

# Analysis Model of Buried Pipeline Network under Multi-Support Seismic Excitations

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## **SUMMARY:**

In this paper, a quasi-static model is proposed to investigate the responses of a buried pipeline network under multi-support seismic excitations considering the spatial variation of ground motions. Firstly, the stiffness equations of the buried pipeline network are established using a finite element method (FEM). Secondly, considering the spatial variation of ground motions, a spectral representation method is employed to establish a ground motion field to describe the ground motions where pipeline network locates. Finally, the proposed method is used to investigate the responses of a cross pipeline network. Thereafter, the effects of different seismic excitations on the seismic responses of pipeline network are studied in detail.

*Keywords: Buried Pipeline Network Finite Element Method (FEM) Multi-Support Seismic Excitations*

## **1. INTRODUCTION**

Water distribution network and gas supply network are two important components of lifeline engineering systems and most of them are buried underground. During many previous strong earthquakes, it was found that buried pipeline systems suffered serious damages. The investigation indicated that seismic wave propagation is a major factor for the damages of buried pipelines. Since these buried pipeline systems are always distributed in a large area and composed of different kinds of components such as straight pipelines, joints, bends and tees, damages to single portions (especially close to junctions) often affect other adjacent portions of the systems under seismic excitations. Therefore, it is necessary to study the systems as a whole and investigate the interactions between different portions. Many researches have been carried out for this problem. In 1983, Singhal and Meng analysed pipe stresses as a static problem by assuming that the pipelines are the beams on elastic foundation. Takada and Tanabe (1987) carried out a seismic response analysis of buried pipeline with branches under sinusoidal waves. Using the FEM and the quasi-static analysis method, Wang and Lau (1989) studied the seismic responses of buried pipeline systems under seismic traveling waves. In recent years, Kuwata et al. (2008) developed a FEM analysis method to simulate the seismic behaviour of complicated pipeline network with several branches.

Apparently, among the previous researches, the ground motion is usually simplified as a sinusoidal wave or a travelling wave, ignoring the spatial variation of ground motions. In this paper, a pseudo-static model is proposed to investigate the responses of buried pipeline network under multi-support seismic excitations considering the spatial variation of ground motions. Meanwhile, a cross network is used to validate the proposed method.

## **2. FEM MODEL**

Usually, the seismic responses of buried pipelines can be obtained by quasi-static approach. Herein, the buried pipeline is idealized as a beam on elastic foundation, and its axial and lateral motion equations are given by

$$EA \frac{\partial^2 u(x,t)}{\partial x^2} - k_A u(x,t) = -k_A u_g(x,t) \quad (2.1)$$

and

$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + k_L v(x,t) = k_L v_g(x,t) \quad (2.2)$$

where  $EA$  and  $EI$  are the axial and bending stiffness of pipeline respectively,  $k_A$  and  $k_L$  are the spring constants of the soil surrounding pipeline in axial and lateral directions,  $u(x,t)$  and  $v(x,t)$  are axial and lateral displacements of pipeline,  $u_g(x,t)$  and  $v_g(x,t)$  are axial and lateral displacements of ground motions.

The finite element stiffness matrices can be obtained from the differential equations (2.1) and (2.2) by variational principal. The element stiffness matrices of pipeline  $[K_P]$  and soil  $[K_S]$  are given by

$$[K_P] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & \text{Symmetric} & & \frac{EA}{L} & 0 & 0 \\ & & & & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \quad (2.3)$$

and

$$[K_S] = \begin{bmatrix} \frac{1}{3}\alpha & 0 & 0 & \frac{1}{6}\alpha & 0 & 0 \\ & \frac{13}{35}\beta & \frac{11}{210}L\beta & 0 & \frac{9}{70}\beta & -\frac{13}{420}L\beta \\ & & \frac{1}{105}L^2\beta & 0 & \frac{13}{420}L\beta & -\frac{1}{140}L^2\beta \\ & \text{Symmetric} & & \frac{1}{3}\alpha & 0 & 0 \\ & & & & \frac{13}{35}\beta & -\frac{11}{210}L\beta \\ & & & & & \frac{1}{105}L^2\beta \end{bmatrix} \quad (2.4)$$

where  $L$  is the length of element, and  $\alpha=k_A L$ ,  $\beta=k_L L$ , in which  $k_A$  and  $k_L$  are the spring constants of the soil surrounding pipeline in axial and lateral direction respectively.

