Damage Detection Method by Using Inverse Solution of Equations of Motion in Frequency Domain

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SUMMARY:
This paper presents a new damage detection method using direct solution of the equations of motion in the frequency domain. The method utilizes directly the acceleration frequency response of the selected degree of freedoms, some natural frequency of the damaged structure and mode shapes of the undamaged analytical model of structure to identify the structural stiffness of the damaged structure including the damping ratio. The least square method is used to minimize the residual force in the frequency domain. The validity and efficiency of the proposed method is tested on a simple truss structure subjected to sweep harmonic forces. It is shown that the proposed method can estimate the structural member's stiffness with a high level of accuracy. In the case of white noise RMS of equal to 5% of the maximum RMS of the structure response, the stiffness of members can be identified with less than two percent error.

Keywords: Damage detection, inverse problem, frequency domain, structural health monitoring, optimization

1. INTRODUCTION

Vibration-based Structural health monitoring methods estimate the change in the modal or/and structural properties can detect the probable damage location and severity in order to identify structural integrity (Doebling and et al., 1996; Salawo, 1997; Yan, 2007; Antonino and Vestroni, 2008; Sohn and et al., 2004). One of these methods which is “Model Updating” can be classified as: modal-based and response-based. The modal-based model updating techniques utilize the modal characteristics data extracted from the experimental modal analysis (Yuen, 1985; Stubbs and et al., 1990; Hearn and Testa, 1991; Narkis, 1994; Amani and et al., 2007). Considering that: 1) The numerical identification of the modal parameters is accompanied by the imprecision that can exceed the level of required accuracy to update FE models (Sestieri and D’Ambrogio, 1985); 2) For structures with closely spaced modes, the identified modal properties will be associated with high levels errors; and 3) Some type of damage have low effect on the modal properties (Farrar et al., 1994); thus the modal-based-updating techniques cannot always result to reliable estimation of damage.

In both time and frequency response-based updating methods, the measured data are directly utilized to identify some of the structural parameters. The identified parameters will then be used to update and improve the finite element model (FEM) of the structure. In this method the measured responses which reflect more data points and the structural properties can better update the structural model, in comparison with the modal-based where only few eigen-properties are identified. Recently some have used the combination of the aforementioned methods to cover the effect of the unmeasured data (Esfandiari and et al., 2009).

In the response-based model updating methods, frequency domain and FRF-based methods have been more widely developed and applied due to the ease of handling data and separating the intense noise in the frequency domain. These model updating techniques are used to minimize a residual error between analytical and experimental input force/output response (Natke, 1988; Fritzen, 1992; D’Ambrogio and et al., 1993; Friswell and Mottershead, 1995; Pothisiri and Hjelmstad, 2003). One of the restrictions of the FRF model updating techniques is the incompatibility between the degrees of freedom (DOF) of the FEM and the tested structure due to the incomplete measurements and data acquisition problems, such as limitation of the sensors and installation point. In this case, the objective function of the input-residuals or output-residuals cannot be assembled and minimized. Many researchers have utilized the reduction of DOF of the FEM (Conti and Donley 1992) or expansion of the measurement vector (Esfandiari and et al., 2009; 2010) in order to become compatible with the analytical and tested structure. Both techniques convert a linear model updating problem (full measurement) to a nonlinear problem due to matrix inversion containing unknown parameters. The nonlinear problem can be solved by the nonlinear optimization procedures or by the iterative linear solving techniques. The frequency range of the FRF data and weighting techniques of the equation are considered in the FRF based the research (Wang and et al., 1997; Park and Park, 2003).

In this paper, a frequency domain finite element model updating algorithm has been presented to identify the damage location and severity. The system property matrices of the damaged structure are correlated to the input force and responses of the damaged structure in the frequency domain. The damage reduction factors of the elements are obtained using least square minimization of the residual errors of the equations of the motions in the frequency domain. ‘‘Damage” can be defined as an actual damage, deterioration, or errors in initial section stiffness property. Here, it is assumed that damage is uniformly distributed along the element and it is modeled as stiffness reduction factor for the element. So the identified stiffness reduction factor will be an equivalent reduction factor for the partially distributed damage. The unmeasured responses of the damaged structure are estimated by the mode shapes of analytical FEM of the undamaged structure and the identified frequencies and damping loss factors of the damaged structure. The accuracy of the estimated responses will be improved during the iterative damage detection process. A simple truss structure with the noisy measured response is used to demonstrate the efficiency of finite element model updating and damage assessment. Figure1 shows the summary of the proposed algorithm for the assessment of the structural condition.

