Vulnerability curves for RC substandard buildings

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SUMMARY

This paper presents a proposed probabilistic analytical seismic vulnerability assessment framework for deficient non-seismically designed reinforced concrete structures. To capture the realistic elastic and inelastic behaviour of poor quality structures low strength concrete stress-strain and bond-slip models of different bar types are included in structural modelling. To address the uncertainty in vulnerability predictions, key capacity parameters are generated stochastically to produce the building population and a cyclic pushover analysis is conducted. Non-linear static method by FEMA440 is used with a simple technique to realistically model the complex degrading behaviour of brittle structures by maintaining the true characteristics of degrading curves after the capacity drop. A time period based damage index is used to quantify damage. The framework is currently implemented on the mid-rise case structures and the vulnerability curves showed abrupt accumulation of damage at low peak ground acceleration levels for buildings with no seismic design.

1. INTRODUCTION

Recent worldwide experience indicates that even though new design codes have been introduced in most seismic regions, they did not contribute as much to the minimization of earthquake losses, primarily as a result of the fact that the majority of the existing building stock pre-dates modern codes. An unfortunate verification of the above is given in the 2011 annual report of the Centre for Disaster Management and Risk Reduction in Germany (CEDIM, 2012), which concluded that economic losses from earthquakes and their consequences have peaked in 2011 estimated at a staggering $365 billion U.S. dollars.

The quantification of the damage potential to the existing building stock can be achieved through vulnerability curves which represent the mathematical relationship between damage and seismic hazard. In recent years, analytical procedures are used to derive vulnerability curves, which in general use Finite Element Analysis (FEA) of structural models to simulate the buildings response and determine the damage distribution through an appropriate response-based damage index. In order to produce such curves for non-seismically designed buildings it is important to consider their deficiencies both in design and detailing and accommodate their brittle failure modes in the structural modeling.

In the current study, a framework is developed for conducting probabilistic analytical vulnerability assessment of low and mid rise Reinforced Concrete (RC) structures. The technique is based on improved modelling assumptions, use of new capacity models for low strength structures, improved performance evaluation method for brittle structures and the probabilistic assessment of capacity related uncertainties. The effect of predominant failure modes in non-seismic RC structures such as bond and shear are addressed through cyclic pushover analysis. For simplicity and efficiency of the framework, the improved capacity-spectrum procedure (MADRS) proposed in FEMA (2005) was used in line with a simple proposed methodology to model complex degrading behavior of degrading structures. The brittle failure modes were successfully captured for the case structures using a damage index having secant period as a response parameter. Vulnerability curves were derived as a function of peak ground acceleration (PGA).

2. VULNERABILITY FRAMEWORK

The flowchart diagram outlining the proposed probabilistic analytical vulnerability framework is shown in Figure 2.1. Initially, the building category to be examined is selected. In most cases this depends on the number of stories and period of construction. Secondly, the key capacity parameters are selected and their corresponding probability density functions (PDF's) are defined. To limit the number of simulations, the LHS technique is used in step 3 (Figure 2.1) to generate the variables P_{ij} for the analysis (step 4). Using the generated values of the P_{ij} variables and the deterministic values from the design process of the frame, members capacities in flexure, shear and bond are computed and the hysteretic material models are calibrated. In step 5, the mathematical models for each building category are constructed using a finite element software i.e. DRAIN-3D. In step 6, the population of probabilistic buildings for each building category are generated. Subsequently, cyclic nonlinear analysis is undertaken to produce the capacity envelopes of the Multi Degree of Freedom Systems (MDOF) with top displacement (u) vs base shear (bs) as output (step 7). These capacity curves of the MDOF system are transformed into the corresponding ones of an equivalent SDOF system in the SA vs SD space (Step 8). In Step 9, the secant period (T_{sec}) corresponding to each point on the equivalent SDOF curve is calculated using equation 3.2 (see Figure 3.1). Step 10 selects the type of energy balance of each defined equivalent system as discussed in section 4. This leads to the evaluation of ductility (μ) and initial period (T_{ini}) for each system. Step 11 evaluates the T_{eff} and β_{eff} according to FEMA440 (2005) provisions. Step 12 evaluates the reduction factors η and M according to FEMA440 (2005) provisions. In Step 13 the PGA at every point on the SA vs SD curve is calculated using the EC8 (1998) spectrum (see section 5). Step 14 uses the damage index and defines its necessary parameters such as $T_{initial}$ and $T_{collapse}$ and thus the corresponding DI is calculated. Step 15 evaluates the interpolated values of PGA corresponding to DI from 0 to 100 and the mean PGA is evaluated in step 16. Finally the 95% probability of exceedance (POE) and 5% POE vulnerability curves are plotted. Further information on the most important steps of the framework is given in subsequent sections.

