Nonlinear Dynamic SSI Analysis with Coupled FEM-SBFEM Approach

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ABSTRACT:
This paper presents a coupled FEM-SBFEM approach for the dynamic SSI analysis considering the nonlinearity in the near-field. In order to model the soil nonlinearity, basic HiSS-δ_0 model is used. The implementation of HiSS model in the developed MATLAB program is verified by solving the problem from literature. A problem of an elastic half-space under dynamic load was analyzed to show the importance of radiation damping in SSI analysis. An incremental calculation scheme for dynamic analysis is presented wherein the nonlinear HHT-α method with full Newton-Raphson iteration is employed for numerical integration.

Keywords: Soil-structure interaction, Coupled FEM-SBFEM approach, HiSS model, Nonlinear HHT-α Method

1. INTRODUCTION

The direct method and the sub-structure method are the two major methods available for the dynamic soil-structure interaction (SSI) analysis (Wolf 1985). In the sub-structure method of dynamic SSI analysis, the model consists of a bounded irregular soil domain (near-field) and an unbounded regular soil domain (far-field) separated by an interface called the interaction horizon (Fig. 1.1). The near-field is discretaized using finite element method (FEM) and can incorporate all irregularities including geometrical and material nonlinearities. The dynamic property of far-field, which is always assumed to be linear, can be represented by the force-displacement relationship formulated on the interaction horizon which satisfies the radiation condition at infinity.

Global procedures such as boundary element method (BEM) (Hall and Oliveto 2003), thin layer method (Kausel 1994), consistent infinitesimal finite-element cell method (CIFECM) (Wolf and Song 1996, Emani and Maheshwari 2009) and scaled boundary finite element method (SBFEM) (Wolf 2003) are employed for modeling the unbounded soil domain. These global procedures are rigorous in the sense that the response at a specific location and time depends on the response at all other locations (spatially global) and at all previous times from the start of the excitation onward (temporally global). In the present work, the SBFEM is used to model the unbounded soil domain.

Wolf (2003) has developed the SBFEM for the dynamic analysis of unbounded domains, taking the advantages of both the FEM and the BEM approaches and evading their respective drawbacks. Combined models based on coupling of FEM-SBFEM (Wolf and Song 2000, Bazyar and Song 2006 etc.) have also been proposed for dynamic SSI analysis.

Most of the coupled FEM-SBFEM models for the dynamic SSI analysis include only linear behavior of the near-field. However, Doherty and Deeks (2005) applied adaptive coupling of the FEM-SBFEM interface considering nonlinearity in the near-field in the form of an ideal elastic-plastic Tresca material. Bransch and Lehmann (2011) employed the elastic-plastic cap model given by DiMaggio and Sandler (1971) in the coupled FEM-SBFEM approach to model the nonlinearity in the near-field.

Although the cap model has been used in the characterization of materials that exhibit continuous yielding, they suffer certain limitations in handling number of important attributes of the behavior of materials such as non-associative response of many frictional materials (Desai 2001). The hierarchical single-surface (HiSS) plasticity models provide a general formulation for the elastoplastic characterization of the material behavior. They provide hierarchical adoption of models of increasing sophistication, say, linear elastic to nonassociated elastoplastic to elastoplastic with softening. In the present work, the basic and simplest version of the HiSS models, the HiSS-δ_0, which allows for isotropic hardening and associated response, has been used in the dynamic SSI analysis carried out using FEM-SBFEM approach.

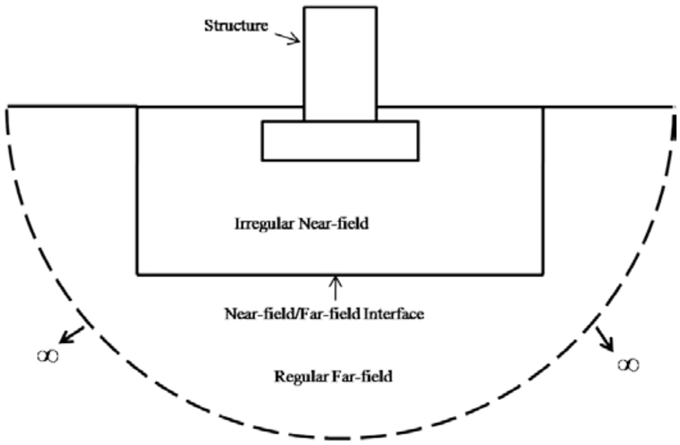

Figure 1.1. Model for soil-structure interaction

2. FEM-SBFEM COUPLING IN TIME DOMAIN

A detailed description of SBFEM is given in Wolf (2003). In the present section, the coupling procedure of the two numerical methods viz. FEM and SBFEM is discussed.