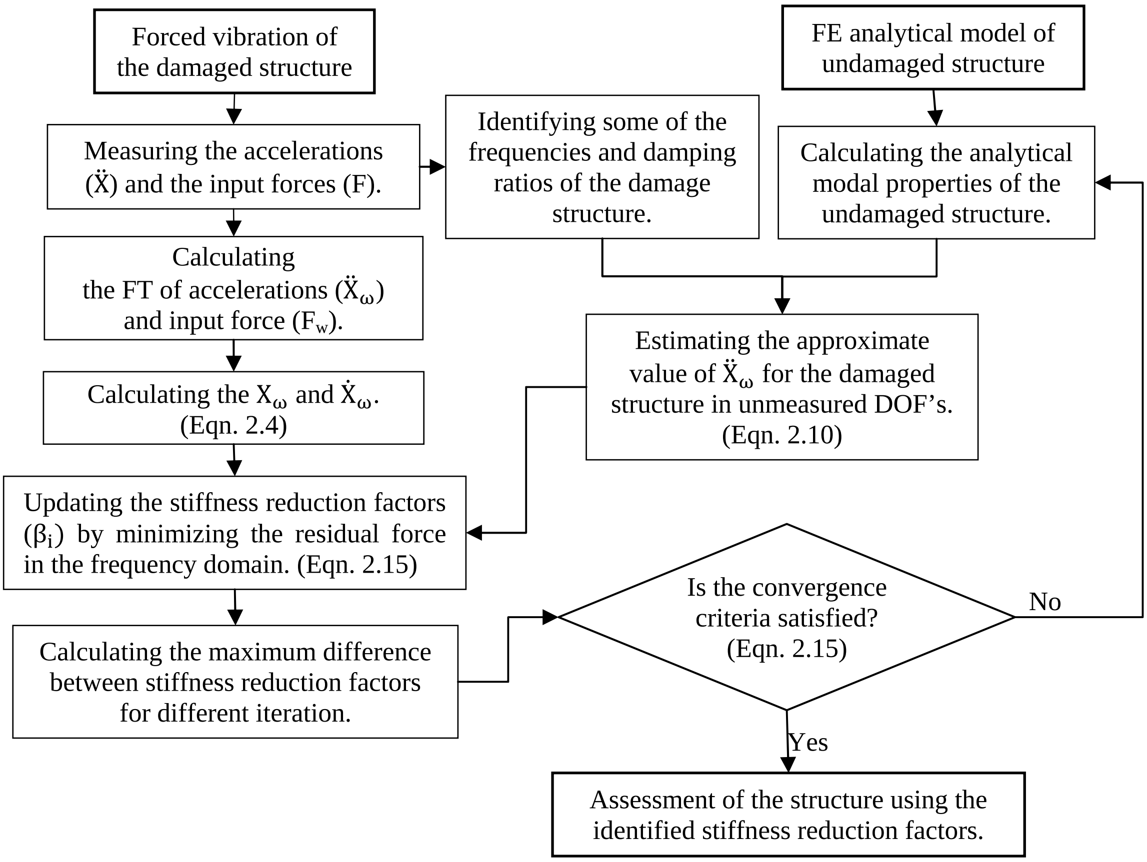


Figure 1. Process of the proposed method of the structure assessment

2. PROPOSED DAMAGE DETCTION ALGORITHM

In this study, it is assumed that the responses of the damaged structure are measured through the force vibration test and the undamaged structure is modelled using the finite element model regarding to the initial structural properties. In the proposed algorithm the modal properties of the undamaged structure are required to approximate the unmeasured response of the damaged structure. These modal properties can be calculated using the analytical model of the initial structure.

