JOINT TIME-FREQUENCY ANALYSIS OF SEISMIC RECORDS

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15 WCEE
LISBOA 2012

SUMMARY
The spectral characteristics of transient signals are generally described by the classical power spectrum. This latter spectrum is usually calculated from the Fast Fourier Transform (FFT). Unfortunately, such approach provides no temporal information and is unsuitable for examining seismic records, which exhibit transient nonstationary behaviour. A classical approach to obtaining the desired time-frequency information contained in a seismic record is to use a Short-Time-Fourier-Transform (STFT), which gives rise to a spectrogram (SP). The uncertainty principle precludes to simultaneously obtaining arbitrary fine resolution in both time and frequency. The Cohen's class, of time-frequency energy distributions has been developed in recent years. This class includes the Wigner-Ville (WV), Choi-Williams (CW), reduced interference distribution (RID) and Adaptive Optimal kernel (AOK). Each possesses certain advantages and disadvantages. The aim of this paper is to compare the performance of these time-frequency distributions when applied to seismic records associated with seismic data collected from free-field stations.

Keywords: *Joint Time-Frequency, Seismic Records, accelerograms, Strong Motion*

INTRODUCTION

Fourier analysis has been used for decades for representing the plane of frequency in seismic signals. The Fourier spectrum provides the content of frequency in time series; nevertheless the identification of temporal location of maximum frequency, as well as, the change of frequencies in time is not defined in the spectrum aforementioned. Time Frequency distributions map a one-dimensional signal into a two-dimensional function of time and frequency, and describe how the spectral content of the signal changes with time. Furthermore Time-Frequency distribution shows how the energy of the signal is distributed over time and frequency domain simultaneously. (Boashash, 2003). The time frequency analysis dates from the early work of Wigner (Wigner, 1932). This work does not look like a direct consequence of a Spectrogram improvement, but as a quantum mechanics problem joining momentum with position (like time and frequency in signal analysis). (Cano, 2008). The Time-Frequency analysis of a signal is nounique in that different time-frequency methodologies yield different time-frequency distributions. In other words, many different time-frequency representations can be associated with the same data (Huerta et al, 2000).
The Seismic waves recorded in accelerograms exhibit clearly nonstationary characteristics. It shows a time-evolving frequency composition due to the scattering of propagating seismic waves. The transient signals of earthquakes are nonstationary in sense that the frequency amplitude vary with time. The nonstationary characteristics of earthquakes are related with the time of the intensity of the ground motion, and the variation in time of the frequency content. The content of frequency in an earthquake is a complex phenomenon; it involves the intensity of the ground motion, the magnitude of the earthquake, the source, and path and site effect.
Appropriate numerical results of time frequency distributions previously mentioned are given for earthquakes recorded in free-field. A comparison among the different Time frequency distribution has been carried out on horizontal components of two different seismic records.

1. JOINT TIME FREQUENCY ANALYSIS

The Time-Frequency Representation (TFRs) of signals map a one-dimensional signal of time *x (t)*, into a two-dimensional function of Time and Frequency *T (t,ω)* (Hlawatsch, and Boudreaux, 1992). Any time-frequency representation that fulfill the Eqn 1.1. Is considered a Time-Frequency Energy Distribution (TFED) *P (t,ω)*. (Quian, and Chen, 1996)

$$|x(t)|^2 = \int_{-\infty}^{\infty} P(t,\omega)d\omega$$

$$|S(\omega)|^2 = \int_{-\infty}^{\infty} P(t,\omega)dt \quad (1.1)$$

Where:

$|x(t)|^2$: Instantaneous Energy
$|S(\omega)|^2$: Intensity per unit frequency at frequency ω

If the marginal conditions are satisfied, the total energy of the signal is:

$$|S(\omega)|^2 = \int_{-\infty}^{\infty} P(t,\omega)dt \quad (1.2)$$

1.1 Cohen's Distribution

In 1989, Cohen proposed (Cohen, 1989) that any time-frequency representation that fulfills equations (1.1 and 1.2) can be represented by the Eqn. 1.3. The Cohen Distribution can be expressed as the convolution of a kernel and a time varying autocorrelation function.