The equation of motion of the structure in total displacements in time domain for the soil-structure system is formulated as (Wolf 1985)

$$\begin{bmatrix} M_{ss} & M_{sb} \\ M_{bs} & M_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{u}_s^t \\ \ddot{u}_b^t \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sb} \\ C_{bs} & C_{bb} \end{bmatrix} \begin{Bmatrix} \dot{u}_s^t \\ \dot{u}_b^t \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb} \end{bmatrix} \begin{Bmatrix} u_s^t \\ u_b^t \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_b(t) \end{Bmatrix} = \begin{Bmatrix} P_s(t) \\ P_b(t) \end{Bmatrix} \tag{2.1}$$

where [M], [C] and [K] are mass, material damping and stiffness matrices, respectively; u^t, $\dot{u}^t$ and $\ddot{u}^t$ are displacement, velocity and acceleration vectors, respectively. The subscripts s and b denote the nodes of the bounded domain and nodes associated with the interface, respectively and the superscript t denotes the total motion of the structure. The ground interaction force vector, R_b^t, is obtained solving the convolution integral (Lehmann 2005)

$$R_b(t) = \int_0^t M_{bb}^{\infty}(t-\tau)(\ddot{u}_b^t(\tau))d\tau \tag{2.2}$$

where M_{bb}^{∞} is the acceleration unit-impulse response matrix in the time domain. Its derivation is discussed in Wolf and Song (1996).

The convolution integral in Eqn. 2.2 can be written in the discrete form as:

$$R(t) = \sum_{j=1}^{n} M_{n-j}^{\infty} \int_{(j-1)\Delta t}^{j\Delta t} \ddot{u}(\sigma)d\sigma = \sum_{j=1}^{n} M_{n-j}^{\infty} (\dot{u}_j - \dot{u}_{j-1}) \tag{2.3}$$

When the α-parameter of the HHT- α method (Hilber et al. 1977) is introduced and the unknown acceleration vector $\ddot{u}_n$ for the time step n is separated, the interaction force $R(t)$ is calculated with:

$$R(t) = \alpha\Delta t M_0^{\infty} \ddot{u}_n + \sum_{j=1}^{n-1} M_{n-j}^{\infty} (\dot{u}_j - \dot{u}_{j-1}) = \alpha\Delta t M_0^{\infty} \ddot{u}_n + \hat{R} \tag{2.4}$$

$\hat{R}$ relates to the loads on the near-field/far-field interface which occur at the infinite domain. Substituting Eqn. 2.4 into Eq. 2.1, the coupling of FEM and SBFEM is done as

$$\begin{bmatrix} M_{ss} & M_{sb} \\ M_{bs} & M_{bb} + \alpha\Delta t M_0^{\infty} \end{bmatrix} \begin{Bmatrix} \ddot{u}_s^t \\ \ddot{u}_b^t \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sb} \\ C_{bs} & C_{bb} \end{bmatrix} \begin{Bmatrix} \dot{u}_s^t \\ \dot{u}_b^t \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb} \end{bmatrix} \begin{Bmatrix} u_s^t \\ u_b^t \end{Bmatrix} = \begin{Bmatrix} P_s(t) \\ P_b(t) - \hat{R} \end{Bmatrix} \tag{2.5}$$

Eqn. 2.5 represents the coupled FEM-SBFEM calculation. For a long simulation time, the calculation of Eqn. 2.5 needs a large computational effort. To reduce this computational effort, a recursive algorithm (Lehmann 2005**)** has been implemented in the present work.

3. IMPLEMENTATION OF FEM-SBFEM COUPLING FOR ELASTIC HALF-SPACE

The verification of the coupled FEM-SBFEM approach for the dynamic SSI analysis for plane strain problems and three dimensional problems has been presented in Syed and Maheshwari (2011) and Maheshwari and Syed (2011), respectively. Doherty and Deeks (2003) presented the formulation of SBFEM for axisymmetric problems. Here, the FEM-SBFEM coupling is employed to study the axisymmetric problem of elastic half-space under dynamic load. Fig. 3.1 shows the investigated geometry, its discretization and the applied load. The number of four-node plane elements used in FE discretization is 40 whereas the total degrees of freedom in the system is 108. The total number of SBFE is 18 with a total number of 38 degrees of freedom. The unit weight considered is 1800 kg/m^3, Poisson's ratio value is 0.35 and shear-wave velocity is 200 m/s. The results from the dynamic analysis with FEM-SBFEM approach is compared with those with the FEM analysis wherein the boundary nodes are fixed in all the directions. No material damping is considered in order to highlight the effect of radiation damping present in the FEM-SBFEM analysis.

