SUMMARY:
The objective of this study is to investigate the effect of three-dimensional configuration of the soil-structure system from the viewpoint of elastic wave propagation. Two three-dimensional configurations have been considered. The one is a two-dimensional slope ground with a small canyon that penetrates perpendicularly into the upper part of the ground, and the other is a rigid foundation embedded in a slope ground. Both are subject to incident waves. The analysis method is a combination of three-dimensional finite element and 2.5-dimensional thin layered and finite element methods in conjunction with a substructure technique. It was found from the study that the wave field becomes very complex when there exists a irregularity, causing a significant difference between 2.5-dimensional and three-dimensional results. The frequency dependency is also affected by an irregularity. These differences are significant not only in its vicinity but they are also fairly large even in a distant place.

Keywords: Irregular Ground, Incident Wave, Finite Element Analysis, 3-D Analysis, 2.5-D Analysis

1. INTRODUCTION

It is essential to know the condition of the ground when considering earthquake disaster mitigation. It is well known that the surface soil condition and micro topography, or landform, influence the seismic intensity of the ground and hence impact structural damage to the buildings and civil infrastructure during earthquakes. For example, dense array observations of strong ground motion have revealed that peak ground accelerations can vary by a factor of two due to an irregular ground such as land reclamation or slope (Nishizaka et al., 1998; Nagata et al., 2007). However, obtaining information on the ground condition, such as soil profiles, over a wide area is not an easy task from a practical perspective. One of the most popular approaches to estimate the ground condition is to conduct microtremor (ambient vibration) measurements on the ground surface, from which natural frequencies of the ground are obtained (e.g., Nakamura, 1989; Arai et al., 2000, 2004). It is also possible to obtain soil profiles from microtremor array measurement results by applying inversion techniques based on the surface wave propagation theory (e.g., Aki, 1957; Capon, 1969; Cho et al., 2006). All the approaches proposed so far, however, are based on a parallel layer assumption. A difficulty arises when the ground has an irregularity, which is often the case in an actual situation. For example, Photo 1 shows a typical landscape found in a central part of Chiba City, or the eastern part of Tokyo metropolitan area, Japan. As can be seen in the picture, this area consists of terrace and lowland (Nakai et al., 2007). It is often the case that there exist fairly steep slopes along the boundaries between these two landforms.

A number of researches regarding wave propagation in an irregular ground have been reported so far (e.g., Hisada et al., 1996; Kawase, 1996). However, a completely flat ground surface assumption for the far field is made in almost all three-dimensional studies (e.g., Bielak et al., 1998, 2003). In the previous study, the authors have looked at surface wave propagation in an irregular ground from the viewpoint of the applicability of microtremor measurement to soil investigations, which has had its
basis on the parallel layer assumption, and pointed out that H/V spectral ratios and phase velocity dispersion curves can be influenced to some extent by the existence of ground irregularities even in the distant locations (Nakai et al., 2011a, 2011b).

![Photo 1. Typical landscape in Chiba area](image)

In this paper, the effect of a three-dimensional ground irregularity on the wave propagation is studied. More specifically, a two-dimensional slope ground with a small canyon (hollow) and a two-dimensional ground on which a massless rigid foundation with embedment is placed, both subject to an incident wave are considered, as schematically illustrated in Figure 1. Incident waves considered in the study are Rayleigh and shear waves. The analysis method used in the study is a combination of three-dimensional and 2.5-dimensional finite element methods (Nakagawa et al., 2010) in conjunction with a substructure technique (Nakai et al., 1985).

![Figure 1. Irregular ground under study](image)

2. PROBLEM UNDER STUDY

As mentioned above, the main objective of this study is to examine the characteristics of wave propagation in an irregular ground in three dimension in order to evaluate its influence on the ground motion. As an attempt to address this issue, two simplistic configurations were studied. The ground is basically a two-dimensional slope ground but has either a slit-like narrow canyon that penetrates perpendicularly into the terrace (upland part of the landform) or a massless rigid foundation that is embedded in the ground, as shown in Figure 1. The problem under study is the wave field of this landform subject to an incident wave. The problem considered in this study is defined by the following statements.

- The ground has a two-dimensional landform, i.e., a slope ground.
- The ground, however, is in three-dimension in that it has a small canyon that penetrates
perpendicularly into the terrace (the upland part of the slope ground) or a massless rigid foundation that is embedded into the ground.

- The soil is two layered throughout the landform.
- The ground is subject to a surface (Rayleigh) wave coming from a variety of directions or a vertically incident shear wave with a variety of vibration directions.

3. METHOD OF ANALYSIS

The method of analysis is basically a three-dimensional finite element method in conjunction with a substructure technique. It features, however, a couple of points so that it can handle the problem under study. One major point is that a far field ground involves a topographic irregularity, which is rarely seen in the previous literatures.