The general equation of motion for viscously damped linear model of damaged structure with "n" number of degrees-of-freedom can be written as:

$$\mathbf{M_d}\ddot{\mathbf{x}}(\mathrm{t}) + \mathbf{C_d}\dot{\mathbf{x}}(\mathrm{t}) + \mathbf{K_d}\mathbf{x}(\mathrm{t}) = \mathbf{f}(\mathrm{t}) \tag{2.1}$$

where $\mathbf{M_d}$, $\mathbf{C_d}$ and $\mathbf{K_d}$ are $n \times n$ matrices of the mass, damping and stiffness of the damaged structure, respectively. $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$, $\mathbf{x}$ and $\mathbf{f}(\mathrm{t})$ are $n \times 1$ vectors of the acceleration, velocity, displacement and input force, respectively. This equation for "m" time step of input and responses can be written as:

$$\mathbf{M}_{\mathrm{n\times n}}\ddot{\mathbf{X}}_{\mathrm{n\times m}} + \mathbf{C}_{\mathrm{n\times n}}\dot{\mathbf{X}}_{\mathrm{n\times m}} + \mathbf{K}_{\mathrm{n\times n}}\mathbf{X}_{\mathrm{n\times m}} = \mathbf{F}_{\mathrm{n\times m}} \tag{2.2}$$

Assuming the acceleration and input force vectors as:

$$\mathbf{F}(\mathrm{t}) = \mathbf{F}(\omega)\mathrm{e}^{\mathrm{j}\omega\mathrm{t}} \quad \text{and} \quad \ddot{\mathbf{X}}(\mathrm{t}) = \ddot{\mathbf{X}}(\omega)\mathrm{e}^{\mathrm{j}\omega\mathrm{t}} \tag{2.3}$$

where $\mathbf{F}(\omega)$ and $\ddot{\mathbf{X}}(\omega)$ are the input force and acceleration of the damaged structure in the frequency domain. ω is the frequency of the excitation input force and $\mathrm{j} = \sqrt{-1}$. The velocity and displacement frequency response can be estimated as:

$$\dot{\mathbf{X}}(\omega) = \ddot{\mathbf{X}}(\omega)/\omega\mathrm{j} \quad \text{and} \quad \mathbf{X}(\omega) = \ddot{\mathbf{X}}(\omega)/(-\omega^2) \tag{2.4}$$

When the frequency response of acceleration is accompanied by measurement noise, the Eqn. 2.4 can amplify or reduce the noise effect in the velocity and displacement frequency response regarding to the frequency domain. The equation of motion in Eq. 2.1 for the damaged structure in the frequency domain can be written as:

$$\mathbf{M_d}\ddot{\mathbf{X}}(\omega) + \mathbf{C_d}\dot{\mathbf{X}}(\omega) + \mathbf{K_d}\mathbf{X}(\omega) = \mathbf{F}(\omega) \tag{2.5}$$

Considering that in most of the structures, the damage will not result to any changes in the structural mass and it can be assumed that $\mathbf{M_d} = \mathbf{M}$.

Eventhough, the damping of the damaged structure which is related to the material properties, structural geometry, structural connections, non-structural elements, etc. are not modelled in the analytical model of the structure; here only the linear viscous damping matrix of the damaged structure, C_d, has been assumed as a function of the modal properties:

$$\mathbf{C_d} = \mathbf{\Phi_d^{-1}} \begin{bmatrix} c_{d_1} & 0 & 0 & 0 \\ 0 & c_{d_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_{d_n} \end{bmatrix} \mathbf{\Phi_d} \tag{2.6}$$

where $\mathbf{\Phi_d}$ $(n \times n)$ is mode shape and c_{d_i} is i^{th} modal damping of the damaged structure defined as:

$$c_{d_i} = 2m_{d_i}\xi_{d_i}\omega_{d_i} \tag{2.7}$$

in which ω_{d_i} and ξ_{d_i} are the frequencies and damping ratio of the ith mode of the damaged structure. The ith modal mass, m_{d_i}, of the damaged structure can be written as:

$$m_{d_i} = {\phi_{d_i}}^T \mathbf{M_d} \phi_{d_i} = {\phi_{d_i}}^T \mathbf{M} \phi_{d_i} \tag{2.8}$$

where ϕ_{d_i} is $n \times 1$ the ith mode shape of the damaged structure.