Usually, a segmented pipeline is connected by segments and joints. The joint here is also simulated by axial and bending springs. Meanwhile, its lateral relative displacement is restrained by an infinite stiffness spring. Correspondingly, the matrix is given by

$$[K_J] = \begin{bmatrix} k_{JA} & 0 & 0 & -k_{JA} & 0 & 0 \\ & k_{\infty} & 0 & 0 & -k_{\infty} & 0 \\ & & k_{JR} & 0 & 0 & -k_{JR} \\ \text{Symmetric} & & & k_{JA} & 0 & 0 \\ & & & & k_{\infty} & 0 \\ & & & & & k_{JR} \end{bmatrix} \quad (2.5)$$

where  $k_{JA}$  and  $k_{JR}$  are the axial and the bending spring constants of the joint respectively,  $k_{\infty}$  is the infinite spring constant.

Before assembling the system stiffness matrix, the element stiffness matrices  $[K_P]$ ,  $[K_S]$  and  $[K_J]$  induced in local coordinates must be transformed to the corresponding matrices in global coordinate, and then the system stiffness equations can be written as

$$[\bar{K}_{sys}] \{\bar{u}_j\} = [\bar{K}_s] \{\bar{x}_g\} \quad (2.6)$$

where  $[\bar{K}_{sys}] = [\bar{K}_p] + [\bar{K}_s] + [\bar{K}_j]$  is the system stiffness matrix in global coordinate, the symbol ‘-’ denotes matrix and vector in global coordinate,  $\{\bar{u}_j\}$  is the pipeline displacement vector in global coordinate and  $\{\bar{x}_g\}$  is the ground motion displacement vector in global coordinate.

### 3. SIMULATION OF GROUND MOTIONS

#### 3.1. Ground Motion Locations

Figure 1 shows a buried pipeline network subjected to a seismic wave propagating with an incident angle of  $\theta$ . In order to reduce the computation time, the network is assumed to be far away from the epicentre. Therefore, the wave surface can be approximated as a plane vertical to the wave travelling direction, and the two-dimension problem becomes a one-dimension one. Projecting the pipeline network nodes on the wave travelling direction, some locations can be obtained and the corresponding earthquake histories can be described as  $f_1(t), f_2(t), \dots, f_N(t)$ , where  $N$  is the number of nodes. Thereafter, the ground motions at these locations can be simulated by a spectral representation method.

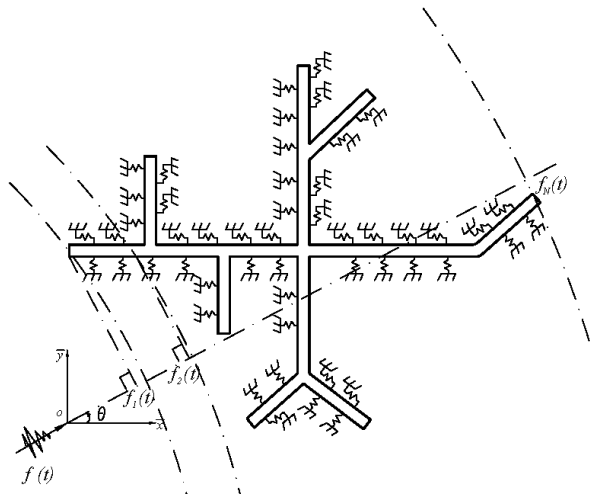


Figure 1. Locations for simulating ground motions

#### 3.2. Spectral Representation Method

Using the spectral representation method (Deodatis, 1996), a one-dimension and multivariate stochastic processes can be described by