$$P(t,\omega) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j\theta t - j\tau\omega + j\theta u}\phi(\theta,\tau)x^*\left(u-\frac{1}{2}\tau\right)x\left(u+\frac{1}{2}\tau\right)du\,d\tau\,d\theta \quad (1.3)$$

Where $\phi(\theta,\tau)$ is the kernel, which the Cohen Class Distribution is independent of time and Frequency. $x^*(t)$ is the complex associate of the real signal *x (t)* of the real signal, the analytical is defined as:

$$x(t) = x_r(t) + jHT(x(t)) \quad (1.4)$$

Where *HT(x(t))*: Hilbert Transform of *x(t)*

The complex analytical signal is used to avoid the aliasing problem in the time-frequency plane, it satisfy the Shanon's sampling theorem.

Table 1.1. Kernel for some Cohen's Distribution of Joint Time-Frequency

Time Frequency Representation	Kernel $\phi_x(\theta,\tau)$
Spectrogram (SP)	$\int_{-\infty}^{\infty} h^*\left(u-\frac{1}{2}\tau\right)h\left(u+\frac{1}{2}\tau\right)e^{-j\theta u}du$
Wigner-Ville (WVD)	1
Choi-Williams (CWD)	$e^{-\frac{\theta^2\tau^2}{\sigma}}$
Reduced Interference (RID)	2D low pass filter in (θ,τ) space
Radial Gaussian Kernel (RGKD)	$e^{-\frac{r^2}{2\sigma^2}(\psi)}$
Adaptive Optimal Kernel (AOKD)	$A(t:\theta,\tau)$ applied to the optimization of $e^{-\frac{(\theta^2+\tau^2)}{2\sigma^2}(\psi)}$

1.3 Joint Time-Frequency Representation

The Joint Time-Frequency Representation can be used to assess the change of frequency content in time of a multicomponent signal such as earthquake data. There are many Joint Time-Frequency Representations. Theoretically, it is possible to construct infinite Time-Frequency Representation by selecting different frequency kernels and independent time.
Figure 1.1 shows two different cases of signals with Time-Frequency representation in each particular case. The sinusoidal signal has a dominant frequency; it can be observed by means of each time-frequency representation. In the case of monocomponent signals the instantaneous frequency (IF) is equal to the first derivative of the phase. Equation.1.5. (Quian and Chen, 1996).

$$IF(t) = \frac{d\varphi(t)}{dt} = \frac{x(t)d\frac{HT(x(t))}{dt} - d\frac{x(t)}{dt}HT(x(t))}{x^2(t) - HT^2(t)} = \frac{d}{dt}\tan^{-1}\frac{HT(x(t))}{dt} \tag{1.5}$$

where:

x(t): Real signal,
*$\phi(t)$:*phase of signal
HT(x(t)): Hilbert Transform of *x(t).*
For multicomponent signals, like earthquake signal, the instantaneous frequency of the signal has a bandwidth; therefore the weighted average of IF is defined as equation 1.6. (Quian and Chen, 1996).

$$IF(t) = \int_{-\infty}^{\infty} \left(\frac{d\varphi(t)}{dt} \frac{|x(t)|^2}{\int_{-\infty}^{\infty} |x(t)|^2 dt} \right) dt \tag{1.6}$$

The concept of IF is interwoven with study of non-stationary signals, it serves as a tool to compute the frequencies present in a signal at each time instant *t.* (Spanos, Giaralis and Politis, 2007)

