

# Optimal Topology Design for Replaceable of Reticulated Shell Based on Sensitivity Analysis



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## SUMMARY:

The control approach taken in this paper is to replace selected bars in reticulated shell with passive viscoelastic dampers. Sensitivity method is proposed to determine the optimal topology of dampers in the shell. Based on the eigenvalue perturbation and earthquake spectrum concept, the sensitivity of shell is calculated. Contrast the sensitivities of all elements; meanwhile, considering the symmetry topology of elements, the reasonable topology of damper is selected. The optimal shell is analyzed under earthquake actions. The results show that: the sensitivity method is effective for getting optimal topology. The displacement control effect of optimal topology dynamic responses reaches to 5%-30% and the axial force control effect reaches to 16%-45%.

*keywords: Topology optimization, Reticulated shell, Finite element, Sensitivity analysis, Replaceable elements*

## 1. GENERAL INSTRUCTIONS

The trend in development of long-span shells has been towards higher, longer and more elaborated structural configurations. As a result, new challenges have arisen to ensure the safety and performance for these shell structures when subjected to strong earthquakes and severe winds (Cao and Zhang, 2000). Various vibration control methods have been proposed for shell control, including rubber-bearing isolators (Shingn and Niki, 2001), tuned mass and tune liquid dampers (Shen and Lan, 2001), viscous dampers (Fan et al., 2005; Kasai et al., 2001), controllable fluids devices (Onoda et al., 1996; Xu et al., 2001; Oh and Onoda, 2002), and replaceable dampers (Ni, 2001; Yang et al., 2011). Four prototype viscous dampers are used for reticulated cylindrical shell for laboratory testing by Liang (Liang et al., 2003) proposed the effective topologies and parameters codes for replaceable damper of cylindrical shell. To evaluate the parameter effects of damper topologies and damping coefficient for reticulated spherical shell, various damper topologies and examples were simulated by Yang (Yang et al., 2011).

The location and amount of dampers are important parameters in research of replaceable bar-type damper in reticulated shell. The suitable location of dampers can obtain better effects while the amount of dampers is fixed or obtain better effects using fewer dampers. Agrawal and Yang (Agrawal and Yang, 1999) developed a new combine algorithm for optimal location research of passive dampers in space structures suffer earthquake action or wind load. This algorithm is useful for space structure such as reticulated shell, but the algorithm is difficult to be realized. Genetic algorithm is used by Singh and Moreschi (Singh and Moreschi, 2002) to determine the optimal location and size of dampers in structure. Some researchers have considered sensitivity method in optimal location study of dampers. Ni (Ni, 2001) uses this concept to determine the location and amount of replaceable dampers in double-layer reticulated cylindrical shell. The results indicate that it is the most effective location to replace the elements where sensitivities of shell natural frequency and deformation can be affected easily. Furthermore, the authors develop the optimal rule corresponding to that method.

Structural design sensitivity analysis concerns the relationship between design variables available to the design engineer and structural responses determined by the laws of mechanics. Chen (Chen, 1991) develops the sensitivity theories for structure vibration analysis. Kyung and Nam (Kyung and Nam, 2005) develop the sensitivity analysis theory for structural optimization and introduced analysis methods deeply. Habib (Habib et al., 2007) adapts the sensitivity method to solve the optimum shape design for shell structures. Amini and Ghaderi (Amini and Ghaderi, 2011) develop a new algorithm in optimal topology study for structure with MR dampers, the effective is demonstrated in the analysis.

In this study, sensitivity is used to determine the topology of control dampers in double-layer reticulated spherical shell. To calculate the sensitivity of structural natural frequency various, generated by small perturbation of every element section, eigenvalue perturbation and earthquake spectrum concept are used. Contrasting the elements sensitivities, then considering the symmetry topology, the reasonable location of damper is selected. Base on theory research and formula deduction, ANSYS Parametric Design Language (APDL) incorporated in the ANSYS finite element is used for the work. Moreover, parameter effects of sensitivity analysis are the required results for optimal design.

## 2. SENSITIVITY THEORY

### 2.1. Eigenvalue Design Sensitivity Analyses

Design sensitivity analysis is commonly used to represent a structural parameter that can affect the results of the analysis. When the cross-sectional area of a truss component changes, the dynamic results vary for the applied vibration load because the stiffness matrix changes. In such cases, the location and the related parameters of the truss component can be a design. The natural frequency of vibration load is eigenvalue of a generalized eigenvalue problem; hence, it depends on the design. In this paper, the expansion into power series is used to obtain derivatives of such eigenvalues in which repeated eigenvalues appear as an efficient solution to the optimal location design problem.