3.1. Substructure Method

There exist a variety of substructure approaches that deal with wave propagation in an elastic medium. The method used in this study follows the following procedures (Nakai et al., 1985):

1. Subdivide the entire ground under study into two parts; a near field that involves three-dimensional irregularities, and a far field that is basically a two-dimensional slope ground.
2. Compute an impedance matrix \([K^c_\ast]\) of the far field from which the near field is excavated.
3. Compute a displacement vector \(\{u_c\}\) and traction vector \(\{p_c\}\) of an equivalent far field which does not have an excavation and is subject to an incident wave.
4. Compute a driving force vector \(\{f^\ast\}\) at the boundary by the following expression:
   \[
   \{f^\ast\} = [K^c_\ast]\{u_c\} + \{p_c\}
   \] (3.1)
5. Compute a response of the near field by attaching the impedance matrix at its boundary and by applying the driving force to the boundary.

3.2. Response of a Far Field: 2.5-Dimensional Analysis

The substructure analysis described above requires a three-dimensional analysis of a two-dimensional slope ground subject to an incident wave. This type of analysis is called a 2.5-dimensional analysis (Khair et al., 1989; Nagano et al., 1985). Since irregularity (slope) is already involved in this analysis, another sub-structuring is considered, i.e., 2.5-dimensional thin layered elements and 2.5-dimensional finite elements are combined to obtain the response of a slope ground due to an incident wave. Figure 2(a) illustrates the method of analysis (Nakagawa et al., 2010).

![Figure 2. Analysis method (left: 2.5-D, right: 3-D)](image-url)
In the 2.5-dimensional analysis of surface wave incidence, the fixed condition was set at the bottom boundary of the model and the depth to the bottom boundary was changed depending on the analysis frequency in such a way that the depth to the bottom boundary is twice as thick as the wavelength. Whereas, in the case of shear wave incidence, the dashpot was attached to the bottom boundary and the thickness of the underlying layer was set to 105 meters, which is greater than the wavelength of shear wave of the layer for the lowest frequency of analysis.

Figure 3 gives a comparison between the proposed method and the one found in the literature (Nagano et al., 2002). The analysis model is a so-called basin edge model consisting of a soft surface soil and a bedrock. The displacement is measured at the location which is distant from the basin edge with the distance equal to the depth of the surface soil. From the figure, it is understood that the proposed method correctly handles the problem.

In the analysis model of the target, i.e., a slope ground with a tiny canyon or an embedded foundation, requires the impedance matrix \([K_c^*]\) and the driving force vector \(\{f_c^*\}\) as described earlier. In this study, the impedance matrix \([K_c^*]\) at the boundary of the analysis model is computed as dashpots attached to the boundary. The displacement vector \(\{u_c\}\) of the equivalent far field, found in Eq. (3.1), can be computed from the 2.5-dimensional analysis described in the previous section by the following expression:

\[
\{u_c\} = \{u_{2.5}\} \exp(-ik_y y), \quad k_y = k \sin \phi
\]  

(3.2)

in which, \(\{u_{2.5}\}\) is a displacement vector obtained from the 2.5-dimensional analysis. Eq. (3.2) states that the displacement wave field in the \(y\)-direction is expressed in an analytic form once the displacements on the \(x-z\) plane are obtained. \(k\) is the wave number of an incident wave, and \(\phi\) is the angle of incidence of the surface wave and the angle of vibration direction for the vertically incident shear wave. Figure 2(b) illustrates the three-dimensional analysis.

4. ANALYSIS MODEL

4.1. Slope Ground with a Tiny Canyon

Figure 4(a) shows the finite element mesh layout of the analysis model of a slope ground with a tiny canyon. As shown in the figure, the angle of inclination of the slope is 45º and its height is 12 meters. Eight node linear elements are used in the three-dimensional analysis, while eight node quadratic elements are used in the 2.5-dimensional finite element analysis and three node quadratic elements are used in the 2.5-dimensional thin layered element analysis. In the model, there exists a small
canyon-like hollow that penetrates perpendicularly into the slope, as shown in Figure 4(a). The configuration of the canyon has the depth of 12 meters, the width of 20 meters and the length of 20 meters. The ground surface inside the canyon is on the same level as the ground surface of the lowland part of the landform. The soil is two-layered throughout the model and the thickness of the surface soil is 12 meters, meaning that there exists no surface layer in the innermost area of the canyon because the height of the slope is also 12 meters. The shear wave velocity of the surface soil is half of that of the underlying half space.

The reason why the ground is two-layered throughout the model is because it is possible to concentrate on the effect of topography, in this case a slope, by adopting the same configuration in terms of soil layering.