3. DAMAGE INDEX

A calibrated damage index (DI) based on the change in period due to damage is used in the proposed framework (Kyriakides, 2007). The damage index involves period elongation with increased displacement demand and assumes the fundamental mode as the predominant mode for structures having small inelastic deformations. The proposition that damage is related to the increase in period was recently verified by Calvi et al. (2006) using experimental data. Zembaty et al. (2006) moved a step forward by producing a damage scale that can be used for the definition of the degree of damage from the recorded drop in natural frequency of a structure.

The DI defined based on the above, normalised for the initial condition of zero damage at initial period Tinitial is given in equation 3.1.

$$DI = \frac{T_{\sec}}{T_{initial}} - 1 = \frac{T_{\sec} - T_{initial}}{T_{initial}} \tag{3.1}$$

'$T_{initial}$' is the period of vibration of the structure with no damage, whereas 'T_{sec}' is the secant period which can be evaluated from the capacity envelope using the equation 3.2.

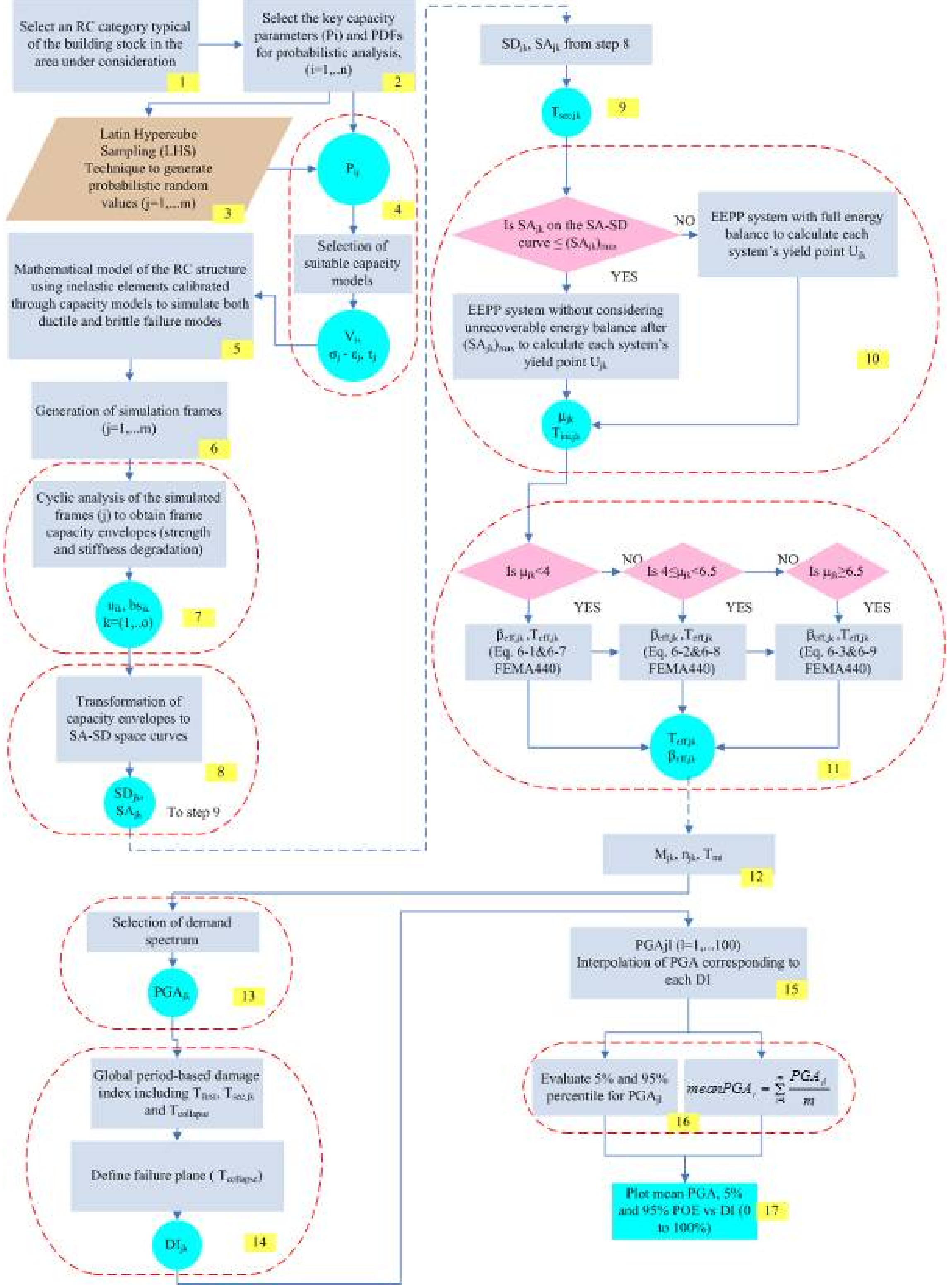

Figure 2.1. Proposed framework for the probabilistic analytical assessment of RC structures

$$T_{\sec} = 2\pi\sqrt{\frac{SD_i}{SA_i}} \tag{3.2}$$

Where:
SD_i spectral displacement at point 'i' on capacity curve
SA_i spectral acceleration at point 'i' on capacity curve

