

# Maximum Likelihood based Parameter Estimation of Constitutive Models for Earthquake Engineering

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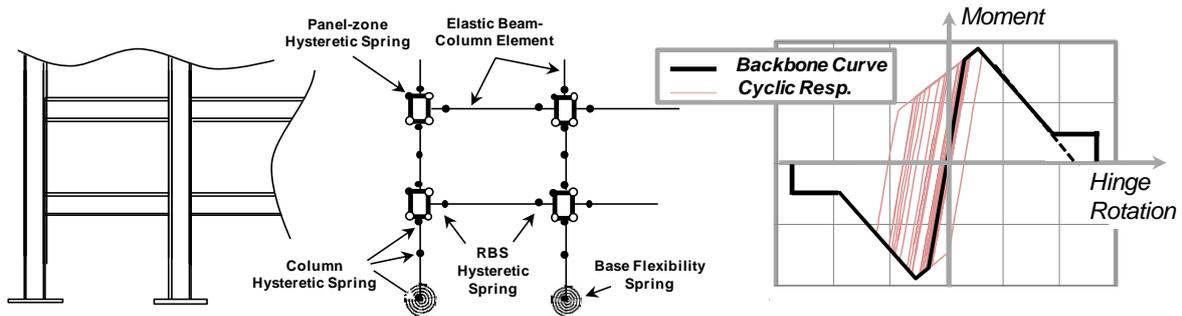
## **SUMMARY:**

Accurate simulation of earthquake-induced collapse in structures is contingent on the effectiveness of constitutive models used to characterize component response. These constitutive models simulate several aspects of response including accumulated damage leading to deterioration and failure, such as due to low-cycle fatigue. While the models themselves are sophisticated, the methodology to calibrate their parameters is subject to several improvements. A key issue is that current calibration techniques consider only the instant of (or deformation corresponding to) observed failure in the calibration test, disregarding the preceding loading excursions that provide important non-failure data. An investigation of this issue indicates that neglecting the non-failure excursions produces systematic bias in the fitted model parameters. To rectify this, a new methodology based on Maximum Likelihood Estimation is presented to calibrate the model parameters. The methodology provides a rational way to incorporate the effect of non-failure cycles on the calibrated parameters. The approach, similar to that used for characterizing life-expectancy in medical studies (in the presence of non-fatalities) also provides a probabilistically robust framework for the application of this methodology in predicting collapse. The Ibarra-Krawinkler constitutive model is used as an illustrative test-bed for the application of this methodology. Results are compared with those obtained from traditional calibration techniques, and limitations of the approach are discussed along with suggestions for future work.

*Keywords: Seismic, Collapse, Maximum Likelihood Estimation, Probabilistic, Uncertainty*