$$f_j^0(t) = 2 \sum_{m=1}^j \sum_{l=1}^n |H_{jm}(\omega_{ml})| \sqrt{\Delta\omega} \cos[\omega_{ml}t - \theta_{jm}(\omega_{ml}) + \varphi_{ml}], \quad j = 1, 2, \dots, N \quad (3.1)$$

where  $f_j^0(t)$  is the stationary ground motion at point  $j$ ,  $\varphi_{ml}$  is an independent random phase angle distributed uniformly over the interval  $[0, 2\pi]$ ,  $\Delta\omega = \omega_u/n$  where  $\omega_u$  represents an upper cut-off frequency of power density function,  $\omega_{ml}$  represents two-index frequency, in which  $\omega_{ml} = (l-1)\Delta\omega + m\Delta\omega/n$ ,  $l=1, 2, \dots, n$ .

In Eqn. (3.1),  $\mathbf{H}(\omega)$  is a lower triangular matrix by decomposing cross-spectral density matrix  $\mathbf{S}_a^o(\omega)$  using Cholesky's method, and can be described as

$$\mathbf{S}_a^o(\omega) = \mathbf{H}(\omega) \mathbf{H}^{T*}(\omega) \quad (3.2)$$

where superscripts  $T$  and  $*$  denote the transpose and conjugate of the matrix, and

$$\mathbf{S}_a^o(\omega) = \begin{bmatrix} S_1(\omega) & \sqrt{S_1(\omega)S_2(\omega)}\gamma_{12} & \cdots & \sqrt{S_1(\omega)S_N(\omega)}\gamma_{1N} \\ \sqrt{S_2(\omega)S_1(\omega)}\gamma_{21} & S_2(\omega) & \cdots & \sqrt{S_2(\omega)S_N(\omega)}\gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{S_N(\omega)S_1(\omega)}\gamma_{N1} & \sqrt{S_N(\omega)S_2(\omega)}\gamma_{N2} & \cdots & S_N(\omega) \end{bmatrix} \quad (3.3)$$

where  $S_i(\omega)$  is the power spectral density function of point  $i$ ,  $\gamma_{ij}(i \neq j)$  is the complex coherence function between points  $i$  and  $j$ , and can be given by

$$\gamma_{ij}(\omega) = |\gamma_{ij}(\omega)| \exp\left[-i \frac{\xi_{ij}\omega}{v_a}\right] \quad (3.4)$$

in which  $|\gamma_{ij}(\omega)|$  is the logged coherency function,  $\xi_{ij}$  represents the distance between points  $i$  and  $j$ ,  $v_a$  is the velocity of wave propagation.

According to Eqns. (3.2) to (3.4), the off-diagonal elements of  $\mathbf{H}(\omega)$  can be written in polar form as

$$H_{jk}(\omega) = |H_{jk}(\omega)| e^{i\theta_{jk}(\omega)}, \quad j = 2, 3, \dots, N; \quad k = 1, 2, \dots, N-1; \quad j > k \quad (3.5)$$

where

$$\theta_{jk}(\omega) = \tan^{-1} \left\{ \frac{\text{Im}[H_{jk}(\omega)]}{\text{Re}[H_{jk}(\omega)]} \right\} \quad (3.6)$$

and Im and Re denote the imaginary and the real part of a complex number respectively.

Submitting Eqns. (3.5) and (3.6) in Eqn. (3.1) and using FFT method, the stationary ground motions can be given. In order to describe the non-stationarity of ground motions, by multiplying modulating functions, the stationary ones can be transformed to the non-stationary ground motions.

$$f_j(t) = A_j(t) \cdot f_j^0(t), \quad (j = 1, 2, \dots, N) \quad (3.7)$$

## 4. EXAMPLES

### 4.1. Background

Figure 2 shows a buried pipeline network composed of continuous steel pipes. The parameters of the pipelines are  $E=2.05 \times 10^{11} Pa$ ,  $A=0.0134 m^2$ ,  $I=0.00014 m^4$  and the soil spring constants surrounding the pipelines are  $k_A=k_L=5.65 \times 10^7 N/m$ , the boundaries of the network are free.