This DI is bound by an additional condition (DI=100) for the value of period at complete damage defined herein as T_{100} (Kyriakides, 2007). The definition of T_{100} is based on the top storey drift and an example is given at the case study at the last part of the paper. Therefore a final adjustment is applied to equation 3.1 to produce the final relationship for the DI (standardised for no damage at DI=0 and collapse at DI=100) at each SA_i, SD_i coordinate (eq. 3.3).

$$\mathrm{DI} = 100\left(\frac{\mathrm{T_{sec}} - \mathrm{T_{initial}}}{\mathrm{T_{100}} - \mathrm{T_{initial}}}\right) \tag{3.3}$$

The backbone curves obtained from the monotonic or degrading cyclic response of a structure can be used by this method for damage quantification at each displacement point. The threshold values for drift at collapse, which is used for evaluating the collapse period T_{100} and defining the failure plane, can be found in different codes for various categories of structures. T_{100} can be defined by the radial line corresponding to the limit value of SD (corresponding to collapse drift) as shown in Figure 3.1. Any building with capacity envelope crossing the failure plane is regarded as collapsed.

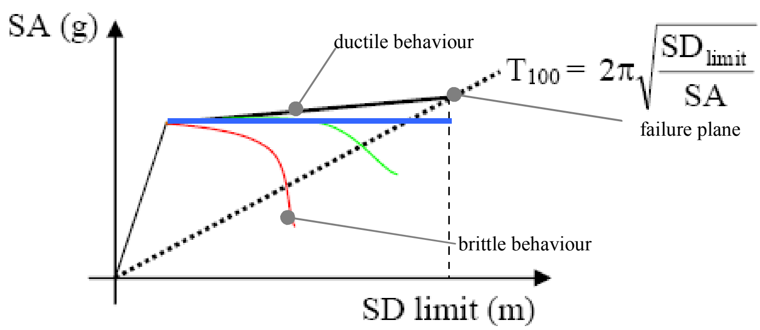

Figure 3.1. Definition of the failure plane (Kyriakides, 2007)

Apart from collapse failure plane, different intermediate performance planes (Tslight, Tmoderate, Tsevere) corresponding to other damage states can also be defined on the capacity spectrum as shown in Figure 3.2. Using these degraded cyclic envelopes the abrupt damage, due to the brittle failure modes, can be quantified efficiently. Multiple secant period planes (slight, moderate, severe) shown in Figure 3.2 can be used to evaluate the POE of a damage state at a particular ground motion level.