## **1. INTRODUCTION**

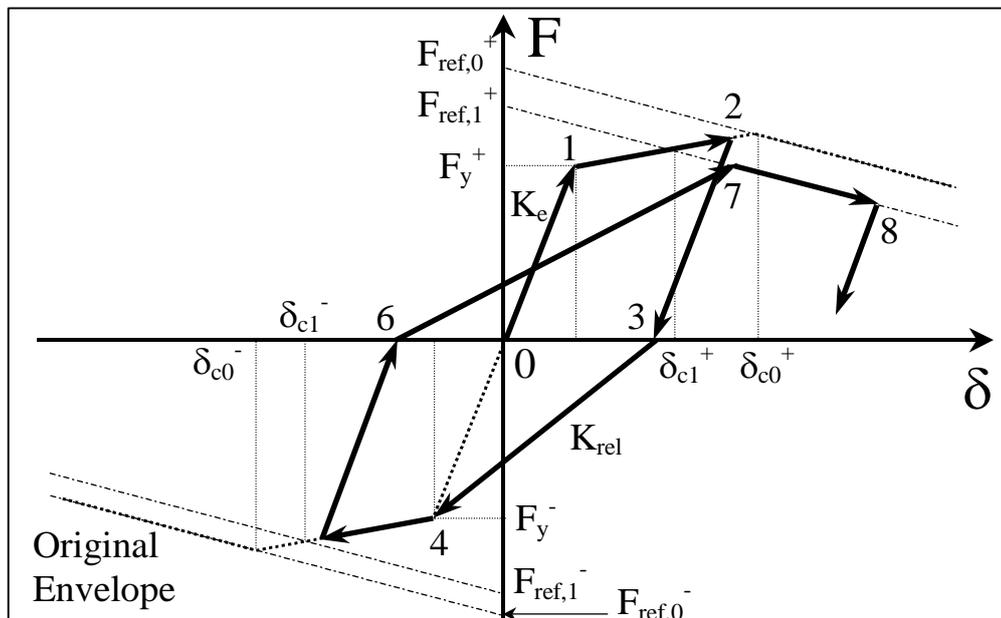
Recent years have seen the proliferation of deteriorating-hinge models for simulation of various components in structures subjected to earthquakes. Examples of such models include Ibarra et. al (2005), Rahnama and Krawinkler (1993), Sivaselvan and Reinhorn (2000), Song J., Pincheira J. (2000), and Clough and Johnston (1966). These models are commonly used to simulate the constitutive response of beam and column plastic hinges in moment frame buildings. In addition, they may be used to simulate the constitutive response of materials such as concrete or steel, as implemented within a fiber or continuum based modeling framework. Figure 1 schematically illustrates the response simulated by the Ibarra-Krawinkler (IK) model as applied to the cyclic response of a plastic hinge in a steel moment frame. While the models may be used to phenomenologically represent a range of physical phenomena (such as local buckling, fracture or collapse), they all share some basic characteristics. For instance, (1) cyclic deterioration of various properties such as strength and stiffness and (2) simulation of sudden failure, through a “post-capping stiffness”, also indicated in Figure 1.



**Figure 1.** Representation of Ibarra-Krawinkler model used to represent the constitutive response of various structural components in a moment frame building (Zareian and Krawinkler, 2010).

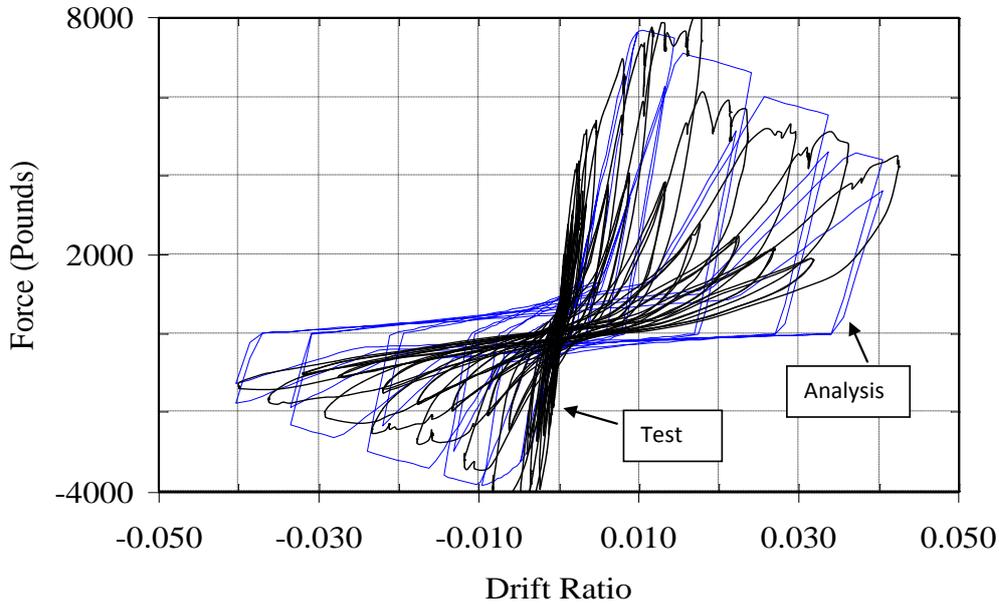
Lignos and Krawinkler (2011), as well as Kanvinde (2004) suggest that accurate simulation of these deterioration and failure modes is necessary for simulation of extreme limit states (such as earthquake induced collapse) that are controlled by interactions of geometric nonlinearities with these deterioration modes.