For a discrete system such as reticulated shell, the freedom is  $N$ ,  $\{q\}$  is the generalized coordinate matrix.  $[K]$  and  $[M]$  are the stiffness and mass matrixes according to matrix  $\{q\}$ .  $\omega$  is the natural frequency of structure, which is defined as function  $\lambda = \omega^2$ .

The oscillation equation of MDOF undamped structure is shown as:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\} \quad (2.1)$$

where  $\{\ddot{q}\}$  is the generalized acceleration vector of MOD structure.

The natural vibration of structure is harmonic oscillation, the function is given as:

$$\{q\} = \{u\} \cos(\omega t - \phi) \quad (2.2)$$

where  $\{u\}$  is the vibration mode matrix.

After substituting Eq. (2.2) into Eq. (2.1), the derivative of Eq. (2.1) with respect to the eigenvalue problem of discrete system is described as:

$$[K]\{u\} = \lambda[M]\{u\} \quad (2.3)$$

The variations of structure are given by stiffness and mass matrixes. The alternative stiffness and mass matrixes are shown.

$$[M] = [M_0] + \varepsilon[M_1] \quad (2.4a)$$

$$[K] = [K_0] + \varepsilon[K_1] \quad (2.4b)$$

where  $\varepsilon$  is a perturb design parameter. The system is original system according with  $\varepsilon = 0$ .  $[K_0]$  and  $[M_0]$  as and mass matrixes of original system.  $\varepsilon[K_1]$  and  $\varepsilon[M_1]$  are the variations for stiffness and mass matrixes of system. Furthermore, while  $\varepsilon[K_1]$  and  $\varepsilon[M_1]$  approach zero,  $[K]$  and  $[M]$  approach  $[K_0]$  and  $[M_0]$  separately.

In the derivatives, it is assumed that all eigenvalues are simple and not repeated. Under these conditions, the eigenvalues and vibration modes are subtle changed with a small number of  $\varepsilon[K_1]$  and  $\varepsilon[M_1]$ .

## 2.2 Dynamic Sensitivity Analyses

Here, for reticulated shell,  $i$  is the number of mode and  $j$  is the number of element.  $\{u_i\}$  and  $\lambda_i$  are the mode and eigenvalue of mode  $i$ . They meet the equation below:

$$([K] - \lambda_i[M])\{u_i\} = 0 \quad (2.5)$$

Commonly, stiffness and mass are seldom expressed as the display function of variables. In this study, for the reason of programming easier, the differential formulas of sensitivity are expressed as perturbation form.  $[\Delta K]$  and  $[\Delta M]$  are respectively the increment of stiffness matrix and mass matrix generated by the small perturbation in the design variable  $[\Delta b_j]$ .  $\Delta\lambda_i$  and  $\{\Delta u_i\}$  are the increment of eigenvalue and eigenvector respectively. The modal sensitivity  $\lambda_{i,j}$  and  $\{u_{i,j}\}$ , the derivatives of  $\lambda_i$  and  $\{u_i\}$  to variable  $b_j$  ( $j=1,2, \dots, L$ ), can be expressed as follows.

$$\lambda_{i,j} = \frac{\Delta\lambda_i}{\Delta b_j} \quad (2.6)$$

$$\{u_{i,j}\} = \frac{\{\Delta u_i\}}{\Delta b_j} \quad (2.7)$$

where  $\Delta\lambda_i$  and  $\{\Delta u_i\}$  can be calculated by first-order perturbation equation.

In finite element analysis problems,  $[\Delta K]$  and  $[\Delta M]$  are the total increment of element stiffness and mass matrix respectively, shown as follows.

$$[\Delta K] = \sum_e [\Delta K^e] \quad (2.8)$$

$$[\Delta M] = \sum_e [\Delta M^e] \quad (2.9)$$

Therefore, the modal sensitivity equations are transformed to finite element sensitivity equations. Thus,  $\lambda_{i,j}$  and  $\{u_{i,j}\}$  can be reduced as:

$$\lambda_{i,j} = \sum_e \lambda_{i,j}^e \quad (2.10)$$

$$\{u_{i,j}\} = \sum_e \{u_{i,j}^e\} \quad (2.11)$$

where  $\lambda_{i,j}^e$  and  $\{u_{i,j}^e\}$  are the sensitivity of  $\lambda_i$  and  $\{u_i\}$  of element  $e$ .