In order to use the adopted DI for the scope of this work its predictions should be correlated to Mean Damage Ratio's (MDR, eq. 3.4) since this is the most widely used economic damage indicator. In brief, it represents the ratio of the repair to the replacement cost i.e. the fraction of cost that has to invested for repair in comparison to replace a structure. Thus, the shape of the function relating DI with MDR needs to be defined.

$$\mathrm{MDR} = f(\mathrm{DI}) \tag{3.4}$$

For that purpose, empirical results of damage at various PGA levels observed in Cyprus and Pakistan are used as reference. It is observed that a linear increase in the damage grade causes an exponential increase in the MDR. Since the adopted DI increases exponentially with an increase in damage grade (due to the exponential increase in period due to the nonlinear behaviour) it is decided to assume that the DI is linearly correlated to the MDR with a correlation coefficient equal to 1. This assumption requires further justification using more empirical data.

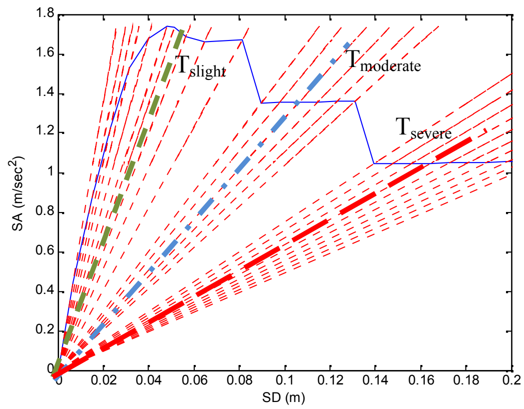

Figure 3.2. Secant periods corresponding to each performance point on the capacity curve

4. MODELLING OF DEGRADATING BEHAVIOUR

In order to assess the detailed characteristics (μ, T_{sec}) of the capacity spectrum especially for degraded curves obtained from the cyclic pushover analysis, each point (SA_i and SD_i) on the curve can be considered as a performance point (PP). As one option, each of the assumed PP can be considered either as an equivalent bilinear (BLN), stiffness degradation (STDG) or strength degradation (STRDG) system in accordance with the MADRS capacity spectrum method in FEMA 440 (2005). An example of BLN and STRDG idealisation of the capacity curve is given in Figure 4.1. The post elastic stiffness ratio (α) can be evaluated for each system using energy balance. Values of α are either positive or negative depending on the location (hardening and softening region) of the PP on the capacity curves, and these values describes different hysteretic behaviours. Coefficients corresponding to different α values and hysteretic behaviours in FEMA 440 (2005) can be used to evaluate equivalent damping and period of a system at a particular ductility level. However, due to un-conservative predictions of STDG and STRDG at high PGA for a RC building case structure this approach is not considered suitable for degrading structures.

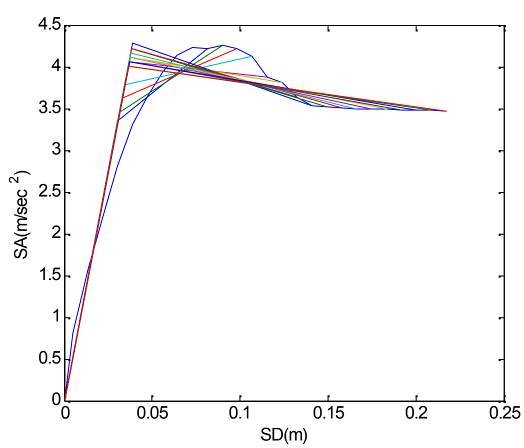

Figure 4.1 Series of equivalent BLN, STRDG systems for evaluation of α and μ

To address the matter of seismic demand prediction of brittle degrading structures, a simple and effective approach to simulate the complex degrading behaviour that ignores the unrecoverable energy (energy above PP after capacity drop) and maintains special characteristics of the degrading RC structures is proposed. Similar to the previous approach, each point on the degraded capacity curve is assumed to be the PP and for these PP's, Equivalent Elastic Perfectly Plastic systems (EEPP) for each PP are assumed as shown in Figure 4.2a. After the capacity drop on a degraded capacity curve, the unrecoverable energy is not accounted in the energy balance as shown in Figure 5b. A detailed discussion on this idealisation procedure can be found in Kyriakides (2007). This is expected only to enhances slightly the ductility level, due to the reduction in the yield displacement.