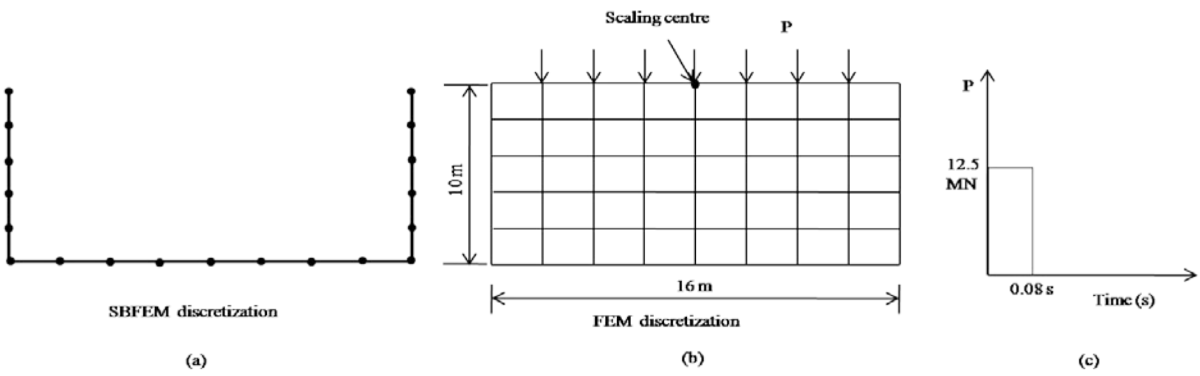

Figure 3.1. Elastic half-space: (a) SBFEM discretization (b) FEM discretization and (c) Step-load

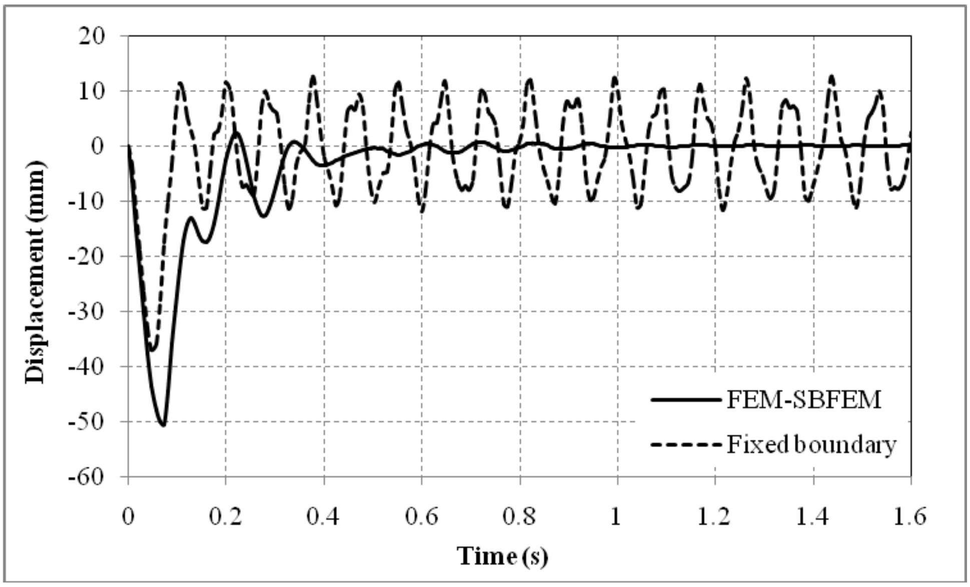

Figure 3.2. Vertical response at one of the loaded nodes

In Fig. 3.2, the time histories of the vertical response at one of the loaded nodes are plotted. As evident from the figure, in case of FEM-SBFEM approach, the displacement almost vanishes after 0.8 s, clearly highlighting the effects of radiation damping. On the other hand, in the FEM analysis with fixed boundary, the vibrations continue without any diminution since the system considered is an undamped system and there is no radiation damping present.