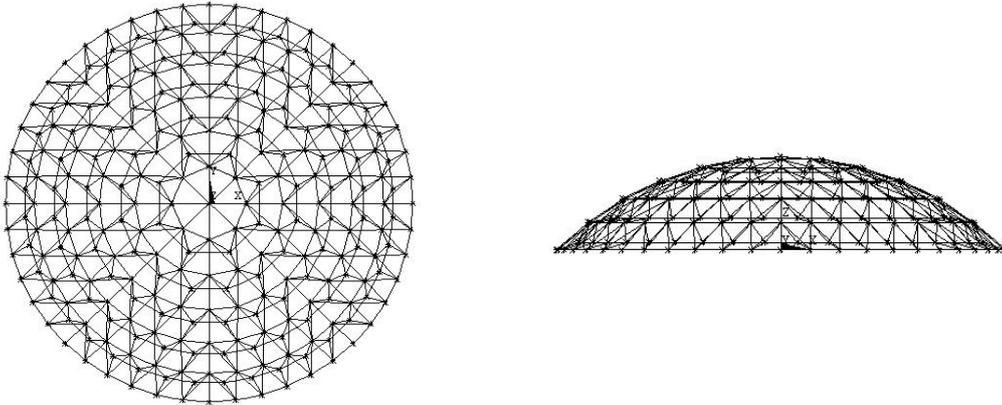
The sensitivity of structural natural frequency is set as the optimal objective function. The elements at the location where the variety of structural natural frequency is max, while the shell suffers a small perturbation, are replaced by the bar-types dampers. The structural dynamic sensitivity indicates the effects of structural design variable to structural dynamic characteristics. The effect is more remarkable at the location where the sensitivity is bigger. In this study, the section area of element is set as design variable. The elements at the location where the sensitivities of natural frequency are bigger than the others are replaced by the bar-types dampers, where section area is suffered from a small perturbation. The optimal objective function of dampers location can be written as

$$J(j) = \sum_{i=1}^n \alpha_i \lambda_{i,j} \quad (2.12)$$

where  $\alpha_i$  is the weight parameter, which can be set as the eigenvalue  $i$  corresponding to earthquake response spectrum.  $\lambda_{i,j}$  is the sensitivity of structural  $i$ -order natural frequency generated by the section area perturbation of element  $j$ .  $J(j)$  indicates the influence degree of element  $j$  to the vibration mode.

### 3. MODEL GENERATION

#### 3.1 Design Parameters of Shell



**Figure 1.** Plane and elevation view of double-layer reticulated shell

The Kiewitte-8 spherical reticulated spherical shell is commonly used in engineering. The model shown in Figure 1 is 40m span and the height is 8m. The thickness of shell between upper and lower grid is 0.8m. The number of nodes is 289. The number of elements is 936. In calculation, the distributed mass is 200kg/m<sup>2</sup> and modeled by MASS21 ELEMENT in ANSYS®. Nodes in lower layer have no external load. The bar element of shell is made by steel pipe and modeled by PIPE20 ELEMENT in ANSYS®. PIPE20 is Plastic Straight Pipe Element. The element has six degrees of freedom at each node: translations in the nodal  $x$ ,  $y$ , and  $z$  directions and rotations about the nodal  $x$ ,  $y$ , and  $z$  axes. Stress stiffening and large deflection capabilities are included. The bars in the upper and lower shell have an inner diameter of 123.5mm and an outer diameter of 127mm which stiffness is about  $4 \times 10^7$  N/m. The bars in-between upper and lower shells have an inner diameter of 118mm and an

outer diameter of 121mm, the stiffness is about  $3 \times 10^7$  N/m. The length of latitudinal bar on upper layer is 2.6-2.8m. The length of slanting bar on upper layer is 3.7-4.4m. The length of costal bar on upper layer is 3.7m. The length of bar on lower layer is 2.8-4.4m. The length of bar between upper and lower layer is 2.0-2.4m.

The shell is hinged at the support, which is modeled by NODE ELEMENT with 3 rotation degree of freedom in ANSYS®. The element joints of shell are rigid, which is also for node and damper. The elements in lower and middle layer of double layer reticulated shell have only axial pressure but no bending moment or the bending moment is very small. So we can replace these elements with dampers. Because of the complexity of shell with support column or other support structures, which could affect the control responses, the shell studied here is placed on the ground. The controlled shell with substructure will be studied in the follow-up work.