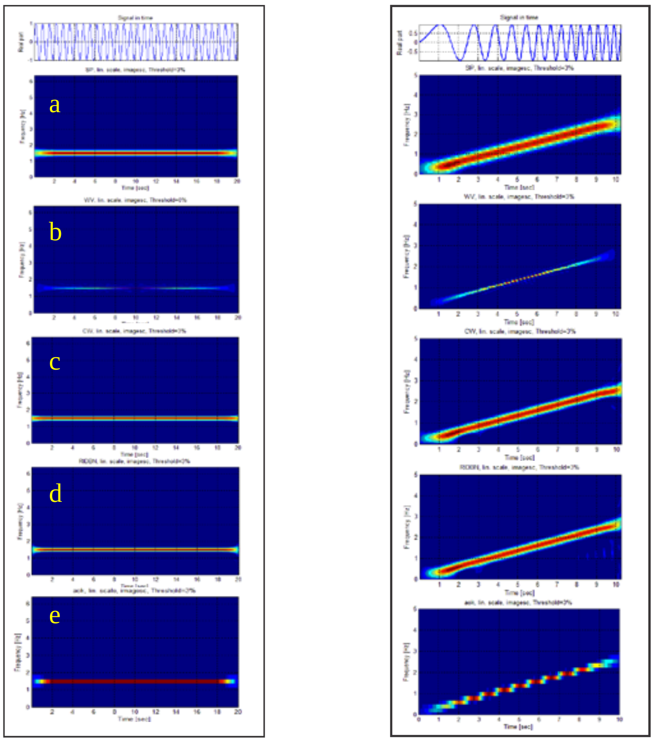

Figure 1.1. Join Time Frequency Representation of a sample signal (Left: Sinusoidal with predominant frequency - Right: Sinusoidal with modulated frequency) .a.) Spectrogram, b) Wigner-Ville, c.) Choi-Williams, d.) RID Reduce Interference Distribution and d.)AOK

2. JOINT TIME FREQUENCY ANALYSIS OF SEISMIC RECORDS

2.1 Data

The accelerations time series used in this analysis were recorded on free-field stations of the Puerto Rico Strong Motion Program (PRSMP). Only free-field stations seated on soil or rock were used in this study. Alluvial deposits, limestone, and beach deposits are the predominant type of soils in the study zone. In the first stage, the accelerograms were analyzed following the well established data processing procedure for acceleration strong motion records through volume I to III, and the Power Spectral Density (PSD's) was computed. The information of earthquakes used in this study is listed below in table 2.1. Among the strong motion stations deployed around the Puerto Rico Island, only free-field stations were used in this analysis. In this particular case the stations chosen were: MY10 (Mayagüez, PR, Lat. 18.21501N – Long. 67.13873W) and AR02 (Arecibo, PR, Lat. 18.47089N – Long. 66.74028W).

Table 2.1. Information of Earthquakes used

Earthquake Name	Location		Depth (Km)	Magnitude
	Lat N	Long. W		
Puerto Rico, May 16, 2010	18.400	67.070	113	5.8
Puerto Rico, Dec 17,2011	18.172	67.371	17	5.1

2.2 Analysis of Joint Time-Frequency

The Analysis of Joint Time Frequency was carried out on horizontal components of the earthquakes aforementioned. The fig. 2.1 shows at the top time series of the December 17, 2011 Puerto Rico Earthquake, Mw=5.1., The SP (fig. 2.1a) shows a concentration of time frequency amplitude in 3 Hz, the resolution in frequency is more than acceptable, however SP has an unacceptable resolution for identification of frequencies changing in time. The WV (fig. 2.1b) shows that frequencies are crossed by the case of multi-component signal, which is a well known characteristic of the earthquake signals. The interpretation of changes in time and frequency can be difficult with the WV distribution. CW(fig. 2.1c), and RID(fig. 2.1d) show clearly the change in frequencies, from low to high frequencies in the time of maximum time-frequency amplitude, however cross term and synchronization effects may lead to misinterpreted features as part of the signal. The CW distribution shows vertical lines related with interferences in resolution of frequency. The RID has a good time-frequency resolution; nevertheless interference lines appear in both, time and frequency. The AOK (fig. 2.1e) clearly shows only one significant maximum amplitude at 3 Hz, it that can be associated with fundamental vibration frequency of site. The other features shown in the AOK T-F plane may be associated to very short time duration transients of low and moderate amplitude.

The fig.2.2 shows at the top time series of the May 16, 2010 Puerto Rico Earthquake, Mw=5.8., The SP (fig. 2.2a) shows a concentration of time frequency amplitude in 7 Hz, the WV (fig. 2.2b) increases resolution in frequency, nevertheless the interference makes almost impossible the interpretation of frequencies in a multi-component signal. CW (fig. 2.2c) shows concentration of time frequency amplitude in 7 Hz and 9 Hz, those spots of concentration in energy are clearly identified, the vertical lines are related with the interference in resolution in the frequency band. RID (fig. 2.2d) shows a trend of change in frequencies, from low to high frequencies in the time of maximum time-frequency amplitude, some dispersive characteristics can be interpreted, the cross term and synchronization effects may lead to misinterpreted features as part of the signal. The AOK (fig. 2.2d) shows concentration of energy from 7 Hz to 9 Hz, and the change of frequencies are clearly interpreted in time. In this case the suppression of numerical artifacts is without a doubt evident regarding the distributions beforehand analyzed.