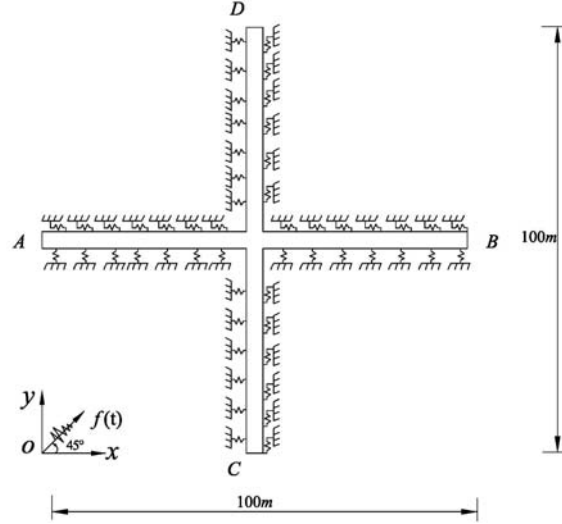


Figure 2. A buried cross pipeline network

### 4.2. Simulation of Ground Motions

#### 4.2.1 Power spectral density model

The Clough-Penzin acceleration spectrum(1975) is selected to model the power spectral density function.

$$S(\omega) = S_0 \frac{\left(\frac{\omega}{\omega_f}\right)^4}{\left[1 - \left(\frac{\omega}{\omega_f}\right)^2\right]^2 + 4\zeta_f^2 \left(\frac{\omega}{\omega_f}\right)^2} \frac{1 + 4\zeta_g^2 \left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\zeta_g^2 \left(\frac{\omega}{\omega_g}\right)^2} \quad (4.1)$$

where  $S_0$  is a constant which determines the intensity of acceleration,  $\omega_g$  and  $\zeta_g$  are characteristic frequency and damping ratio of the field respectively, and  $\omega_f$  and  $\zeta_f$  are filtering parameters. According to Deodatis' paper (1996), for stiff soil condition,  $S_0=62.3 cm^2/s^3$ ,  $\omega_g=8\pi rad/s$ ,  $\zeta_g=0.6$ ,  $\omega_f=0.1\omega_g$  and  $\zeta_f=\zeta_g$ .

#### 4.2.2 Coherence function model

The Harichandran –Vanmarcke model (1986) is chosen to describe the coherence function.

$$|\gamma(\xi, \omega)| = A \exp\left(-\frac{2B|\xi|}{\alpha v(\omega)}\right) + (1-A) \exp\left(-\frac{2B|\xi|}{v(\omega)}\right) \quad (4.2)$$

where  $\xi$  is the distance between two points,  $B=1-A-\alpha A$ ,  $v(\omega)$  is the frequency-dependent correlation distance and can be written as

$$v(\omega) = \kappa \left[ 1 + \left( \frac{\omega}{\omega_0} \right)^b \right]^{-\frac{1}{2}} \quad (4.3)$$

and other parameters are  $\omega_0=6.85 \text{ rad/s}$ ,  $\alpha=0.147$ ,  $A=0.736$ ,  $\kappa=5210m$  and  $b=2.78$  respectively.

Finally, noticing the Eqn. (3.4), the velocity of wave propagation  $v_a$  is set as  $v_a=1000 \text{ m/s}$ .

#### 4.2.3 Modulating function model

Amin-Ang model (1968) is selected as the intensity modulating function to transform the stationary ground motions to non-stationary ones.

$$A(t) = \begin{cases} (t/t_1)^2 & 0 \leq t < t_1 \\ 1 & t_1 \leq t < t_2 \\ \exp[-c(t-t_2)] & t \geq t_2 \end{cases} \quad (4.4)$$

where  $t_1=1 \text{ sec}$ ,  $t_2=7 \text{ sec}$ ,  $c=1.15$  (Li and Li, 1992).