The damping coefficient of global shell fits the Rayleigh Damping theory and the damping ratio is 0.02. The steel is Q235 type which has a modulus of elasticity of  $2 \times 10^{11}$  N/m<sup>2</sup>. The yield strength  $\sigma$  of steel is  $2.35 \times 10^8$  N/m<sup>2</sup>. The buckling code of steel fits the Von Mises Isotropic Hardening Code.

### 3.2 Design Parameters of Replaceable Bar-Damper

The damping coefficient of VE damper is  $1 \times 10^5$  Ns/m and the stiffness is  $6 \times 10^4$  N/m, which is modeled by COMBIN14 ELEMENT in ANSYS®. The model sketch of replaceable bar-damper is shown in Figure 3. The degree of element COMBIN14 is according to the amount of dampers. Every shell bar is separated into three finite element parts and every damper is one FEM element. The large displacement geometric non-linearity analysis method is adapted in calculation.

The replaceable VE dampers are modeled as a linear Kelvin-Voigt element, i.e., linear stiffness and viscous damping

$$F = k \cdot u + c \dot{u} \quad (3.1)$$

where  $c$  is the damping coefficient,  $k$  the stiffness of damper,  $u$  the relative deformation of damper, and  $\dot{u}$  the relative velocity of damper. The material used in damper is piezoelectric and steel, which detail characteristic of damper is presented in National Science Foundation report (Yang Y. 2009). The preliminary step principal and test of damper is finished. This kind of damper can provide the VE characteristic for the adjustable feature of piezoelectric. The parameters of dampers is:  $k$  is equal to  $6 \times 10^4$  N/m, and  $c$  is equal to  $1 \times 10^5$  Ns/m. The elasto-plastic characteristic of steel used in shell is: the modulus of elasticity is  $2 \times 10^{11}$  N/m<sup>2</sup> and the yield strength  $\sigma$  of steel is  $2.35 \times 10^8$  N/m<sup>2</sup>. The buckling code of steel fits the Mises Isotropic Hardening Code.

The equation of motion of the original reticulated shell is given by the following second-order differential equation

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = P \quad (3.2)$$

where  $[M]$  is the mass matrix,  $[K]$  is the stiffness matrix,  $[C] = \alpha[M] + \beta[K]$  is the damping matrix,  $\{u\}$  the nodal displacement vector,  $P$  the input load vector and  $\alpha = \frac{2\xi\omega_1\omega_2}{\omega_1 + \omega_2}$ ,  $\beta = \frac{2\xi}{\omega_1 + \omega_2}$ .

The Rayleigh Damping is calculated by the first two frequencies of the shell. The damping ratio  $\xi$  is 0.01.  $\alpha$  and  $\beta$  are used in ANSYS® to model Rayleigh Damping.

For the reticulated shell with replaceable dampers, the equation of motion is given by

$$[M]\{\ddot{u}\} + ([\bar{C}] + [\Delta C])\{\dot{u}\} + ([\bar{K}] + [\Delta K])\{u\} = P \quad (3.3)$$

where  $[\bar{K}]$  is the stiffness matrix of shell not include dampers,  $[\Delta K]$  the stiffness matrix associated with the VE dampers,  $[\bar{C}]$  the damping matrix of shell not include dampers and  $[\bar{C}] = \alpha_1[M] + \beta_1[\bar{K}]$ ,  $[\Delta C]$  the damping matrix associated with the VE bar-type damper, which is considered in COMBIN14 element,  $\alpha_1$  and  $\beta_1$  are according to the first two frequencies of the controlled shell. The damping matrix of structure with VE damper is considered to satisfy the orthogonal modes. The equivalent damping ratio of vibration mode  $j$  of controlled shell is shown as

$$\xi_j = \frac{E_d^j}{4\pi E^j} \quad (3.4)$$

where  $E_d^j$  is the energy dissipation of vibration mode  $j$  of controlled shell, which is related to the material characteristic of VE damper and the vibration mode of shell.  $E^j$  is the maximum structural strain energy of vibration mode  $j$ , which is related to vibration mode of shell and the equivalent stiffness of shell. As a result of the decrease of damper stiffness compared to original shell bar, the damper bar has bigger deformation than shell bar. The energy dissipation of damper is increased with the increased deformation. That will reduce the dynamic responses of shell.