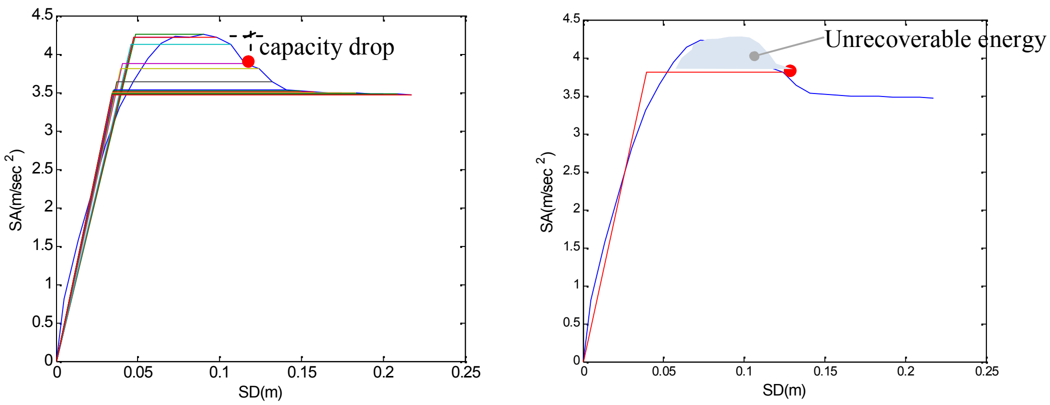

Figure 4.2. a) Series of equivalent EEPP systems at each PP b) Unrecoverable energy at higher displacement

Figure 4.3a shows the cumulative area under the capacity curve at a SD(j) corresponding to the maximum capacity point. EEPP corresponding to this point is shown in Figure 4.3b. Equal area rule (equations 4.1 and 4.2) is applied to evaluate the yield displacement U(j) using equation 4.3.

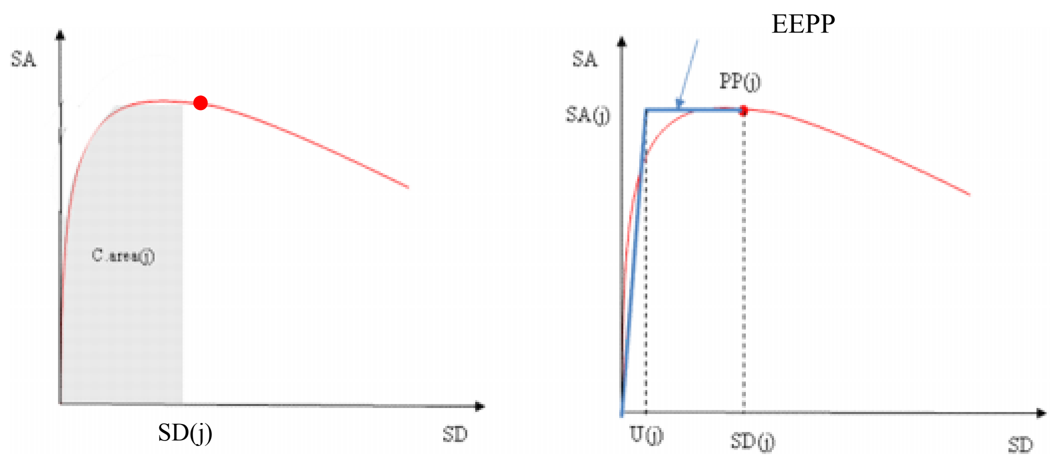

Figure 4.3. Evaluation of yield displacement for EEPP a) Cumulative area at a particular spectral displacement b) Implementation of equal energy rule for yield displacement evaluation using proposed methodology

$$C.area(j) = \frac{U(j)SA(j)}{2} + (SD(j) - U(j))SA(j) \tag{4.1}$$

$$C.area(j) = \frac{U(j)SA(j) + 2SD(j)SA(j) - 2U(j)SA(j)}{2} \tag{4.2}$$

$$U(j) = -\frac{2(C.area(j) - (SD(j)SA(j)))}{SA(j)} \tag{4.3}$$

The performance of the proposed methodology was assessed in predicting the seismic demand of brittle low strength structures. For this purpose a simulation study was conducted and a small building population (10 buildings for each building type) was generated from the probabilistic data related to key capacity parameters (concrete strength, steel yield strength, cover and development length) as described in Ahmad (2011). The error in seismic demand predictions using the MADRS capacity spectrum method in FEMA440 (2005) and calibrated EEPP method (using cyclic analysis) for each building category are summarized in Table 4.1. Details of this study can be found in Ahmad (2011).