#### 4.2.4 Ground motion displacements

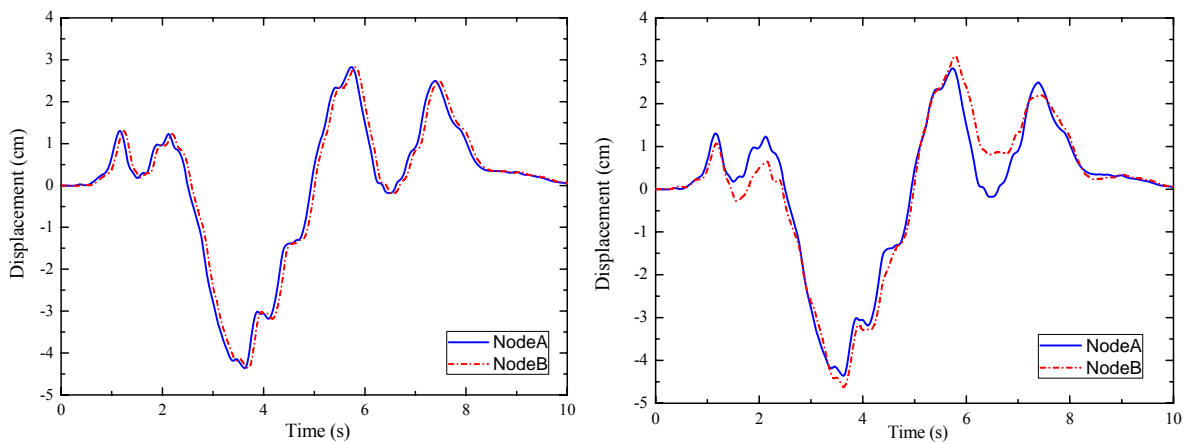
The ground motions for seismic response analysis of the buried pipeline network is displacement histories, therefore, the previous ground motions simulated by spectral representation method must be integrated twice.

$$\dot{u}(t) = \int f(t) dt \quad (4.5)$$

$$u(t) = \int \dot{u}(t) dt \quad (4.6)$$

#### 4.2.5 Samples of simulated ground motion

Two methods are used to simulate the seismic wave propagation. One is a travelling wave and the other is a wave generated by the spectral representation method. In two waves, the time history of node A is assumed the same. In figure 3-a), the seismic wave propagates as a travelling wave with the propagation velocity of  $1000 \text{ m/s}$  and without any change in its shape between node A and node B. In figure 3-b), considering the spatial correlation, the ground motion, which is generated by the spectral representation method, changes obviously from node A to node B.



a) Travelling waves

b) Spatial correlated seismic waves

**Figure 3.** Samples of time histories of displacement

### 4.3. Seismic Responses

Discretizing the buried pipeline networks showed in Figure 2 to elements with 1 meter in length, the system stiffness equations can be obtained. The seismic response analysis of the pipeline network under excitations of travelling waves and spatial correlated waves are carried out respectively, and the two models are defined as model 1 and model 2.

Without loss of generality, the peak stresses distributions along pipeline AB of model 1 and model 2 are drawn in Figure 4 and Figure 5, respectively. In Figure 4, the axial and bending peak stresses of model 1 distribute periodically along the pipeline AB and the maximum values are at the junction. But in Figure 5, the seismic responses of the pipeline network of model 2 are very different from that of model 1. The axial and bending peak stresses distribute irregularly along the pipeline AB, and the maximums of the peak stresses are much larger than those of model 1. For example, the maximal axial peak stress of model 1 is about 30MPa while that of mode 2 is two times larger, about 60MPa. This indicates that the ground motions have important influences on the responses of buried pipeline network.