Incident waves considered for this model are the Rayleigh wave of fundamental mode traveling from the terrace part of the landform with the incidence angle of 45º and the vertically incident shear waves with the angles of vibration direction (hereafter called the azimuth angles) of 0º, 45º and 90º. Here, the azimuth angle of 0º corresponds to the vibration direction perpendicular to the depth direction of the slope and that of 90º is parallel to its depth direction.

4.2. Slope Ground with a Massless Rigid Foundation with Embedment

Figure 4(b) shows the second model in this study, that is a slope ground in which a massless rigid foundation is placed. The size of the foundation is 20 meters by 20 meters with the embedment of 10 meters. The rest of the configuration of the model is the same as the one shown in Figure 4(a), except that there exists no canyon. Vertically incident shear waves were considered for this model.

5. RESULTS AND DISCUSSIONS

5.1. Wave Field due to an Incident Rayleigh Wave

Figure 5 compares the displacement wave fields due to an incident Rayleigh wave of the frequency of 5.0 Hz and the incidence angle of 45º for a slope ground and a slope ground with a tiny canyon. From this figure, it is noted that:

- Due to the existence of a canyon, the displacement field changes by a great deal and becomes very complex.
- Displacement amplitude is large in the area located upstream with respect to the canyon and is small in the back.
The affected area is fairly large when compared to the size of the canyon.
- Amplitude and its variation of the displacement is large on the upland. The reason for this is because the wave comes from left and it reflects at the slope.
- Displacement amplitude on the ground surface inside the canyon is small.

**Figure 5.** Wave field due to an incident Rayleigh wave of fundamental mode, 5 Hz

Although these characteristics vary depending on the frequency, hence the wavelength, which is not shown in this paper due to the space limitation, it can be said that the influence appears over large areas even in the low frequency range.

5.2. Wave Field due to an Incident Rayleigh Wave

**Figure 6.** Wave field due to a vertically incident shear wave, 5 Hz

Figure 6 compares the displacement wave fields due to a vertically incident shear wave of the
frequency of 5.0 Hz and the azimuth angle of 45º for a slope ground and a slope ground with a canyon. From this figure in conjunction with Figure 5, it can be pointed out that:

- Due to the existence of a canyon, the displacement field changes by a great deal as in the case of surface wave incidence.
- In the vicinity of the canyon, amplification is remarkable and the vertical displacement is generated due to the existence of the canyon edge.
- The lowland part of the ground is less affected when compared to the upland part.

5.3. Effect of a Foundation on the Wave Field

Figure 7 compares the displacement wave fields due to a vertically incident shear wave of the frequency of 5.0 Hz and the azimuth angle of 45º for a two-layered horizontal and slope grounds. A massless rigid foundation with the size of 20 by 20 meters and with an embedment of 10 meters is placed in the ground. The foundation is located in the center of the two-layered horizontal ground or is located either in the upland or the lowland part of the slope ground. As can be seen from the figure, it is noted that:

- The wave field is affected by the existence of a foundation in wide-ranging areas.
- Its influence appears even in the lowland even if the foundation is placed in the upland. Vice versa also holds.

![Figure 7. Wave field due to a vertically incident shear wave, 5 Hz](image)

5.4. Effect on Transfer functions

Figures 8 through 11 show the transfer functions at the four particular locations on the ground surrounding the irregularity with respect to the vertically incident shear wave that is defined at the
bottom of the model. The following points can be made from these figures:

- As is expected from Figures 5 through 7, the transfer function from the three-dimensional analysis differs from those from one-dimensional and 2.5-dimensional computations.
- Difference includes amplification factors, predominant frequencies and frequency dependency.
- Influence appears differently depending the location with respect to the irregularity, a canyon or a foundation.
- Generally speaking, influence is large near the predominant frequencies of the horizontal two-layered soil.

**Figure 8.** Transfer functions at four locations surrounding a tiny canyon, vertical incidence of shear wave

**Figure 9.** Transfer functions at four locations surrounding a rigid foundation placed on a horizontal two-layered ground, vertical incidence of shear wave

**Figure 10.** Transfer functions at four locations surrounding a rigid foundation placed on the terrace of a two-layered slope ground, vertical incidence of shear wave
4. CONCLUSIONS

In order to examine the effect of irregularity of a ground on the elastic wave field by taking a slope ground with a tiny canyon or an embedded foundation as a target, a series of three-dimensional finite element analyses have been conducted with a help of 2.5-dimensional analysis. It was found from the study that:

- It is possible to conduct a three-dimensional analysis of a ground with basically a two-dimensional topography.
- The wave field becomes very complex when there exists a small canyon or an embedded foundation, causing a significant difference between the results in 2.5 and three dimensions.
- The affected area is fairly large when compared to the size of the irregularity.

REFERENCES


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![Figure 11. Transfer functions at four locations surrounding a rigid foundation placed on the lowland of a two-layered slope ground, vertical incidence of shear wave](image)