In the case of the incomplete measurement, only frequencies and damping ratios of the lower modes (n_i modes) of the damaged structure can be accurately identified using common system identification methods. The other modal properties of the damaged structure in Eqns. 2.6 to 2.8 can be initially estimated to be equal to the modal properties of the analytical model of the undamaged structure. Then, these modal properties and the damping matrix will be updated during the iterative model updating procedure and consequently, their precision will be improved. Moreover, the influences of the damping of the structure on the frequency responses rapidly reduce by moving away from the resonance frequencies.

Since measurement of response of structure in all DOF's is not practical; only few $\ddot{X}_\omega$ are measured and the unmeasured $\ddot{X}_\omega$ can be estimated as:

$$\ddot{X}_\omega = -\omega^2 \left(\sum_{i=1}^{n} \frac{\phi_{d_i} {\phi_{d_i}}^T}{m_{d_i}(\omega_{d_i} - \omega^2 + 2j\xi_{d_i}\omega_{d_i}\omega)} \right) F_\omega \tag{2.9}$$

Substituting the unknown modal properties of the damaged structure with the updating modal properties (modal properties of the analytical model of the undamaged structure in the beginning of iteration procedure), the Equation (2.9) can be rewritten as:

$$\ddot{X}_\omega \cong -\omega^2 \left(\sum_{i=1}^{n_i} \frac{\bar{\phi}_i {\bar{\phi}_i}^T}{\bar{m}_i(\omega_{d_i} - \omega^2 + 2j\xi_{d_i}\omega_{d_i}\omega)} + \sum_{i=n_i+1}^{n} \frac{\bar{\phi}_i {\bar{\phi}_i}^T}{\bar{m}_i(\bar{\omega}_i - \omega^2 + 2j\bar{\xi}_i\bar{\omega}_i\omega)} \right) F_\omega \tag{2.10}$$

where $\bar{\omega}_i$, $\bar{\xi}_i$, $\bar{\phi}_i$ and $\bar{m}_i$ represent the unknown natural frequencies, damping ratios, mode shapes and modal mass's of the ith mode of the damage structure which replaced by the modal properties of the analytical model of the undamaged structure. It should be noted that the modal parameters of the analytical model in Eqn. 2.10 are used as the starting point; and they will be updated during the optimization process. It should also be noted that the accuracy of the estimated $\ddot{X}_\omega$ at nearby the natural frequencies of the structure is more than other frequency points.

Using the simulated parameters from Eqns. 2.10, 2.6 and 2.4; the global stiffness matrix of damaged structure from Eqn. 2.5 can be updated as:

$$\mathbf{K_d X_\omega} = \mathbf{F_\omega} - \mathbf{M\ddot{X}_\omega} - \mathbf{\bar{C}_d \dot{X}_\omega} \tag{2.11}$$

where $\mathbf{\bar{C}_d}$ is the approximate value of damping matrix of the damaged structure using the incomplete modal parameters; and the global stiffness of the structure can be written as:

$$\mathbf{K_d} = \sum_{i=1}^{n_e} \mathbf{T_i}^T \mathbf{K^e_{d_i}} \mathbf{T_i} \tag{2.12}$$