Table 4.1. Mean and standard deviation of seismic demand prediction error

No	Building	Error (MADRS) % (μ)	Error (MADRS) % (σ)	Error (EEPP) % (μ)	Error (EEPP) % (σ)
1	2 storey 1bay	18.6	5.3	8.3	5.7
2	2storey 2bay	10.8	5.4	5	2.72.
3	3storey3bay	19.7	5.4	3.2	3.08
4	5storey4bay	15.2	4.1	3.7	2.3

Hence it was concluded that in cases of ductile structure with predominant flexural failure mode, full energy balance of the EEPP system is proposed to be used, whereas for the deficient structures with strength degradation mainly due to the brittle failure mode, the unrecoverable energy will be removed from the energy balance and shown in Figures 4.2 and 4.3.

5. CALCULATION OF PGA

To implement the MADRS method in FEMA440 (2005) in the current vulnerability assessment framework, a reverse procedure (back analysis) is adopted for PGA calculation. The assumption of the multiple performance points (SA_i, SD_i) and equivalent systems on the capacity curve is used to evaluate T_{eff} and β_{eff} corresponding to the ductility at each PP. Since the PP are already known, the reduction factors (η and M) are evaluated in accordance with the method for the highly damped demand spectrum corresponding to each PP. The reduction factor η refer to the demand spectrum β_{eff} in ADRS format and M refers to the β_{eff} in MADRS format. The ADRS and MADRS spectrums for 2 PPs are shown in Figure 5.1. A back analysis procedure is adopted and these parameters are substituted along with other parameters related to the PP (SA, $T_{initial}$) in the EC8 (1998) elastic spectrum relation (equation 5.1) to evaluate the PGA corresponding to each PP. The PGA associated with each PP of the building population capacity curves is schematically shown in Figure 5.2 and can be evaluated using equation 5.1.