4. NONLINEARITY OF SOIL: HiSS-δ_0 MODEL

The HiSS model presented by Desai (2001), is used in the present study. The yield function, *F* (Fig. 4.1), is given by

$$F = \bar{J}_{2D} - (-\alpha\bar{J}_1^n + \gamma\bar{J}_1^2)(1-\beta S_r)^m = 0 \tag{4.1}$$

Here, $\bar{J}_{2D}$ and $\bar{J}_1$ are respectively the second and first invariants of the stress deviators, nondimensionalised with respect to atmospheric pressure p_a. The quantities α (different from the parameter α of HHT- α method), γ, β, and *n* are material constants that can be determined form the laboratory tests. The parameters γ and β are associated with the ultimate yield envelope, *n* is associated with the phase change from contractive to dilative or zero volume change, and α is the hardening or growth function, which is expressed as a function of the plastic strain trajectory ξ. A simple form of α is given by

$$\alpha = a_1 / \xi^{\eta} \tag{4.2}$$

where a_1 and η are the hardening constants. The plastic strain trajectory ξ is composed of the deviatoric plastic strain trajectory ξ_D and volumetric plastic strain trajectory ξ_v. The value of m = -0.5 is often used. S_r is the stress ratio. In case of nonassociative HiSS-δ_1 model, two additional parameters κ_1 and κ_2, which allow for the correction (deviation from normality) with respect to δ_0 model are required. The notations of the Eqn. 4.1 are adopted from Desai (2001), and the detailed description about the material constants and their determination are discussed therein.

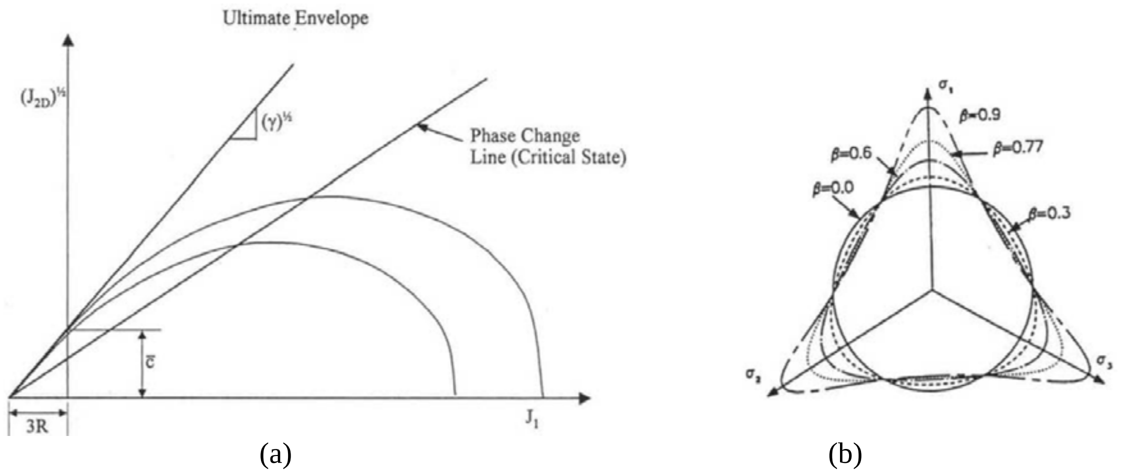

Figure 4.1. HiSS yield surface in (a) J_1-$(J_{2D})^{1/2}$ plane and (b) octahedral plane (After Desai 2001)

5. VALIDATION OF THE ALGORITHM FOR HiSS MODEL

A finite element program in MATLAB is developed for coupled FEM-SBFEM scheme for time-domain analysis of dynamic SSI problems, considering the HiSS-δ_0 nonlinearity in the near-field region. In this section, the developed program for the implemented HiSS model in the FEM analysis is verified.

Along with the basic HiSS-δ_0 model, the HiSS-δ_1 is also implemented in the developed program and the same is used to verify the problem in literature. A two-dimensional axisymmetric problem subjected to the increments of axial displacements in the vertical direction is analyzed using the mesh and boundary conditions shown in Fig. 5.1. The material and model constants used in the problem are listed in Table 5.1.

