Research For Collapse of R/C Frame Composed of Shear And Flexure Column

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SUMMARY:
In this paper, methodologies to evaluate collapse conditions of RC frames composed of shear and flexure columns are proposed. Experimental studies of RC frames including shear column were carried out. Based on the experiments, an analytical model considering P-δ effect to longitudinal bars in shear column is proposed. Moreover, analytical studies based on Shear Column Ratio are carried out to evaluate side-sway collapse.

Keywords: Shear Failure, Collapse, Capacity Spectrum Method

1. INTRODUCTION

Reinforced concrete structures might have shear critical members. Failure of shear critical columns may cause a total collapse of building because of rapid degradation of horizontal and axial capacity. In Japanese seismic code and design standards (AIJ.2004), deterioration in shear resistance in column has not been considered because of complexity and unclearness of such behaviour. Therefore, safety limit state of buildings (maximum deformation point of buildings to prevent collapse) is generally taken at the first shear failure of a structural member.

However, this safety limit state is conservative because buildings might not collapse directly after shear failure if the horizontal and vertical forces can be redistributed from failed members to surrounding members. Main goal of this research is to assess a more reasonable “safety limit state” for RC frames even if shear failure occurred.

In this paper, static cyclic loading experiments of reinforced concrete frames composed of brittle shear and ductile flexural columns were conducted to propose suitable analytical model. Degradations of axial and shear capacity are also considered in the proposed analytical model. After that, computational studies of several RC frame models composed of both shear and ductile members are conducted in order to assess conditions of structure collapse.

2. STATIC LOADING TESTS TO CONSTRUCT ANALYTICAL MODEL

Results of static loading tests of two RC frame specimens are described in this chapter. The purpose of these tests is to develop a suitable analytical model capable of estimating vertical collapse. Vertical collapse is defined as collapse due to excess of vertical load against vertical resistance capacity which is the summation of residual axial capacity of column and vertical capacity of connecting beams (three hinges in beams). Vertical collapse is distinguished from side-sway collapse which occurs when seismic shear force exceeds shear capacity of building. Side-sway collapse is assessed in next chapter.

2.1. Test Specimen and test method

Tests were conducted for two specimens; the drawings are shown in Figure 1 and 2. These specimens are single story and two bays frame structures. Specimen F-01(conducted with Fukuyama et al) was
designed as a one-half scale and BF-01 was designed as a 3/8 scale. Every member except for center column was designed so that flexure yielding precedes shear failure. Center columns were not provided with adequate transverse reinforcement and they were designed to demonstrate shear failure. Transverse reinforcement ratio, $p_w$, which is the area of transverse reinforcement normalized by the hoop spacing and column width are almost same between these specimens as shown in Figure 1 and 2. Center column of F-01 has higher longitudinal reinforcement ratio (shown in Figure 1 as $p_l$) section area of main bars normalized by cross section area of each column) than BF-01. In addition, for center column of F-01, lower initial axial load was applied by vertical hydraulic jack than BF-01. Material properties of the specimens are shown in Table 1.

![Figure 1. Drawing of F-01](image1)

![Figure 2. Drawing of BF-01](image2)

### Table 1. Material Properties

<table>
<thead>
<tr>
<th>F-01</th>
<th>Yield Strength of Steel Bars (Mpa)</th>
<th>BF-01</th>
<th>Yield Strength of Steel Bars (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D6</td>
<td>370</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D10</td>
<td>318</td>
<td></td>
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<tr>
<td></td>
<td>D16</td>
<td>378</td>
<td></td>
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<td></td>
<td>D19</td>
<td>379</td>
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<tr>
<td></td>
<td>D22</td>
<td>396</td>
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<td>Concrete Compressive Strength(Mpa)</td>
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<td>Compressive 26.9</td>
</tr>
<tr>
<td></td>
<td>Tensile</td>
<td>1.39</td>
<td>Tensile 2.31</td>
</tr>
</tbody>
</table>

Horizontal cyclic loads are applied by two hydraulic jacks fixed at the ends of beams. Load cells to measure axial stress and shear stress are installed at middle of side columns of F-01, bottom of center column of BF-01 and middle of beams of BF-01.
2.2. Test results