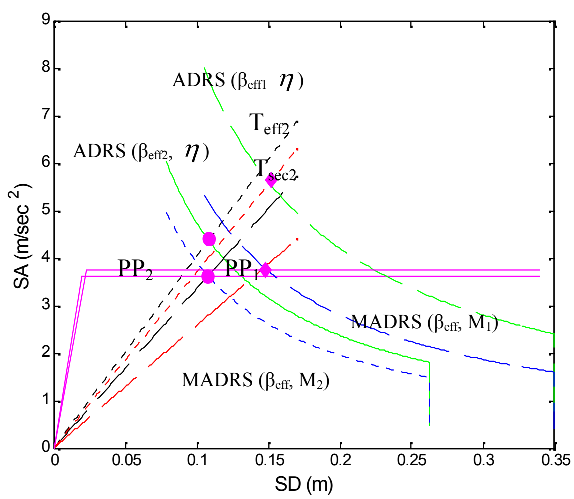

Figure 5.1. Methodology for evaluation PGA from assumed PP's

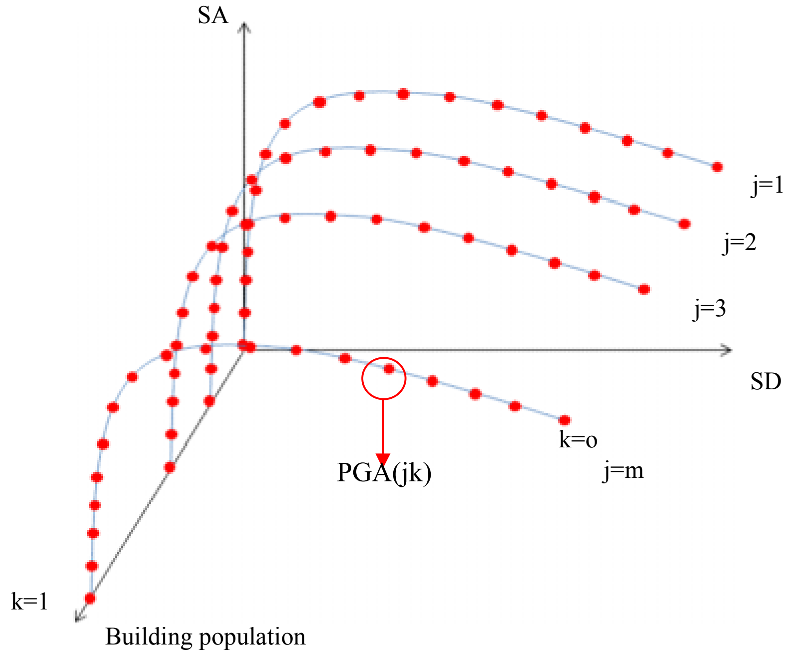

Figure 5.2. Schematic representation of the PGA calculation from a PP on capacity spectrum

$$PGA(jkp) = \frac{SA(jk)T_{ini}(jk)}{\alpha SM(jk)\eta(jk)T_c} \tag{5.1}$$

Where:
j= building population
k=a performance point on capacity curve
SA =spectral acceleration
T_{ini} =initial period of the equivalent system
T_c = site characteristic period
α =spectral amplification coefficient
S =soil factor
η =reduction factor
M =modification factor

The advantage of using back analysis for PGA calculation is the consideration of the demand uncertainty in an indirect manner. It is quicker than using time history analysis where a lot of artificial or natural ground motion record sets corresponding to different PGA levels are required. In the proposed method, there is no need to do either stripe or cloud analysis as done by many researchers in their fragility assessment studies. Moreover, the evaluated PGA by this method is also performance consistent for a particular category of structures.

6. PROBABILISTIC ANALYTICAL VULNERABILITY ASSESSMENT

In order to study the probabilistic aspect of the analytical vulnerability curves, variability of the various significant capacity parameters involved in the calibration of material models was accounted for. The variability of these parameters can cause significant uncertainty in the vulnerability outcome since the dominant mode of failure might be altered. The main source of capacity related variability rises from different parameters involved in the capacity models. These capacity models are used for evaluating concrete stress-strain ($\sigma - \varepsilon$) curves, bond strength (τ_{max}), shear capacity (V_n) etc. The parameters related to capacity uncertainty can be divided into three broad categories, which include key, geometrical and design parameters as shown in Table 6.1. The variability of key parameters can be addressed using expert judgement, code provisions or by using available statistics which set the basis for generation of the probability density function (PDF).

Table 6.1. Calibration parameters for capacity models

Capacity model	Key parameters	Deterministic parameters	Design parameters
Flexure:	$f_c^{'}$, f_y	b, d, $k = \dfrac{f_{ult}}{f_y}$, ε_{su}	ρ
Shear:	$f_c^{'}$, s	b, d, f_{yw}	A_{sw}, s
Bond:	f_{ct}, s, l		s, l, d_b

Stochastic techniques like the Monte Carlo Simulation (MCS), is a very popular technique for generating probabilistic random samples. However, it requires a very large sample to obtain the target accuracy and generating a large sample is also not computationally efficient. For reducing the sample size the Latin Hypercube Sampling (LHS) Method was proposed by McKay et al. (1979). As compared to the Monte Carlo method this it uses a stratified technique for the selection of simulation values from the PDFs which ensures the inclusion of values from all the distribution. In a study contacted in Kyriakides (2007) it was found that 25 and 50 simulations give a converged solution, whereas less (10) simulations showed significant variation. Therefore it is proposed that the use of 25 simulations for the application of the framework is adequate and may regarded as the optimum number given that additional key probabilistic parameters are not added.