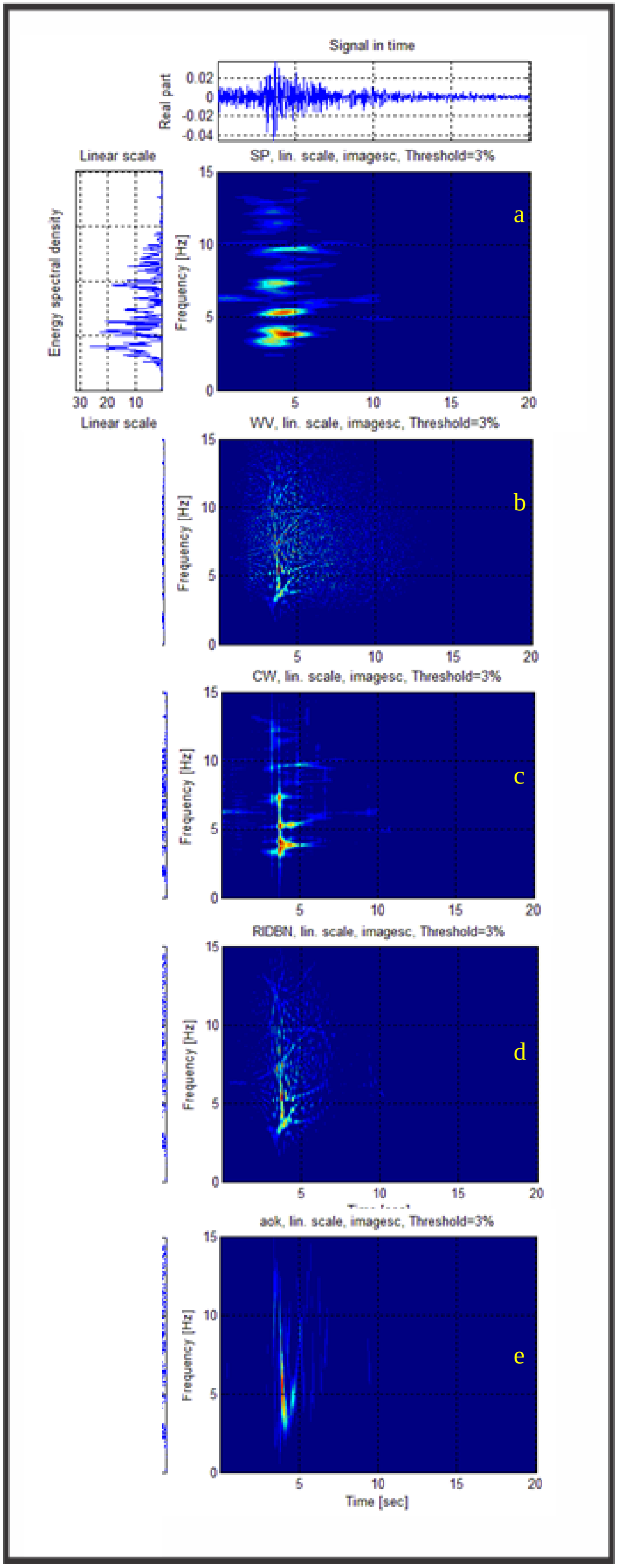

Figure 2.1. Join Time Frequency Representation. Puerto Rico Earthquake, December 17, 2011, Mw=5.1, AR02 station a) SP, b) WVD, c) CWD, d) RID, e) AOK