F-01 was subjected to three cycles of 1/800rad, 1/400rad, 1/200rad, 1/100rad, 1/50rad and 1/33rad of story drift angle which are calculated as average horizontal displacement of the side columns divided by story height which is the height from top of stub to center of top joint of the each column (referred as series1). As shown in Figure 3 (a) and (b), gradual deterioration of shear capacity was observed in series1. After series1, as shown in Figure 3 (c), vertical load of center column was increased from 250kN (initial vertical load) to 500kN and ±1/25rad of cycle was conducted (referred as series2). In series2, vertical displacement ratio of center column (which is the vertical displacement at joint of the center column divided by height from top of spandrel wall to bottom of beams; downward direction is defined as +) was rapidly increased as shown in Figure 3 (d), but axial load was sustained. In addition, shear capacity of center column reached approximately zero in series2. After that, vertical load of center column was increased until vertical collapse occurred (series3). At vertical collapse, vertical displacement ratio exceeded 5.0%.

Blue lines in Figure 3 named as “Pushover” will be explained later.

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**Figure 3.** Experimental Results of F-01

BF-01 was subjected to two cycles of 1/800rad, 1/400rad and 1/200rad of story drift angle (referred as series1). On the way to first +1/200rad, center column failed in shear and shear capacity rapidly decreased to approximately zero with story drift increasing as shown in Figure 4 (a) and (b). Because of rapid shear failure, there’re unmeasured data shown in Figure 4. After first -1/200rad, at +0.45% (+1/222rad) of story drift, vertical displacement ratio was rapidly increased with axial capacity decreasing (see Figure 4 (c) and (d)), the series1 was ended at this point. Then, unidirectional horizontal loads were applied with keeping the axial load as much as possible (referred as series2). In series2, axial force of center column was kept at about 200kN, but vertical displacement gradually increased.

Blue lines in Figure 5 named as “Pushover” will be explained later.
Pictures of vertical collapse of center columns are shown in Figure 5. Flaking of cover concrete and exposure of steel bars caused by shear failure and cyclic loading were observed in these pictures. In specimen F-01, longitudinal bars which were not confined well by transverse reinforcements seem to be deformed by flexure. On the other hand, it is difficult to judge if the longitudinal bars of BF-01 (which were also not confined well by transverse reinforcements) were deformed by either flexure or buckling.

2.3. Analytical model and comparison with test results

For reasonable collapse assessment, analytical model of deterioration of shear and axial capacity is needed. In past, some of models to assess the backbone characteristics of such brittle members considering shear-axial interaction were proposed such as Elwood and Moehle (2004) and Yoshimura (2008). In these models, effects of transverse reinforcement were considered for calculating axial capacity. In this paper, referring to the experimental studies and prior researches mentioned above, an analytical model of residual axial capacity is proposed.
Model of residual axial capacity after shear failure is shown in Figure 6 which is simplified by two assumptions. First assumption is that cover concrete was totally removed due to cyclic loading, the other assumption is that the influence of transverse reinforcements is small and therefore ignorable. In addition, horizontal force of column is assumed to reach zero by capacity deterioration when vertical load N exceeds vertical capacity of column. Additional compressive stress $\Delta \sigma_{PD}$ of longitudinal bars by P-$\delta$ effect is calculated by Eqn (1). Then total compressive stress of longitudinal bars is calculated as summation of $\Delta \sigma_{PD}$ and axial stress as shown in Eqn (2). From equilibrium of stress when steel bars are yielding and column reaches axial collapse, Eqn (3) and (4) are constructed by considering $N$ of Eqn (2) as residual axial capacity $N_R$.

$$\Delta \sigma_{PD} = \delta \cdot \frac{N}{2Z_p}$$  \hspace{1cm} (1)

$$\frac{N}{As} + \Delta \sigma_{PD} = \frac{N}{As} \left( \frac{1}{As} + \frac{\delta}{2Z_p} \right)$$  \hspace{1cm} (2)

$$N_R \left( \frac{1}{As} + \frac{\delta}{2Z_p} \right) = \sigma'_y$$  \hspace{1cm} (3)

$$N_R = \frac{As \cdot \sigma'_y}{1 + \frac{As \cdot \delta}{2Z_p}}$$  \hspace{1cm} (4)

Figure 6. Concept of Constructing of Model

Where; $Z_p$: plastic section modulus of longitudinal bars, $A_s$: total area of longitudinal bars, $\sigma'_y$: yield stress of longitudinal bars.