In which n_e is the number of element of the structure, $\mathbf{T}$ is element geometry transformation matrix, and $\mathbf{K^e_{d_i}}$ is the local stiffness matrix of ith element. The change in the stiffness matrix due to damage can be written as:

$$\boldsymbol{\delta}\mathbf{K} = \mathbf{K} - \mathbf{K_d} = \sum_{i=1}^{n_e} \mathbf{T_i}^{\mathbf{T}} (\mathbf{K_i^e} - \mathbf{K_{d_i}^e}) \mathbf{T_i} = \sum_{i=1}^{n_e} \mathbf{T_i}^{\mathbf{T}} (\boldsymbol{\delta}\mathbf{K_i^e}) \mathbf{T_i} \tag{2.13}$$

where $\boldsymbol{\delta}\mathbf{K_i^e}$ is the reduction in the local stiffness matrix of the i[th] element, which can be expressed as:

$$\boldsymbol{\delta}\mathbf{K_i^e} = \beta_i \mathbf{K_i^e} \tag{2.14}$$

where β_i , the stiffness reduction factor, represents the relative changes in the stiffness matrix of the i[th] element in the damaged state with respect to the undamaged state. Thus, the change in the stiffness matrix can be expressed as the sum of changes in the element stiffness matrix. For frame elements, Eqn. 2.14 can be decomposed to separately update various structural parameters (e.g., EA, EI, GJ, etc.). Substituting Eqns. 2.13 and 2.12 into the Eqn. 2.11, it becomes:

$$\left(\sum_{i=1}^{n_e} (1 - \beta_i) \mathbf{K_i^e}\right) \mathbf{X_\omega} = \mathbf{F_\omega} - \mathbf{M_d} \ddot{\mathbf{X}}_{\boldsymbol{\omega}} - \bar{\mathbf{C}}_{\mathbf{d}} \dot{\mathbf{X}}_{\boldsymbol{\omega}} \tag{2.15}$$

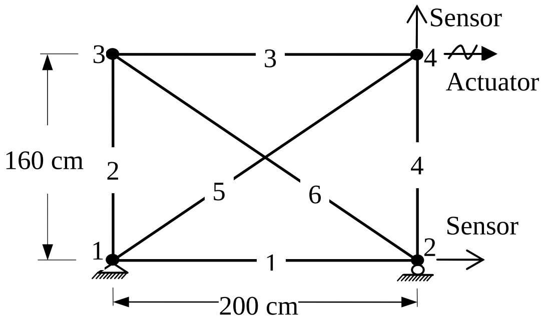
Stiffness reduction factor for each structural element can be obtained using Eqn. 2.15 by several optimization methods such as: the least square method (LS), singular value decomposition method (SVD), and non-negative least square method (NNLS). The process of the updating method is an iterative procedure which the accuracy of model parameters and the unknown simulated parameters will be improved in each iteration. The updating process will be converged if the maximum changes of the stiffness reduction factor in the successive iterations (j and j-1) will be less than proper criteria (ε) as following:

$$\max(\Delta\beta_i) = \max(\beta_i{}^{j} - \beta_i{}^{j-1}) \leq \varepsilon \tag{2.16}$$

The accuracy of predicted damage depends on the several factors such as: the sensor types and locations, excitation types and locations, the quality of measured data (measurement error), selected frequency points for model updating, accuracy of the mathematical model (modeling error), observability of the unknown parameters, weighting techniques applied to the system of equations, and numerical methods used for solution of the system of equations. A balanced attention to these factors is expected to lead to less estimation errors in finite element model updating.

3. NUMERICAL EXAMPLE

The proposed procedure has been tested on several structural models. Here, the result for a high frequency system such as the two-dimensional truss shown in Fig. 3.1 is presented to demonstrate the robustness of the proposed updating method. The methods work much better for the low frequency system such as frame structure. The axial rigidity of elements, EA, (cross-sectional area of elements multiplied by Young's modulus) has been updated through the model updating procedure.



Member	Area (cm^2)
1, 3	18
2, 4	15
5, 6	10
E = 200Gpa	

Figure 3.1. Property, geometry, and the location of actuator and sensor of the truss model.

The acceleration responses at the selected DOF's of the example truss can be measured from a force vibration experimental test; or mathematical simulation of a test. Here, for the simulation, harmonic sweep input forces are applied horizontally at the node 4 and the acceleration responses at the sensor locations were obtained (see Fig. 3.1 for the sensor and actuator placements). Considering that measured responses are noisy, 5% normal distributed random white noise has been added to the input forces and calculated responses. The level of the noise is defined as the root mean square (RMS) of noise with respect to of the RMS of the structure response and input forces.

