \omega^{2} + 2i \Theta_{\zeta} \omega}
\]

where \(\ddot{X}(\Theta, \omega), \dot{U}(\Theta_{b}, \omega)\) are the frequency domain expressions of ground motions at the engineering site and the bedrock, respectively; \(\Theta = (\Theta_{\omega}, \Theta_{\zeta}, \Theta_{b})\) is the random vector characterizing the randomness involved in the ground motion at the surface of the engineering site, which is used to model the randomness inherent in systems. \(\Theta_{\omega}, \Theta_{\zeta}\) are the random parameters denoting the random nature of the site soil, the predominant frequency of the engineering site \(\omega_{0}\) and the equivalent damp-
ing ratio $\zeta$, respectively; $\Theta_b = \{\Theta_b, \lambda_s\}_{s=1}^b$ is the random variables characterizing the randomness involved in the ground motion at the bedrock coming from the properties of the sources and the propagation path, $s_b$ being the number of the random variables involved in this stage. $H_g(\omega, \Theta_\zeta, \omega)$ is a frequency transfer function; $\omega$ is the circular frequency; $i$ is the unit of imaginary number $\sqrt{-1}$. The time history of the stochastic ground motion could then be obtained by the inverse Fourier transformation:

$$\ddot{x}_\zeta(\Theta,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{X}_\zeta(\Theta,\omega)e^{i\omega t} d\omega$$

Figure 2. Flow chart of resolution procedure of integrated design.

Table 1. Parameters of the ten-storey shear frame.

<table>
<thead>
<tr>
<th>Storey no.</th>
<th>0-1</th>
<th>1-2</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (1×10^4 kg)</td>
<td>2.4</td>
<td>2.4</td>
<td>2.0</td>
<td>2.0</td>
<td>1.8</td>
<td>1.8</td>
<td>1.6</td>
<td>1.6</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Stiffness (kN/mm)</td>
<td>18</td>
<td>18</td>
<td>14</td>
<td>14</td>
<td>12</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>9.6</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Totally 221 representative points with corresponding assigned-probabilities are selected using the tangent spheres method (Li and Chen, 2007) and accordingly representative time histories of ground accelerations are synthesized. The predetermined objective of structural optimal control is designing the
parameters and placement of the least number of dampers to reach the control efficiency of dampers fully distributed in the structure. As an assessment objective of structural optimal performance, the fully distributed dampers are placed in the structure simultaneously, and their parameters are designed as the same.

The optimal placement and parameters of newly added viscous dampers at each sequence are shown in Table 2. It is seen that with only three optimally located dampers, a similar performance is achieved to that of a structural system that is fully controlled. The three dampers are distributed in the inter-6-7-storey, the inter-9-10-storey, and the inter-4-5-storey in turn. Furthermore, the parameters vector in the matrix function of generalized optimal control policy is given by

\[
(C_d^*, K_d^*, L^*) = \begin{bmatrix}
0 & 0 & 0 & 0.155 & 0 & 0.252 & 0 & 0 & 0.100 \\
0 & 0 & 0 & 0.098 & 0 & 0.111 & 0 & 0 & 0.100 \\
0 & 0 & 0 & 3 & 0 & 1 & 0 & 0 & 2
\end{bmatrix}^T
\]

where matrices \(C_d^*, K_d^*\) and \(L^*\) have been unfolded into the first, second, and third rows, respectively.

Table 2. Optimal placement and parameters of newly added viscoelastic damper in sequences.

<table>
<thead>
<tr>
<th>Sequence no.</th>
<th>Topology vector</th>
<th>Parameters of newly added viscoelastic damper*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(c_d) (kN×sec/mm)</td>
</tr>
<tr>
<td>0</td>
<td>[0 0 0 0 0 0 0 0 0 0]^T</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>[0 0 0 0 0 1 0 0 0 0]^T</td>
<td>0.252</td>
</tr>
<tr>
<td>2</td>
<td>[0 0 0 0 0 0 0 1 0 0 2]</td>
<td>0.100</td>
</tr>
<tr>
<td>3</td>
<td>[0 0 0 3 0 1 0 0 2]</td>
<td>0.155</td>
</tr>
<tr>
<td>Fully distributed</td>
<td>[1 1 1 1 1 1 1 1 1]</td>
<td>0.374</td>
</tr>
</tbody>
</table>

* Initial values of parameters are \(c_d = 0.1\) kN×sec/mm, \(k_d = 0.1\) kN/mm.