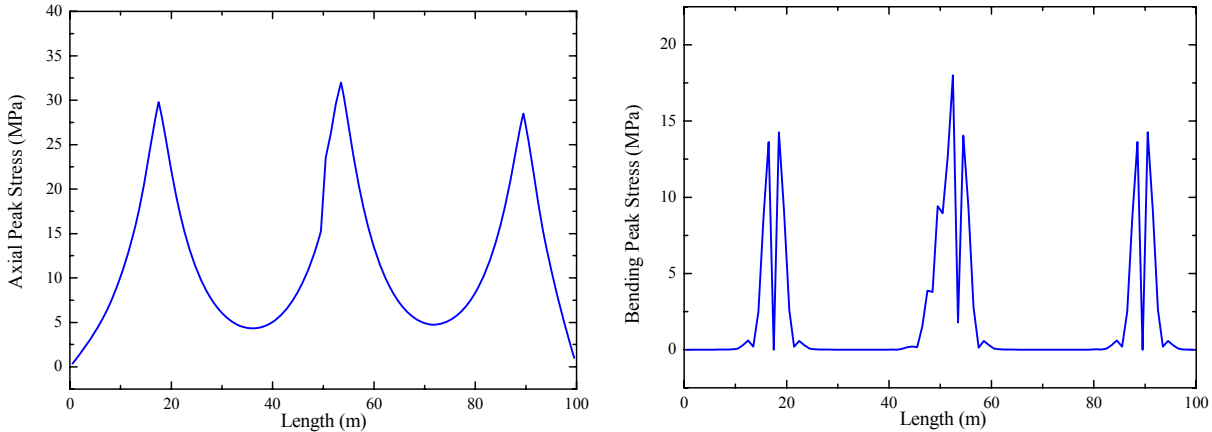


Figure 4. The stress distribution of pipeline AB of model 1

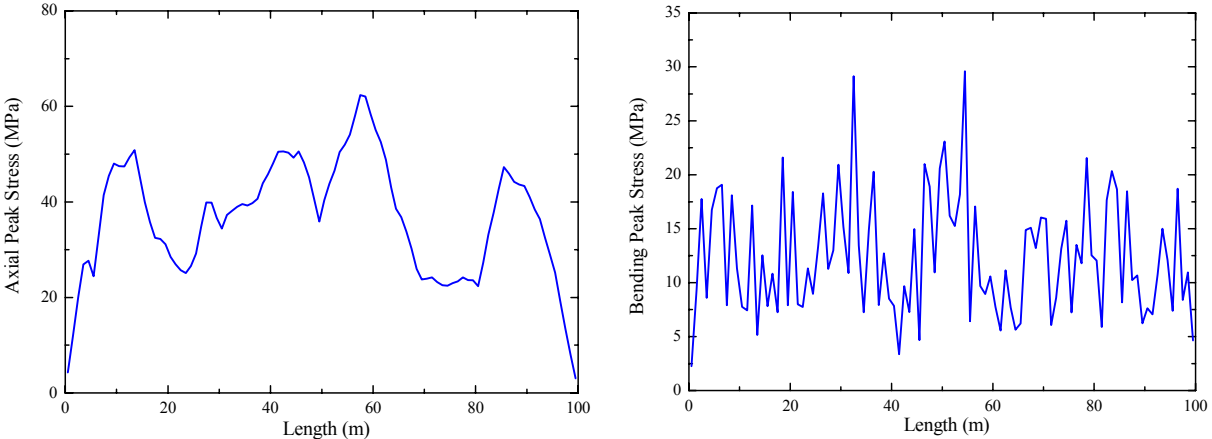


Figure 5. The stress distribution of pipeline AB of mode 2

### 5. CONCLUSIONS

Considering the spatial variation of the ground motions, a method of seismic response analysis of buried pipeline network subjected to multi-support seismic excitations is presented in this paper. Thereafter, numerical examples are given to study the seismic responses of buried pipeline network

under different types of ground motions. Apparently, ground motion is an important factor which influences the seismic response of the buried pipeline network. The responses of the pipeline network subjected to ground motions considering the spatial variation are obviously larger than those without considering. Therefore, the previous analysis methods which simplified the seismic wave as a sinusoidal wave or a travelling wave underestimate the pipeline network's response. Thus, it is necessary to analyse a buried pipeline network as a whole and consider the influences of spatial variation of ground motions.

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