Because these models (and appropriate calibrated values for their parameters) are critical for high-fidelity structural simulation, significant research effort (Liel and Deierlein, 2008; Haselton and Deierlein, 2007) has been expended in developing databases wherein these parameters are calibrated and correlated with configurational properties of structural components (such as dimensional properties of concrete columns). Perhaps the most important aspect of these models is their ability to simulate the onset of catastrophic failure (such as due to fracture), which typically precipitates collapse of the structure. This catastrophic failure is simulated by means of the “post-capping stiffness” as shown in Figure 2. For example, collapse of SDOF systems is assumed to occur when the loading path is on the post-capping stiffness curve and the restoring force approaches zero. Also, as shown in Figure 2, the onset of the cap (indicated by  $F_{ref,i}$  – see Figure 2) for the  $i^{th}$  cycle may be hastened or delayed by the level of damage accumulation in the previous cycles.



**Figure 2.** Schematic illustration of cap degradation due to damage accumulation (Ibarra et. al, 2005)

Typically, the calibration of these types of hysteretic models to observed component response involves “eyeball fitting”, wherein the model response is compared visually to the experimental response to achieve a close fit, as indicated in Figure 3.



**Figure 3.** Fitting of IK Hysteretic model to experimental response of gypsum wallboard partition (Kanvinde and Deierlein, 2006).

The onset of the cap is typically calibrated based on observed points (i.e. instants during the loading history) of failure. While this is a convenient approach, it raises questions about the robustness of such a calibration process. For example, the non-failure cycles (i.e. cycles in which component response was successfully reversed without failure), provide valuable information about the probability space within which the actual failure point lies.

In other words, the current paradigm of calibration obtains only one data point (i.e. the failure data point). The proposed paradigm, which may be used for calibration as well as application obtains multiple data points (failure as well as non-failure points) through a maximum likelihood based probabilistic framework. This framework has been originally proposed by Myers et. al (2009) in the context of fracture simulation in steel. The next section describes this framework in the context of hysteretic models, while briefly outlining a methodology for its application.

## 2. FORMULATION OF MAXIMUM LIKELIHOOD BASED CALIBRATION METHOD FOR IK MODEL

As discussed in the previous section (and shown in Figure 2), the failure of the structural component is modeled through the deterioration of the “cap” towards the origin. Typically, the slope of the post-capping branch is kept constant (Ibarra et. al, 2005). The post-capping branch is moved inwards by an amount equivalent to reducing the reference strength according to the following equation,

$$F_{ref}^i = (1 - \beta_i)F_{ref}^{i-1} \quad (1)$$

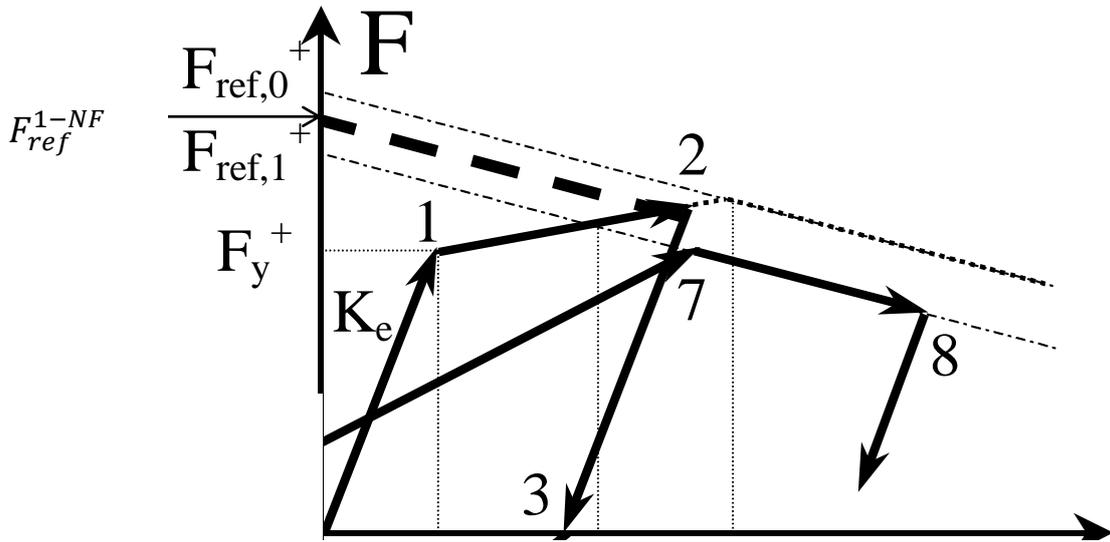
where  $F_{ref}^i$  is the intersection of the vertical axis with the projection of the post-capping branch (Figure 2). There is a positive and negative reference strength parameter for independently deteriorating the positive and negative post-capping strength.