### 3.3 Earthquake Records

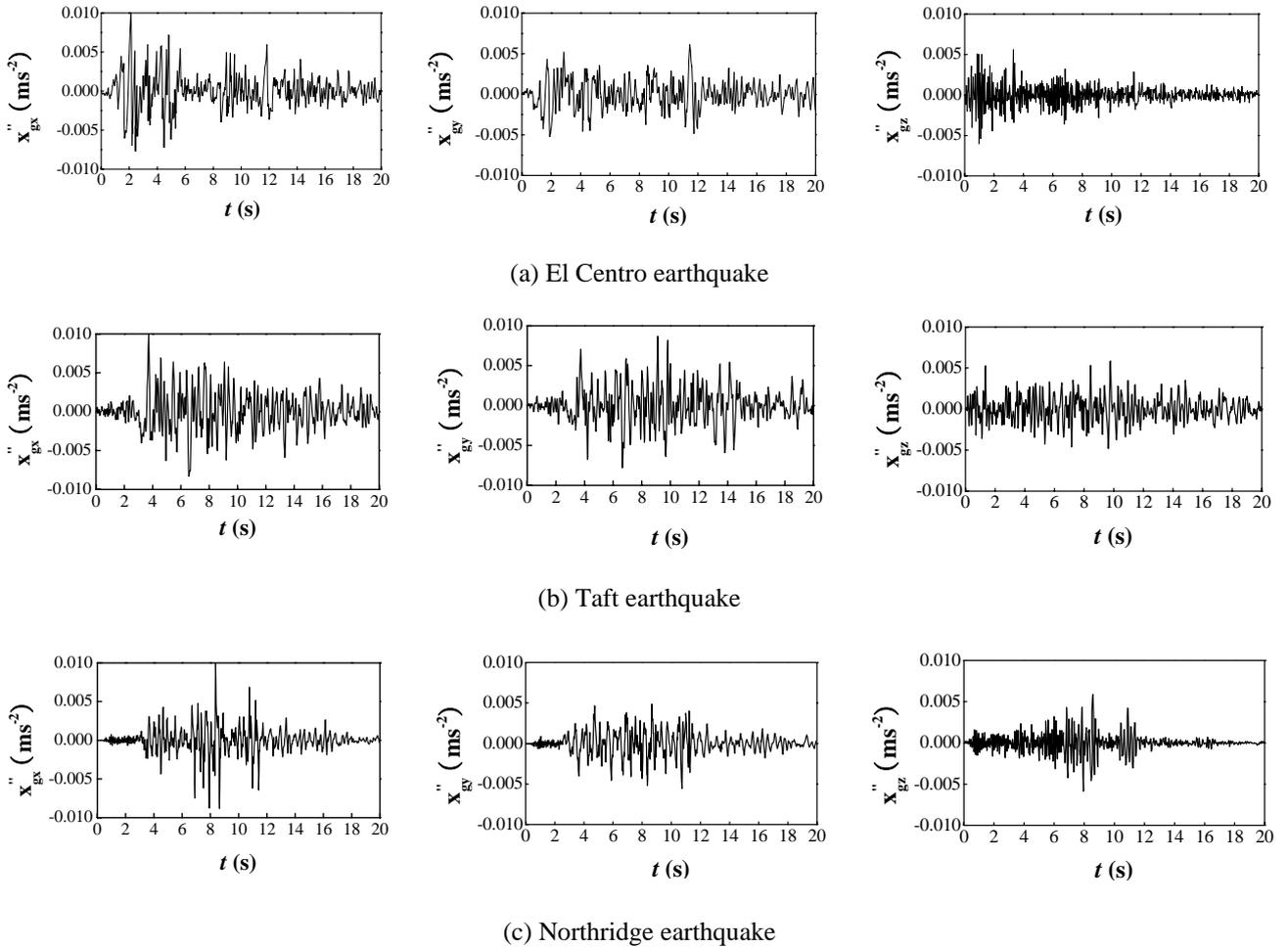


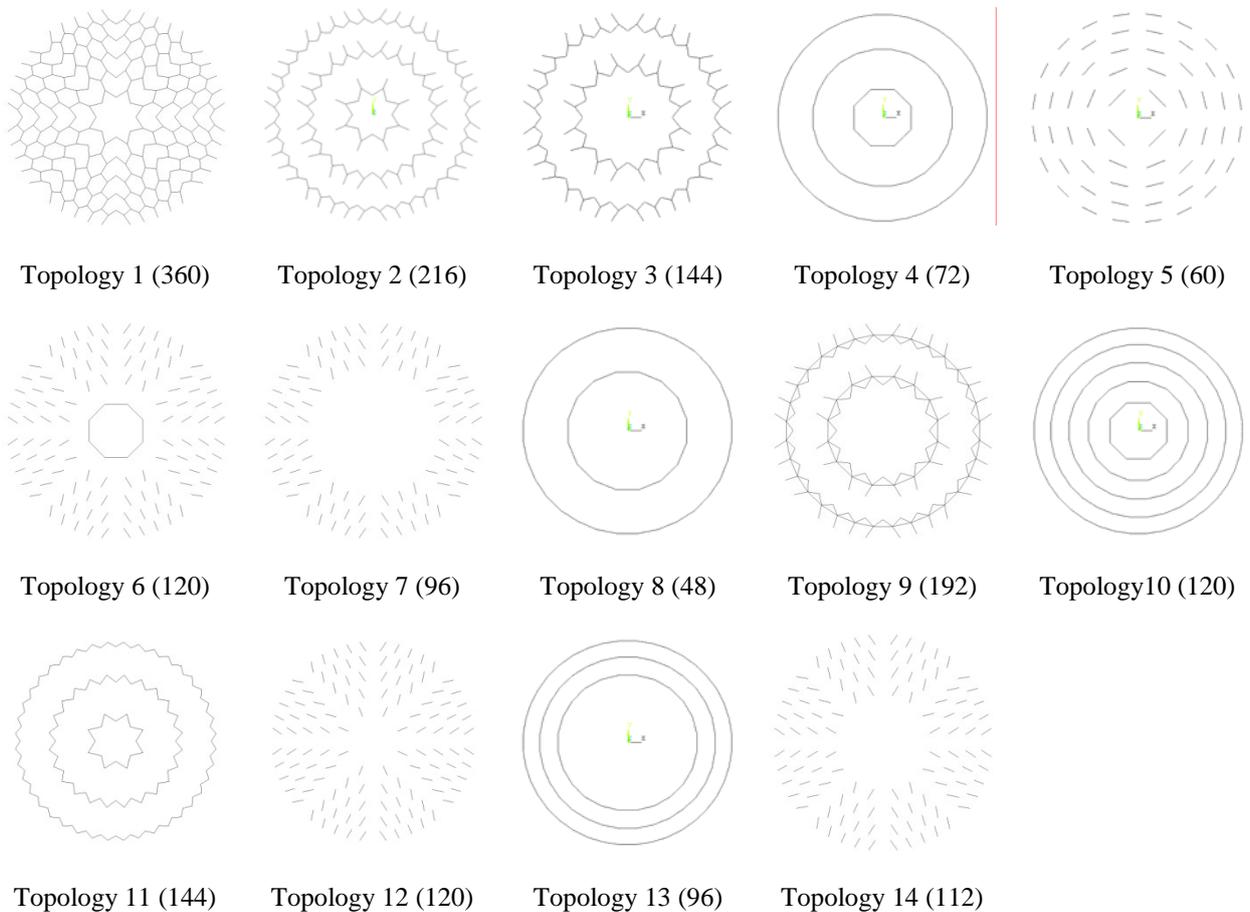
Figure 2. Ground acceleration record of earthquake actions

Three input earthquakes are considered: El Centro (N-S, 1940), Taft (N-S, 1952) and Northridge (N-S, 1995) as shown in Figure 2 where the peak values of earthquake acceleration are normalized to  $0.01\text{m/s}^2$ . The normalized value  $0.01\text{ m/s}^2$  will be timed relative value in analysis, for example, for frequent earthquake analysis, the acceleration time-history should be timed 140; for severe earthquake analysis, it should be timed 400.

## 4. OPTIMIZATION DESIGN

### 4.1 Controlled Shell Model without Optimization

To access the effect of different damper topologies, fourteen different configurations have been considered before, as Figure 3.