Three damage cases are considered to investigate robustness of the proposed parameter estimation method in locating and quantifying damage (see Table 3.1). Moreover, the natural frequencies of the undamaged and damaged structures are presented in Table 3.2. It has been assumed that the first three natural frequencies and damping ratios of the damage structures can be identified exactly. This assumption is not unrealistic due to recently developed system identification methods.

Table 3.1. Damage scenarios and percent of damage (stiffness reduction) of elements.

Case	Element numbers and their damage percentage			
1	Element no. Damage	1 50%		
2	Element no. Damage	1 50%	4 40%	
3	Element no. Damage	1 50%	4 40%	5 30%

Table 3.2 Frequencies of undamaged and damaged trusses.

Mode No.	Undamaged	Case 1	Case 2	Case 3
1	161.7	152.8	147.4	137.9
2	369.3	304.7	302.1	301.8
3	398.8	397.0	330.5	318.9
4	439.5	419.8	415.9	414.3
5	559.2	557.6	557.0	551.9

Using the proper excitation frequency and the selecting of the frequency response points can improve the reliability and accuracy of the results. Some general rules are proposed in the past researches (Pereira and et al., 1995; Ren and De Roeck, 2001; Pascuala and et al., 2007); such as: 1) The frequency of excitation shall not be selected too close to the natural frequencies of the structure in order to see the effect of all structural modes and not get dominated by a specific or single mode of the structure; and 2) The higher frequency excitations will lead to the more accurate results, since in most cases the higher modes are more sensitive to the damage occurrence. Considering that the selection of the excitation frequency can be accompanied by the severe changes in the results; here instead of commonly used harmonic input, the sweeping harmonic load with frequency of 100 to 600 Hz is used to excite the all modes of the structure. The frequency domain of the input force can affect the efficiency and accuracy of the proposed method. It has been seen that the accuracy of the results decreases when a narrow band frequency loading is used.

Selection the proper and enough frequency points in the model updating procedure using the Eqn. 2.15 is important to achieve the accurate results. These frequency points should be selected such that: 1) to eliminate the noisy data in the measurements and simulated data, and 2) to properly select the part of data that are more influenced by the structural damage. In this process following points should be considered:

1. The anti-resonance points should be excluded due to highly relative measurement noise in measured DOF's; and being insensitive to the structural damage.
2. The first portion of the velocity and displacement frequency response is eliminated due to amplification of the noise in the acceleration response, see Eqn. 2.4.
3. The frequency points very close to natural frequencies must be excluded. The reasons are: a) The precision of the approximated damping matrix using the Eqn. 2.6 with substituted undamaged modal properties will decrease in frequency points very close to the natural frequency; and b) Eventhough the accuracy of the simulated $\ddot{\mathbf{X}}_{\omega}$ using the Eqn. 2.10 in the vicinity of the natural frequencies is more than the anti-resonance ranges, a little imprecision in the identification of the natural frequencies of the damage structure will shift the $\ddot{\mathbf{X}}_{\omega}$ and results to large difference between the simulated and exact in the frequency point very close to the natural frequencies. In this study the exact frequencies of the damage structure have been used.

Based on the abovementioned points, the frequency ranges as shown in the Table 3.3 are selected for the model updating process. Moreover, it can be recommended to select the frequency point at the regions in which the occurred damage caused more changes in the natural frequencies, since the change in the frequency responses will be more noticeable in these regions.

Table 3.3 Selected frequency ranges for model updating.

Damage case	Case 1	Case 2	Case 3
Frequency range (Hz.)	160-170	160-170	150-160
	310-330	270-290	270-290

Using the selected frequency ranges at 1 Hz interval, the model has been updated according to the flowchart shown in Fig. 2.1; and results are presented in the Fig. 3.2. Black bars indicate location and severity of the damaged truss member defined in terms of stiffness reduction factor (β), and the white bars indicate the predicted results. The maximum error in the estimated stiffness reduction factor is less than 5% for the all damage cases. Moreover, Fig. 3.3 shows the convergences of the iterative process. It can be seen the calculations are converged through the 20 iterations. The error of predicted stiffness reduction factor (β) calculated based on the differences of the actual and predicted stiffness reduction factor. The model updating procedure is performed using the 100 data sets for three damage cases and COV of the results are calculated. The low level of the COV has been seen in the all cases which show the high level of reliability of the results.