Table 5.1. Material and Model Constants

Elastic constants	Young's Modulus E = 103841.9 kPa, Poisson's ratio v = 0.291
Ultimate constants	m = -0.5, γ = 0.089, β = 0.442
Phase change constants	n = 3.0
Hardening constants	a_1 = 0.18x 10-3, η = 0.85
Non-associative constants	κ_1 = 0.2637, κ_2 = -0.037

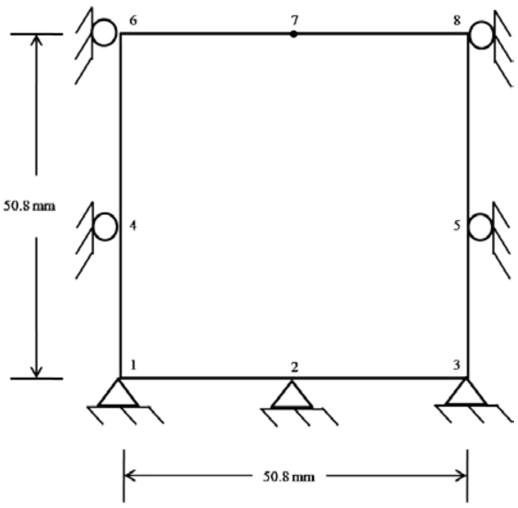

Figure 5.1. Mesh and boundary conditions

Nodes 6, 7 and 8 are subjected to downward vertical displacements and zero horizontal displacements. The incremental displacement value is 0.0508 mm with a total applied displacement value of 0.508 mm in 10 numbers of steps. A hydrostatic in situ stress equal to 137.90 kPa is also applied. Since the results obtained are same for all the four Gauss points, they are reported in Table 5.2 for only one Gauss point.

As evident from Table 5.2, the stresses values obtained using the developed program are exactly the same as with those given by Desai et al. (1991). This verifies the developed MATLAB algorithm.

Table 5.2. Stresses at a Gauss Point

Total downward vertical displacements at nodes 6, 7, and 8 (mm)	Stresses at a Gauss point (kPa) (Desai et al. 1991)		Stresses at a Gauss point (kPa) (Present study)	
	$\sigma_x = \sigma_z$	σ_y	$\sigma_x = \sigma_z$	σ_y
0.002	128.0294	193.0373	128.0294	193.0373
0.004	127.0200	225.7874	127.0200	225.7874
0.006	137.3732	255.5252	127.0200	255.5252
0.008	150.8517	285.0644	150.8517	285.0644
0.010	164.5778	314.2533	164.5778	314.2533
0.012	177.9047	343.0162	177.9047	343.0162
0.014	190.7096	371.3447	190.7096	371.3447
0.016	202.9926	399.2498	202.9926	399.2498
0.018	214.7806	426.7468	214.7806	426.7468
0.020	226.1059	453.8494	226.1059	453.8494

6. TIME INTEGRATION FOR FINITE ELEMENT EQUATIONS

Lehmann (2005) used HHT-α method in the coupled FEM-SBFEM formulation for the dynamic SSI analysis. The nonlinear description for the HHT-α method with modified Newton-Raphson iteration, given in Crisfield (1997), was extended by Bransch and Lehmann (2011) for the full Newton-Raphson iteration. In full Newton-Raphson iteration, the stiffness of the system under consideration is modified at the beginning of each iteration, which can be expensive computationally. To overcome this difficulty, the modified Newton-Raphson iteration can be used, wherein the initial stiffness matrix is kept constant throughout the analysis. However, for highly nonlinear problems, the modified Newton-Raphson iteration can cause convergence problems and may require very large iterations for convergence. With the full Newton-Raphson iteration, better convergence can be achieved even with a larger time step than the modified Newton-Raphson iteration. The detailed discussion of the new nonlinear HHT- α formulation with full Newton-Raphson iteration and the involved equations, can be found in Bransch and Lehmann (2011). Moreover, the algorithm for the implementation of HiSS model can be found in Desai et al. (1991). The algorithm for nonlinear HHT- α formulation with full Newton-Raphson iteration in FEM-SBFEM approach and considering the HiSS constitutive model for modeling the soil in near-field is being developed. Further, the same will be extended considering DSC (Disturbed State Concept) model to deal with soil liquefaction.

7. CONCLUSION

In the present work, a coupled FEM-SBFEM approach for the nonlinear dynamic SSI analysis is discussed. The nonlinearity of soil is modelled using HiSS constitutive model. A problem from literature is analysed for the verification of the algorithm developed. Also, an axisymmetric problem of an elastic half-space under dynamic load was studied. The results highlight the importance of radiation damping in the SSI analysis. The present approach can be used in the dynamic SSI analysis of highly nonlinear problems, since the full Newton-Raphson iteration is considered in the nonlinear HHT-α method of integration. The algorithm is being extended to deal with soil liquefaction.

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