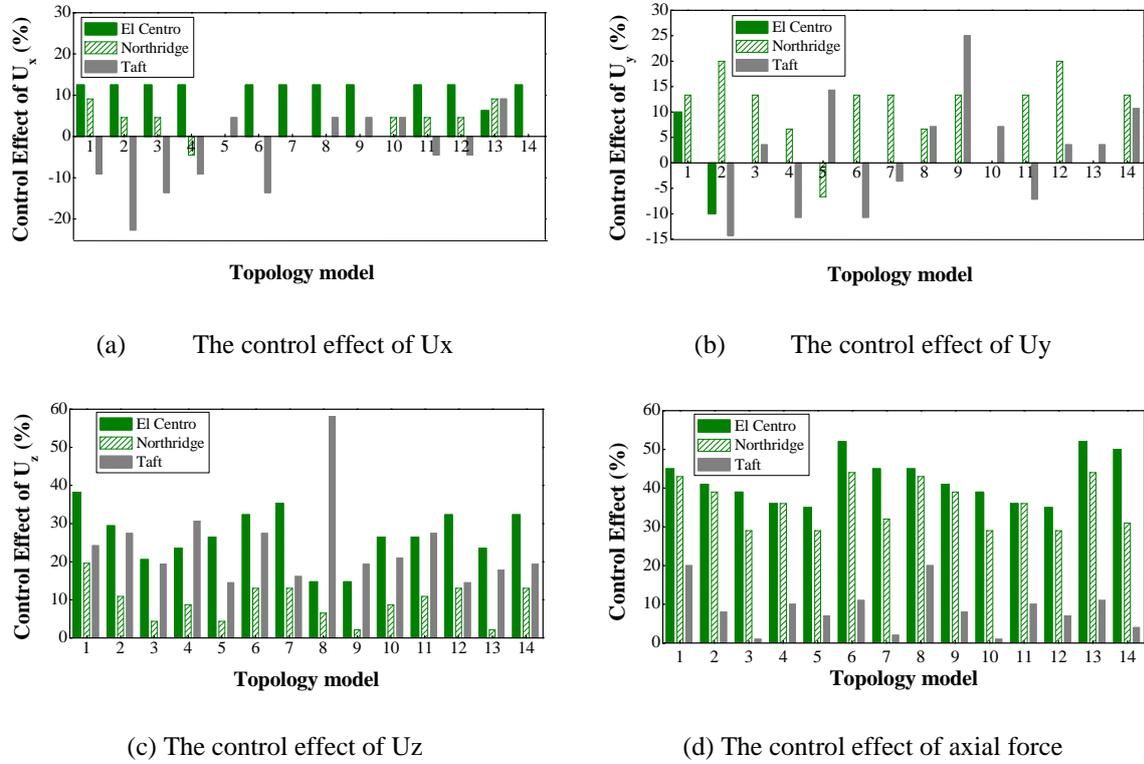


**Figure 3.** Topologies for bar-type VE dampers (The number in brackets is the amount of bar-type dampers)

- Topology 1: All the diagonal and radial elements of the lower and middle layer are replaced with bar-type VE dampers.
- Topology 2, 3: All the elements of the lower and middle layer are replaced with bar-type VE dampers.
- Topology 4, 7, 8, 10, 13: All the perimeter elements are replaced with bar-type VE dampers.
- Topology 5: The discontinuous perimeter elements are replaced with bar-type VE dampers.
- Topology 6: The radial elements of lower and middle layer and the topology 6 as well as some has additional loop elements are replaced with bar-type VE dampers.
- Topology 6, 8, 12, 14: The radial elements of lower and middle layers are replaced with bar-type

- VE dampers.
- Topology 11: The diagonal elements of middle layer are replaced with bar-type VE dampers.

#### 4.2 Effect of Controlled Shells without Optimization



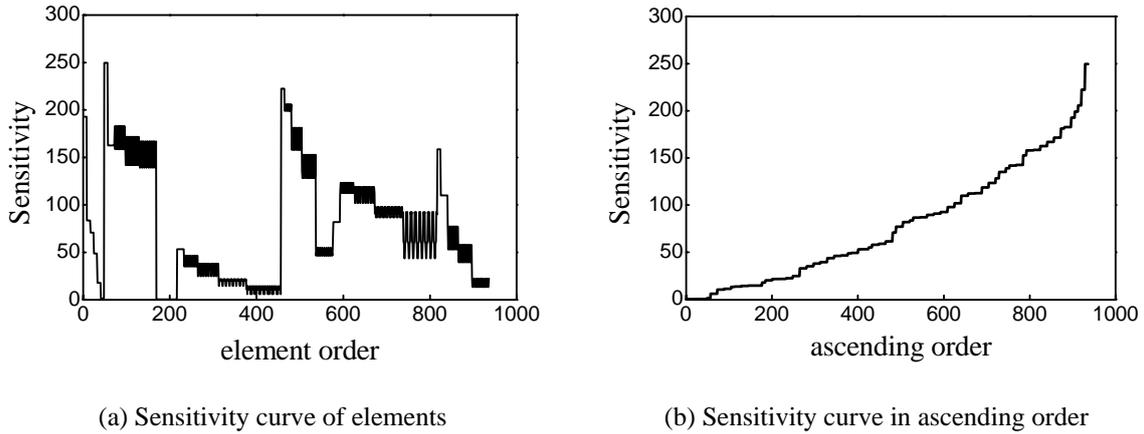
**Figure 4.** Control effect of the maximum displacement of 14 kinds of topologies