The accuracy improvement of the simulated displacement frequency response ($\mathbf{X}_{\omega}$) of the second DOF’s during the updating model has been shown in Fig. 3.4. The scaled error of simulated $\mathbf{X}_{\omega}$ before and after updating the model are compared. The error are scaled to the maximum of $\mathbf{X}_{\omega}$. It can be seen that the error of the simulated $\mathbf{X}_{\omega}$ decreases to the less 1% as through the model updating.

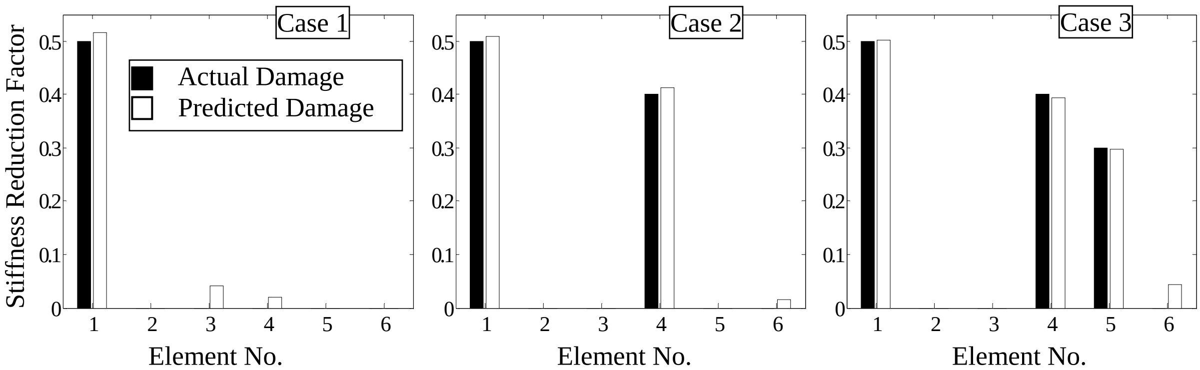


Figure 3.2. Actual and predicted damage (stiffness reduction factors) for the different damage cases.

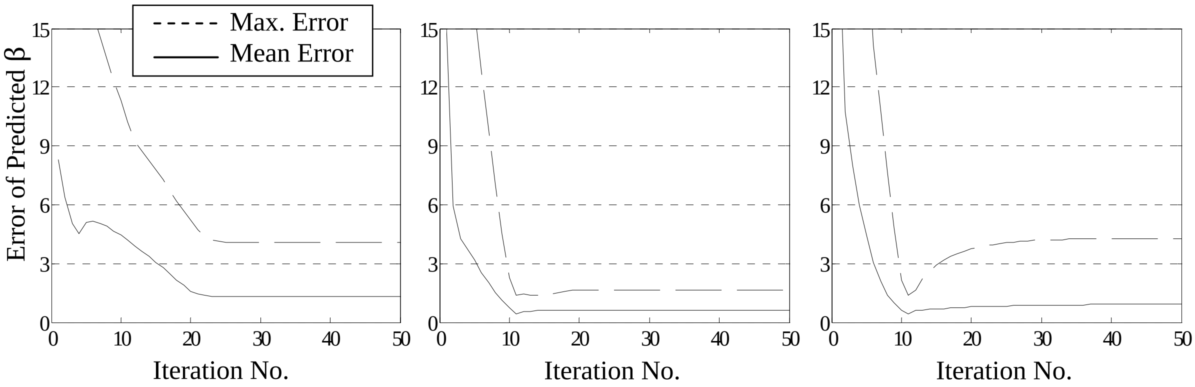


Figure 3.3. Accuracy improvement of the predicted damage (stiffness reduction factors) during the iterative model updating.