The deterioration parameter  $\beta_i$  is expressed as -

$$\beta_i = \left( \frac{E_i}{E_t - \sum_{j=1}^i E_j} \right)^c \quad (2)$$

Where  $E_i$  is the hysteretic energy absorbed in cycle  $i$ , whereas  $E_t$  and  $c$  are parameters that control the rate and extent of deterioration. Finally, the location of the cap before loading begins (i.e. at the beginning of the first cycle), may be defined as  $F_0$ . Thus, a total of three parameters, i.e.  $F_0$ ,  $E_t$  and  $c$  are required to completely characterize the deterioration of the cap in the IK model

The intellectual basis for the new approach of calibration is as follows. For a given loading history, in which failure occurs on the  $n^{th}$  cycle, there are  $n-1$  failure cycles and one (the  $n^{th}$ ) failure cycle. Thus, given an assumed set of parameters (i.e. suitable probabilistic distributions for  $F_0$ ,  $E_t$  and  $c$ ), the probability of observing such a loading history is the probability of observing  $n-1$  non-failures, wherein the cap for each of these cycles was greater than the cap at which the cycle was reversed, and also observing failure at the instant when the cap was actually encountered. Figure 4, which shows only the positive quadrant of Figure 2, indicates a non- load reversal point corresponding to non-failure and the associated non-failure point of the cap  $F_{ref}^{1-NF}$  (see dashed line).



**Figure 4.** Non-failure point for  $F_{ref}^{1-NF}$

Thus, the probability of having observed the  $n-1$  non-failures, given the assumed distributions of these three input parameters may be determined. While all the details of the mathematical process are not outlined here, they are now briefly summarized and provided in detail in Myers (2009).

For the first excursion (non-failure), the probability of non-failure is –

$$P_1^{NF} = P(F_{ref}^1 > F_{ref}^{1-NF}) \quad (3)$$

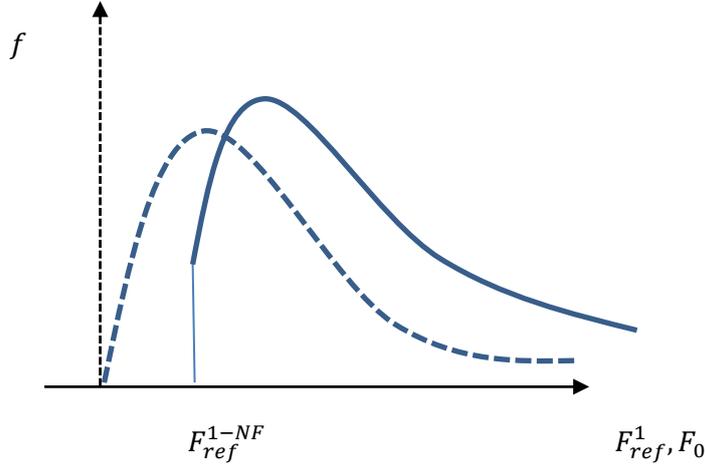
Given the distributions of  $F_0$ ,  $E_t$  and  $c$ , this may be conveniently calculated. Similarly, for the second excursion, the probability of non-failure may be calculated as –