Difference of shear and axial backbone curve between proposed by Yoshimura (2008) and in this paper is shown in Figure 7. In the Yoshimura model, collapse lateral displacement $R_o$ is calculated by empirical formula, composed of parameter $p_w$, $p_f$ and $\eta_0$ (axial force ratio) of column, therefore it is a practical model but has some limitations. In fact, Center column of F-01 is out of the range of the Yoshimura model. On the other hand, model proposed in this paper may be applicable to such specimen using Eqn (5) and backbone characteristics of Yoshimura model (see Figure 7). $R_{o0}$ and $\delta_{o0}$ are defined as displacement (ratio) where residual axial capacity becomes lower than initial axial load shown in Figure 7 and Eqn (5) derived from Eqn (4). Using Eqn (5)

$$R_{o0} = \delta_{o0} / H = \frac{2Z_p}{As} \left( \frac{As \cdot \sigma'_y}{N} - 1 \right) \frac{1}{H}$$  \hspace{1cm} (5)

Where; $H$: story height.

Calculation results of $R_o$ and $R_{o0}$ are also shown in Figure 7. In addition, the model is assumed that column sustains initial axial capacity at a range from the point where shear failure occurs to 1% of drift ratio. Strictly speaking, the assumption isn’t totally correct but several experimental studies (Yoshimura, 2008) showed that possibility that vertical collapse will occur is small before 1% of displacement ratio.

Pushover analyses are carried out to check suitability of the model mentioned above. Structural model is shown in Figure 8. First, members were distinguished as flexure members or shear members. Flexure members have an elastic shear spring, an elastic axial spring and inelastic flexure springs. Tri-linear model is applied to inelastic flexure springs based on AJI standard (2009). Shear members have elastic flexure springs, an inelastic shear spring and an inelastic axial spring. Penta-linear model similar to Figure 7 is applied to inelastic shear spring assuming 10kN as minimum shear capacity. Calculation methodology of stress and displacement of inelastic axial spring is a bit complicated. As
shown in Figure 9, Residual axial capacity shown in Eqn (4) and axial force carried by column are compared step by step. When former becomes lower than later, axial spring becomes inelastic with minute positive stiffness then unbalanced load NP between them will be cancelled.

![Figure 7. Difference with Equation by Yoshimura](image)

![Figure 8. Analytical Model of Specimen](image)

![Figure 9. Methodology to Calculate in Region of Inelastic Axial Spring](image)

Pushover analyses were carried out under constant vertical load. Cases of studies were as follows:

**Case1:** For F01, vertical load for center column is 250kN as used in the experiment of series1 (to compare with series1).

**Case2:** For F01, vertical load for center column is 646kN which is the maximum measured vertical load in experiment of series3 (to compare with series2 and 3).

**Case3:** For BF01, vertical load for center column is 700kN as used in the experiment.

Analytical results of case1 are shown in Figure 3 (a) and (b), and case2 are also shown in Figure 3 (c) and (d). As shown in Figure 3 (a) and (b), maximum story shear force of pushover is about 150kN lower than experimental result even though maximum shear force of center column is approximately the same as experiment. It is caused by difference of stiffness; as shown in Figure 3 (b), displacement at maximum shear force of center column of pushover is smaller than experiment and shear forces of side columns of pushover are still developing at the displacement when shear failure occurred at center column. On the other hands, stiffness at capacity degradation shows good agreement between pushover and experiment.