Figure 3 shows the relationship between the added dampers and the storey controllability index. It is clear that the storey controllability indices drop down rapidly after the damper with present optimal parameters is laid on the current optimal placement. The controllability index of inter-6-7-storey is always maximum among those of the inter-storeys in all the sequences, whereas the optimal placement does not tend to be the inter-storeys near to inter-6-7-storey since the design criterion of the MinSCIG not that of the MaxSCI is employed. Comparison between the two design criteria is illustrated in Figure 4, which shows that using three dampers can reach the control objective when the MinSCIG is used. Four dampers, however, are needed to achieve the same control effectiveness if the MaxSCI is used.

Table 3. System exceedance probabilities of quantities in sequences.

<table>
<thead>
<tr>
<th>Sequence no.</th>
<th>System exceedance probabilities</th>
<th>Objective function (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(P_{f,d})</td>
<td>(P_{f,v})</td>
</tr>
<tr>
<td>0</td>
<td>0.9952</td>
<td>0.8188</td>
</tr>
<tr>
<td>1</td>
<td>0.4195</td>
<td>0.6085</td>
</tr>
<tr>
<td>2</td>
<td>0.3226</td>
<td>0.4373</td>
</tr>
<tr>
<td>3</td>
<td>0.0545</td>
<td>0.1853</td>
</tr>
<tr>
<td>Fully distributed</td>
<td>0.1217</td>
<td>0.1886</td>
</tr>
</tbody>
</table>

The system exceedance probabilities of quantities in sequences including inter-storey drift, inter-storey velocity, storey acceleration and inter-storey control force, are listed in Table 3. It is indicated
that the system reliability of quantities is gradually enhanced with the deployment of dampers. The objective function of sequence 3 reaches the same level as that of the sequence of fully distributed dampers (the objective functions have the same magnitude). It reveals that the structural system has achieved system safety, system serviceability, system comfortability and system workability. Compared with the sequence, meanwhile, of fully distributed dampers, sequence 3 exhibits a better displacement control and a worse acceleration control.

The second-order statistics of extreme inter-storey drifts in different sequences are exposed in Figure 5. It is seen that the mean of inter-storey drift becomes smaller with the dampers being added, whereas the standard deviation of inter-storey drifts of the storeys above the seventh floor increases when the first damper is placed in inter-6-7-storey, while it goes down rapidly when the second damper is placed. It is noted that too few control devices may result in amplification of the local response of the structure. In brief, the proposed sequential procedure can gain the optimal system reliability with a minimum control effort. The controlled system response is significantly reduced and becomes smoother along the height of the structure, which results in a desirable structural performance.

Figure 3. Relationship between added dampers and storey controllability index.

Figure 4. Comparison between design criteria of the MinSCIG and the MaxSCI.
6. CONCLUDING REMARKS

Modern structural designs are not only in demand of accommodating safe residences but also in demand of providing available spaces as possible. The optimal control device placement and optimal parameter design, therefore, should be of the same practical significance. In this paper, an integrated scheme with routine of two-step optimization serving to schedule the optimal topology and optimal parameters of viscous dampers mounted on passively controlled structures is proposed. Numerical results reveal that the proposed integrated scheme can achieve the maximum control effectiveness with the minimum control effort. The system reliability of quantities is gradually enhanced with the deployment of dampers resulting in a desirable structural performance. In addition, using the design strategy of the defined minimum storey controllability index gradient achieves the objective performance more efficiently than that of the maximum storey controllability index currently in use. Besides, following the placement of each damper, one might realize that decision regarding the system reliability of structure can be updated, in a multistage manner, before deciding on whether to proceed with deploying additional dampers.

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