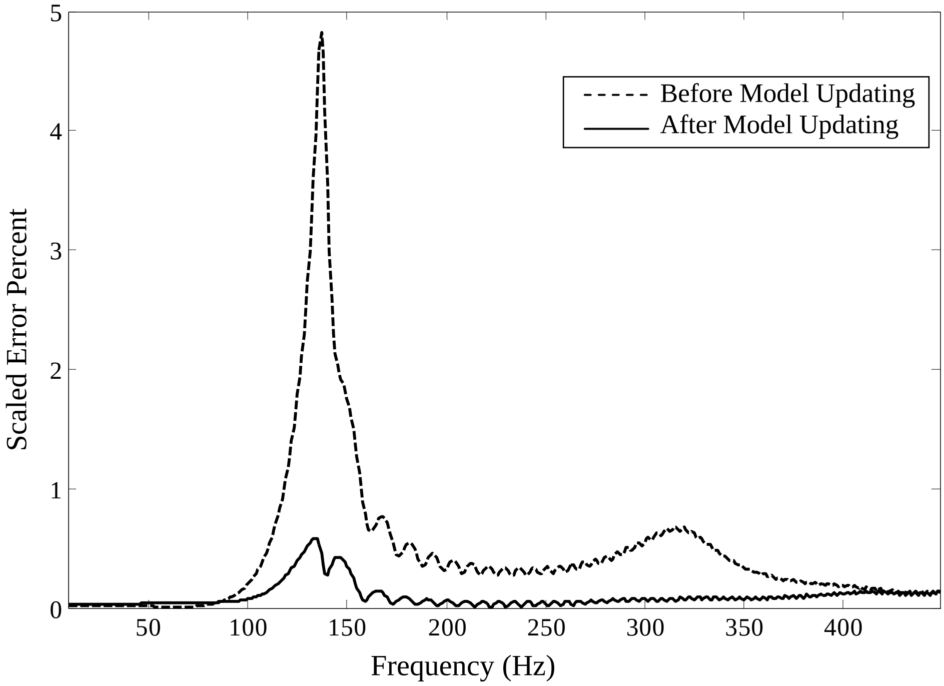


Figure 3.4. Comparison of the relative error of simulated displacement frequency response ($\mathbf{X}_\omega$) before and after the updating model.

4. CONCLUSIONS

In this paper an extension to model updating method has been presented to detect damage of a structure based on the inverse solution of equations of motions of structural system in frequency domain. It is assumed that the acceleration responses of the damaged structure can be measured at the selected DOF's and some of the frequencies and damping ratios of the damaged structure can be identified using the available SI methods. Assuming the viscous linear damping, the damping matrix and unmeasured acceleration response of the damaged structure has been written based on its modal properties. The unknown modal properties of the damaged structure have been initially substituted with modal properties of the analytical model in the first step of model updating procedure. The model parameters and the unknown simulated response of the damaged structure have been updated using an iterative procedure. In each iteration, the velocity and displacement frequency responses are estimated using the measured and simulated acceleration responses and the model parameters are calculated using least square method. The results of the model updating method have been presented for a two-dimensional truss to show the robustness of the method. The sensitivity of the method to the frequency range of the input force is discussed. The sweeping harmonic input is employed to overcome the high sensitivity of the input force to the frequency range in the practical cases. The appropriate criteria for the selection of the frequency point for the model updating procedure are explained. Three damage cases have been presented and it has been shown that the proposed method can predict all damage (stiffness reduction factor) locations and severities accurately. It has been shown that the maximum error of predicted stiffness reduction factor is less than five percent of the initial stiffness. Also, it is shown that the iterative model updating process has been converged in the maximum 20 iterations for all damage cases. One of the advantages of the present method is that the COV the predicted parameters are very low. The error of the simulated responses at the unmeasured DOF's are presented and it has been shown that their accuracy will improve during the model updating process.

AKCNOWLEDGEMENT

The research conducted by the authors has been funded by the International Institute of Earthquake Engineering and Seismology (IIEES) under Grant Number 7137. This support is gratefully acknowledged. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the funding body.

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