In Figure 3 (c), residual axial capacity of F-01 which is calculated based on the model mentioned above is also shown. In series2 of the experiment, axial force of center column exceeded calculated residual axial capacity, and vertical displacement of center column rapidly increased shown as Figure 3 (d). In the series3 of experiment, max axial force of 486kN was observed at center column and this quite matches the calculated residual axial capacity at same displacement. In addition, path of vertical displacement of center column calculated in case2 shows good agreement with series2 and 3 of experiment (see Figure 3 (d)). Result of case3 is also shown in Figure 4. Maximum story shear force of pushover is lower than experiment because of the same reason mentioned before. Although verification of shear capacity degradation stiffness is difficult because of the unmeasured data due to
rapid degradation, $R_{d0}$ which is shown in Figure 7 shows good agreement with the point at which shear force reached approximate zero in experiment. In addition, path of axial force of pushover and vertical displacement show good agreements with experiment.

These results show that the model mentioned above is capable to estimate residual axial capacity and shear capacity degradation sloop based on $R_{d0}$. However, it should be noted that estimation of stiffness until maximum shear capacity needs more development.

3. ANALYTICAL STUDIES OF COLLAPSE FOR RC FRAMES COMPOSED OF VARIABLE SHEAR COLUMN RATIO

3.1. Estimation Methodology of Side-sway Collapse

In past researches of collapse assessment, such as Haselton et al (2011), vertical collapse was distinguished from side-sway collapse that occurs when lateral seismic load exceeds building capacity. To evaluate the displacement of side-sway collapse, usually, nonlinear dynamic analyses were conducted. In this paper, a methodology to evaluate the displacement of side-sway collapse which is based on Capacity Spectrum Method (CSM) is proposed. Simplified concept of the methodology is shown in Figure 10. Ultimate displacement for side-sway collapse in this paper is defined as point of maximum Seismic Capacity Index (SCI) calculated by Eqn (6) for every analytical step. For structures with capacity degradation, maximum SCI point corresponds to maximum displacement which structures can experience. Response spectrum used in this paper is shown in Figure 11.

$$SCI = \frac{I_{\text{standard}}}{I_{\text{Ultimate}}} \quad (6)$$

Where; $I_{\text{standard}}$ is intensity of standard Earthquake. $I_{\text{ultimate}}$ is ultimate intensity of spectrum of Earthquake calculated at step by step.

3.2. Analytical Model

Models of structures similar to Figure 8, shown in Figure 12 are used. The models have first story collapse mechanism. They’re actually three storied structures, but for simplifying, they’re calculated as one storied structures assuming upper story as rigid. Cross sections of beams are 500mm x 700mm and columns are 600mm x 600mm. Shear Column components Ratio (SCR) defined as number of shear columns divided by number of columns. There’re four models with different SCR also shown in Figure 12. In addition, two types of shear columns are used; one has $R_{U0}$ of 1.9% of story drift angle (referred as series”B”). The other has $R_{U0}$ of 5.3% of story drift angle (referred as series”D”). Former
type is called as “B20%”, “B40%” and so on. Later type is called as “D20%”, “D60%” and so on. Percentages shown in the end of the name of types refer to the SCR ratio.

Then cases considering structure modelling uncertainties are carried out for cases from 20% to 60% of SCR. These cases are calculated using 5% non-exceeding probability of shear strength and variation coefficient of 72.6% for $R_{U0}$ (value of Yoshimura model is used because value of the model proposed in this paper is still unknown). These cases are called as “BB20%”, “DB40%” and so on.

Characteristics of backbone curve of columns are shown in Figure 13. In addition, pushover analyses are stopped when story drift reaches $R_{U0}$.