(Control effect = ((response of uncontrolled shell-controlled shell)/response of uncontrolled shell)×100%)

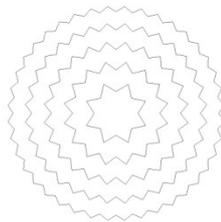
To assess the effects of the various topologies shown in Figure 3, the numerical model was subjected to the three-dimensional earthquakes shown in Figure 2, with each of the records normalized to have a maximum acceleration of  $4\text{m/s}^2$ . The values of the damper parameters are presented in section 3. The control effects of the maximum displacement ( $U_x$ ,  $U_y$  and  $U_z$ ) are presented in Figure 4. From Figure 4, observe that the results of nodal displacement responses are significantly difference in  $x$ -,  $y$ - and  $z$ - direction. Most responses reduction in  $z$ -direction reaches to 5%-50%. But the responses reduction is less than 10% or even less in  $x$ - and  $y$ - direction. Especially for the displacement in  $x$ - and  $y$ - direction subjected to Taft wave, the responses reduction is only 5% or less than that. The control effect of axial force is good except for Taft wave.

#### 4.3 Sensitivity Analysis

In the shell model, as mentioned last part, to achieve the damper location optimization, different program such as structural analysis, automatic mesh generation, sensitivity analysis and mathematical programming, are inter-related. Program modules are developed and communicated by using Matlab<sup>®</sup> language. The sensitivity curve and its ascending order arrangement curve of top 20 vibration modes are shown in Figure 5.



**Figure 5.** Sensitivity generated by section area perturbation of top 20 vibration modes



Topology 12 (240)

**Figure 6.** The optimal topology for bar-type VE dampers(degree of COMBIN14)

#### 4.4 Control Effect of Topology Optimization

Based on the sensitivity results, the optimal topology is selected as Figure 6. The topology is the diagonal elements of middle layer are replaced with bar-type VE dampers. The numerical model of topology optimization is analyzed subjected to the three-dimensional earthquakes. The results are shown in Table 4.1. From Table 4.1, observe that the results of nodal displacement responses are good in  $x$ -,  $y$ - and  $z$ - direction. Responses reduction in  $z$ -direction reaches to 17%-10%. The responses reduction of El Centro and Northridge wave is over 10% in  $x$ - and  $y$ -direction and the response reduction of Taft wave reaches 4%-7%. The effects are better than the model without optimization.

**Table 4.1** The control effect of optimal topology shell

earthquake		Ux(m)	%	Uy(m)	%	Uz(m)	%	F(10 <sup>5</sup> N)	%
El Centro	uncontrol	0.016	--	0.010	--	0.034	--	1.8219	--
	control	0.013	18.75	0.009	10	0.023	32.35	1.0548	42
Northridge	uncontrol	0.022	--	0.015	--	0.046	--	3.1300	--
	control	0.020	9.09	0.012	20	0.038	17.39	1.7218	45
Taft	uncontrol	0.022	--	0.028	--	0.062	--	3.1887	--
	control	0.021	4.55	0.026	7.14	0.045	27.42	2.6766	16

(Control effect = ((response of uncontrolled shell-controlle shell)/response of uncontrolled shell)×100%)

## 5. CONCLUSIONS

Replacing the elements of shell with bar-type dampers is an attractive control method that offers the reliable control but need not vary the grid form of shell. To take full advantage of the optimal topology, the sensitivity method is used for design and analysis. Kiewitt-8 type reticulated shell model has been used for control analysis. Meanwhile, 14 kinds of dampers topologies for this reticulated shell have been presented. Subsequently, the effect of dampers topologies is analyzed subjected to three commonly used 3D earthquake waves. The control effect of 8 kinds of dampers topologies is different and not good for every case. After that, the sensitivity analysis is used to get the optimal topology. The sensitivity orders are calculated based on top 20 vibration mode. The optimal placement of dampers is selected. The numerical model of optimal topology is analyzed. The control effect is better than the other topologies for every case. Responses reduction in  $z$ -direction reaches to 17%-10%. The

responses reduction of El Centro and Northridge wave is over 10% in  $x$ - and  $y$ - direction and the response reduction of Taft wave reaches 4%-7%. The effects are better than the model without optimization.

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