$$P_2^{NF} = P(F_{ref}^2 > F_{ref}^{2-NF}) \quad (4)$$

However, this is more challenging than the first excursion (because the distribution of  $F_{ref}^{1-NF}$ , is in fact identical to the distribution of  $F_0$ , which is already assumed). For the second excursion,

$$F_{ref}^{2-NF} = (1 - \beta_2)F_{ref}^1 \quad (5)$$

In this case, the distribution is not directly known, since  $F_{ref}^1$  is random as well. However, given that a non-failure is observed for the first cycle (until the value of  $F_{ref}^{1-NF}$ ), the updated distribution of  $F_{ref}^1$  may be determined as a truncated and renormalized distribution of the initial distribution of  $F_{ref}^1$  (i.e.  $F_0$ ). This accounts for the redistribution of the probability to account for the fact that failure did not occur before  $F_{ref}^{1-NF}$ . Figure 5 illustrates this schematically –



**Figure 5.** Updated distribution cap location based on first non-failure

A similar process is repeated for each nonfailure cycle, wherein the probability distribution function of the cap at any cycle is conditioned on non-failure at the previous cycles (at their respective peaks). The likelihood of failure at a particular point during the  $n^{th}$  cycle may then be determined from the probability density function and the likelihood of observing the given loading history (i.e. sequence of non-failures followed by failure), is then the probability of non-failure occurring on the previous  $n-1$  cycles multiplied by the probability of failure occurring on the  $n^{th}$  cycle.

This likelihood is calculated for several assumed distributions (i.e. means and standard deviations of  $F_0$ ,  $E_t$  and  $c$ ), and the parameter set that achieves the maximum likelihood is selected.

Calibrating the model in this way has two key advantages –

1. It uses valuable data from all the nonfailure cycles
2. It forestalls illogical but possible situations that may be implied by the current framework, wherein some non-failure cycles actually exceed (in terms of cap magnitude) the final failure cap magnitude. In such a situation, the cap displacement calibrated from a particular test would not be able to predict failure even for that same test, when applied in a forward sense, since the cap displacement has exceeded the calibrated value of failure on a prior cycle (which is, in reality, a non-failure cycle) which is ignored in the current paradigm.
3. Once calibrated, the methodology may be conveniently applied in a probabilistic sense to failure prediction, wherein the probability of failure at any time in the loading history may be evaluated, rather than evaluating failure in a deterministic sense. Myers et. al (2009) outlines such a procedure in the context of ductile fracture in steel structures subject to highly inelastic cyclic loading.

### 3. SUMMARY AND CONCLUSIONS

This paper outlines a new formulation for the calibration and application of Ibarra-Krawinkler type hysteretic model that enables probabilistic predictions of structural failure. The paper first provides an explanation of the theory that underpins failure simulation (i.e. the cap) and then briefly outlines the processes for calibrating and applying the model based on the original deterministic formulation developed by Ibarra et. al (2005). The proposed framework involves first assuming a set of parameters that define the distributions of the parameters  $F_0$ ,  $E_t$  and  $c$  that control cap deterioration. Then, a process is carried out to determine the likelihood of observing the experimental response given these assumptions. The set of parameters that maximizes this likelihood is used as the calibrated set.

The probabilistic formulation proposed in this paper has already been applied in great detail (and supported by experimental data) for the Cyclic Void Growth Model – CVGM. However, the formulation could be applied to any model that predicts a response parameter that depends on cyclic loading and becomes increasingly likely with each loading cycle. Some examples of candidate models include predicting failure by high cycle fatigue using Miner’s rule (Miner, 1945) or degrading Clough type models (Clough, 1966). The calibration performed per this method is more probabilistically sound, more consistent with physics, and more robust overall.

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