To calculate SCI, $h_{eq,m}$ effective damping factor of each member is calculated by summation of viscous damping factor $h_V$ and hysteretic damping factor. Viscous damping factor $h_V$ is calculated by Eqn (7) for each spring. For flexure members, hysteretic damping factor $h_f$ is calculated by Eqn (8) based on plastic ratio of $\mu$ of each inelastic flexure spring. For shear members, based on a model proposed by Takeda et al as shown in Figure 14, $h_s$ is calculated as ratio of hysteretic absorbed energy $\Delta W$ shown in Eqn (9) and potential energy $W$ of each inelastic shear spring. Total damping factor of structure $h_{eq}$ is calculated by weighted average of $h_{eq,m}$ of all springs.

$$h_V = \sqrt{\frac{K_{eq}}{K_0}} \cdot h_0$$

$$h_f = 0.25 \left(1 - \frac{1}{\sqrt{\mu}}\right)$$

![Figure 12. Analytical Model for Side-sway Collapse](image)

<table>
<thead>
<tr>
<th>SCR</th>
<th>C-A</th>
<th>C-B</th>
<th>C-C</th>
<th>C-D</th>
<th>C-E</th>
</tr>
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<tr>
<td>20%</td>
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</tr>
<tr>
<td>40%</td>
<td>Flexure</td>
<td>Flexure</td>
<td>Shear</td>
<td>Shear</td>
<td>Flexure</td>
</tr>
<tr>
<td>60%</td>
<td>Flexure</td>
<td>Shear</td>
<td>Shear</td>
<td>Shear</td>
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<tr>
<td>80%</td>
<td>Flexure</td>
<td>Shear</td>
<td>Shear</td>
<td>Shear</td>
<td>Shear</td>
</tr>
</tbody>
</table>

![Figure 13. Backbone Curve of Columns](image)

![Figure 14. Concept of Energy for Inelastic Shear Spring](image)
\[ \Delta W = \frac{K_{slip} \cdot d_{\Delta}^2}{(1 - K_{slip} \frac{d}{q})} + d_0 \cdot q \quad (d \geq d_y) \]  \hspace{1cm} (9)

Where:
\[ K_{slip} = \frac{q}{d - d_y} \left( \frac{d}{d_y} \right)^{\gamma_1} \]  \hspace{1cm} (10)
\[ d_0 = d - \frac{q}{d_y + d} \left( \frac{q_y + q_y d}{d_y + d} \right)^{\gamma_2} \]  \hspace{1cm} (11)

\( \gamma_1 \) and \( \gamma_2 \) are reduction factors of stiffness, 0.5 and 1 are assumed for each spring.

### 3.3. Analytical Results

Analytical results from B20% to B80% and from D20% to D80% are shown in Figure 15. In cases of D80% and B80%, side-sway collapse occurred just after shear failure of columns. In cases of lower SCR, side-sway collapse just after shear failure was avoided. These results indicate that SCR is important factors to assess side-sway collapse just after shear failure.

![Analytical Results of Side-sway Collapse](image)

**Figure 15.** Analytical Results of Side-sway Collapse

Methodology used in this chapter, based on CSM, may be affected by the period of buildings. Therefore, more studies were conducted to confirm the influence of period to side-sway collapse. Weights of the structures are changed assuming number of stories from 1 to 6 in the cases of BB20%, 40%, 60%, DB20%, 40% and 60% to change period of these structures. The results of studies are shown in Figure 16. Collapse margins of structures are the displacement at side-sway collapse divided by displacement that shear failure occurred. Figure 16 shows that cases of SCR 60% are affected by the period of structure and risk of side-sway collapse will become higher. On the other hand, cases of SCR20% and 40%, side-sway collapse just after shear failure were avoided. Although more studies are
needed, structures of lower than SCR 40% may be allowed to have shear failure.

4. CONCLUSION
Contributions of this paper are shown in as follows;
Experimental studies of RC frames including shear column were carried out to construct an analytical model of residual axial capacity considering P-δ effect to longitudinal bars. Then the proposed model was compared with experimental results. The proposed model was in good agreement with the results of experiment.
To evaluate side-sway collapse, analytical studies with different shear column components ratio (SCR) and $R_{0u}$ were carried out. Main conclusions are summarized as follows;
1) SCR is important factor to evaluate side-sway collapse just after shear failure.
2) Side-sway collapse just after shear failure has greater possibility to occur in the buildings with SCR 60% or larger. On the contrary, building with SCR of 40% or less has greater possibility to avoid sides-way collapse just after shear failure.

Figure 16. Analytical Results of Model Variation of Natural Period

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