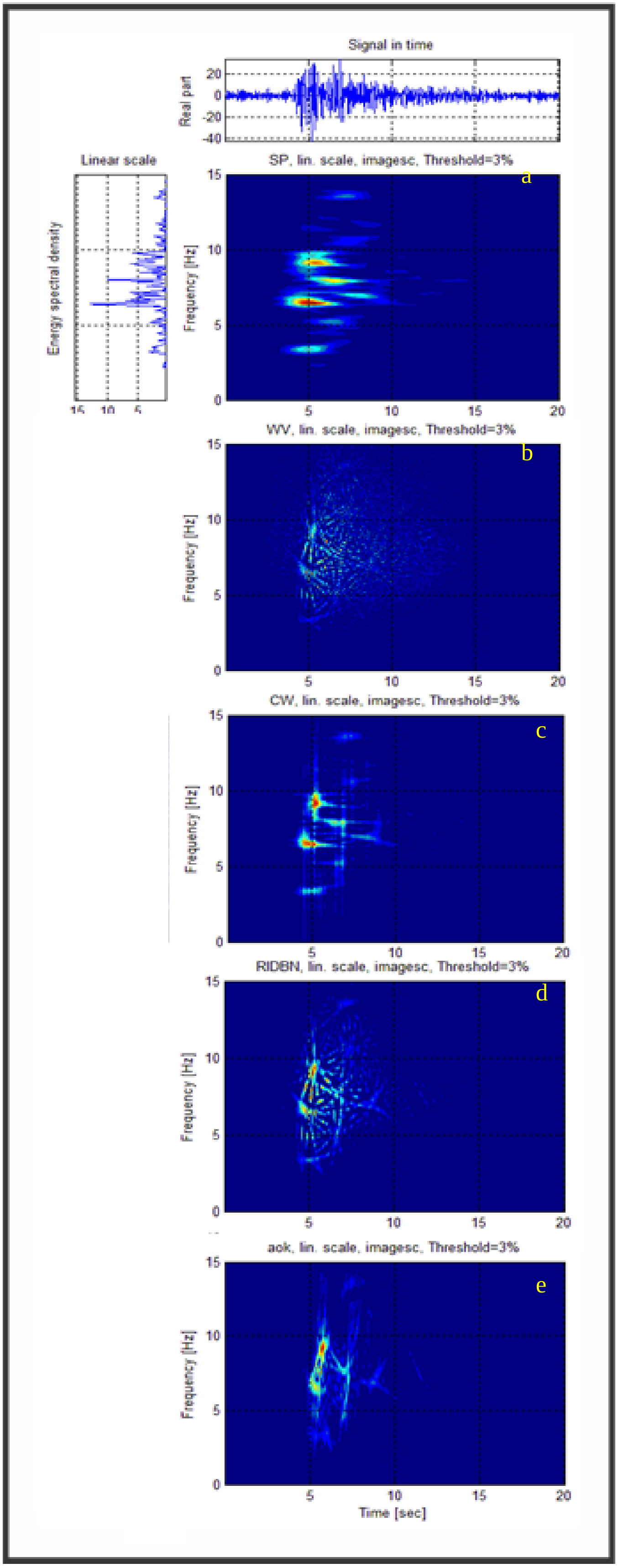

Figure 2.2. Join Time Frequency Representation. May 16, 2010, Mw=5.8, MY10 station a) SP, b) WVD, c) CWD, d) RID, e) AOK

3. ANALYSIS AND CONCLUSIONS

The Joint Time-Frequency analysis was focused on using five of the best known and common used TFD. From these, the Spectrogram (SP), the Wigner-Ville distribution (WVD), the Choi-Williams distribution (CWD), the reduced interference distribution (RID), and the adaptive optimal kernel (AOK).

The frequency resolution of the SP is limited, and restricts its use for highly resolve time-frequency characteristics of multi-component signals such as earthquake data. The resolution of WVD is good, it has advantages over SP; introduces the presence of interference terms (it can be mitigated using RID). The CWD shows a good resolution in time-frequency, it shows interference in the frequency band represented by vertical lines. The RID has a good time-frequency resolution, but some interference terms appear in time and frequency. The AOK has a good resolution in time-frequency, in case of multi-component signals, some interference appear depending of the windowing used.

Due the uncertainty principle, a perfect resolution in time and frequency simultaneously cannot be obtained, thus if increases the time resolution the frequency resolution is bad and the other way around.

The cases showed are affected by the site effect, a different technique to remove the site effect will be considered to improve the analysis.

AKCNOWLEDGEMENT

We express our thanks to Puerto Rico Strong Motion Program. The University of Puerto Rico at Mayagüez, for the computational facilities for the data processing and analysis. We thank additional reviews by anonymous reviewers to further improve the manuscript.

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