7. VULNERABILITY CURVES GENERATION

A very large array of low and mid rise building categories over different construction and design periods (CDP) need to be assessed as far as their vulnerability is concerned in order to arrive at reliable estimates of the possible losses in country scale. In the present paper, and in order to illustrate the vulnerability framework described above, vulnerability curves were developed for Mid-Rise (MR) buildings with Basic and Modern seismic design. Basic seismic frames were designed using BS8110 (1985) provisions and an additional horizontal force equal to the 10% of the base shear, whereas Modern Seismic design buildings were designed using the provisions of EC8 (1998). Each design was based on the corresponding code loading combinations. Basic seismic design buildings are typical of approximately 50% of the present stock worldwide whereas the construction of Modern Seismic designed buildings in Europe initiated in the mid 90's.

For structural modelling purposes, a 2D prototype model of a 4 storey-2 bay RC frame was created on DRAIN-3D using local models to account for most possible failure modes. A complete description of the modelling and the analysis procedure used can be found in Kyriakides (2007). A random population of simulation frames (25 frames) were generated, based on the prototype, using the PDF parameters for the key capacity parameters through LHS. Due to the random assignment of the capacity parameters both brittle (e.g. bond or shear failure) and flexural failure mode were anticipated. The definition of the collapse period T_{100} shown in Figure 3.1 is achieved using the threshold top storey drift values for similar building categories given in HAZUS99 (NIBS, 1999), which corresponds to a 5.5% and 3.5% top storey drift for Modern and Basic seismic designed MR buildings.

The mean, 95% and 5% POE derived vulnerability curves for MR Modern and Basic seismic design buildings are shown in Figures 7.1 and 7.2 respectively. Both curves are compared to the damage limits proposed in HAZUS99 for similar building categories. It can be seen that Basic design buildings fail due to abrupt failure modes which is expected due to their deficient design and detailing mainly as far as non-flexural failure modes are concerned. These modes are not accounted by the complete damage limit given in HAZUS99 (NIBS, 1999). In the case of the Modern design buildings, damage is dominated by the spread in plasticity and the small variation in the derived curves is attributed to material strength variability. Although the response is dominated by the flexural mode, HAZUS99 (NIBS, 1999) limit for complete damage again underestimates the damage potential.

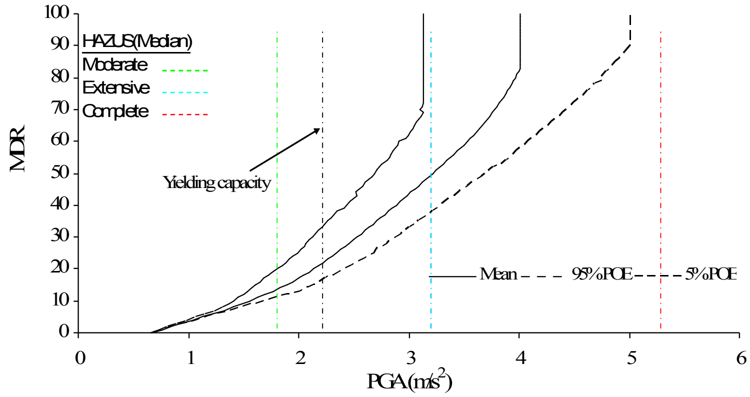

Figure 7.1. Analytical vulnerability curves for MR Basic design RC buildings

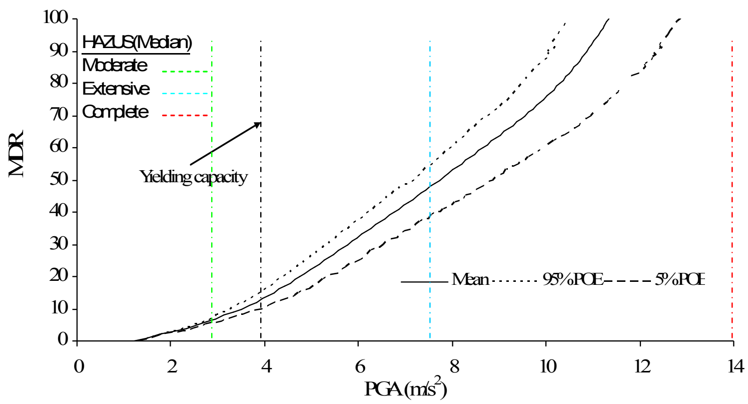

Figure 7.2. Analytical vulnerability curves for MR Modern design RC buildings

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