

# Experimental assessment of PID control for a uniaxial shake table

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## SUMMARY:

Accurate actuator displacement tracking is essential in the seismic verification of structures on shake tables. This paper summarises an experimental study on the effects of proportional, integral, derivative and delta pressure gain on the transfer function of a uniaxial shake table. The study shows that the correct proportional gain component can deliver excellent response in the 0-10 Hz frequency range. The delta pressure gain can simultaneously limit response in the 10-30 Hz frequency range to give a table transfer function closest to unity. The derivative and in particular integral gains have minimal influence on the transfer function. The study concludes that optimal gain settings alone are insufficient to ensure the desired accelerations are achieved. The command excitation should be iteratively updated using the transfer function to minimise error in the single degree of freedom (SDOF) response spectra.

*Keywords: shake table, control*

## 1. INTRODUCTION

Shake tables are a valuable tool in the seismic verification of civil engineering structures. They can directly subject structural models to dynamic excitations resembling those expected to be encountered in a real earthquake event. The full dynamic loading property makes shake table testing an attractive tool in the research field.

There are two significant challenges to the shake table technique – dynamic similitude scaling requirements, and the control of table motions to ensure target accelerations are reproduced. Dynamic hydraulic equipment generally has a lower force capacity than static apparatus of similar cost, so shake table systems have traditionally been assembled with capacity too low to test full scale civil engineering structures. Instead small scale replica models are constructed and tested, but maintaining full dynamic similitude with the real structure of interest is challenging.

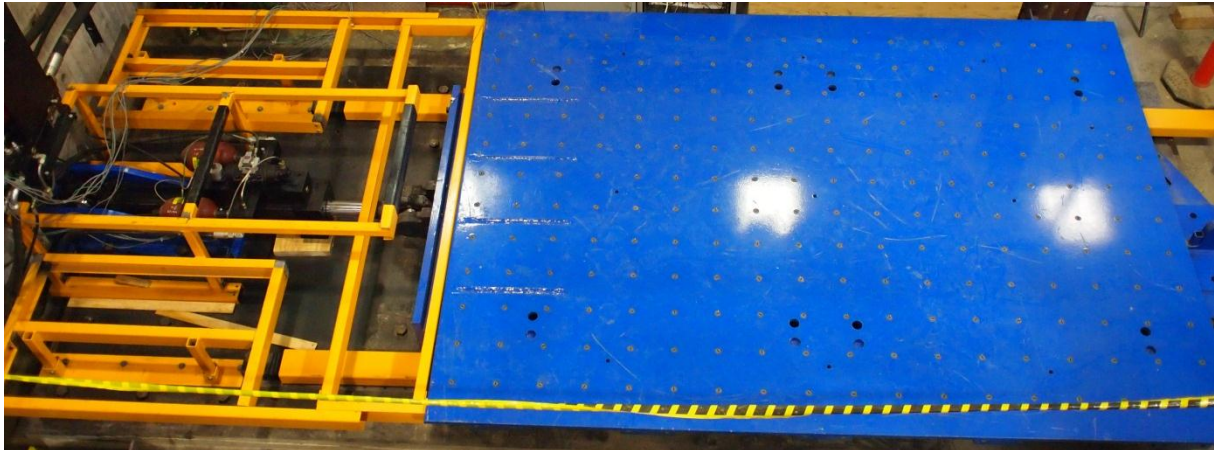
Traditionally shake table response has been controlled by Proportional Integral Derivative (PID) algorithms. Under the PID scheme the controller responds to the difference between the table command and feedback displacements, the displacement error, according to predefined ratios of the magnitude (proportional), integral, and derivative of the error. As the three gains in the conventional controller are predefined prior to test commencement the control scheme is considered out of real time (Crewe 2001). There is an interaction between the specimen response and shake table control, particularly under conditions close to the force capacity of the actuator (Trombetti 1998). The purpose of experimental verification is to assess nonlinear softening behaviour of a real structure. To do so the largest possible specimen for the available hardware is tested, making accurate acceleration reproduction difficult using PID control.

The purpose of this paper is to explore the experimentally driven PID tuning process for a uniaxial shake table and qualitatively assess the nature and influence of the PID and delta pressure gain settings. The University of Auckland has limited experience with shake tables and control so has little

internal knowledge to draw on. The Shore Western controlled shake table system relies on PID control by default, with a subsequent Real Time Adaptive Control (RTAC) function available (Shore Western Mftg 2011). Thus a thorough understanding of the qualities and limitations of PID control must be obtained before progressing to more advanced control algorithms.

## 2. THE SHAKE TABLE SYSTEM

A shake table provides a rigid platform to transmit an absolute acceleration to a mounted structural model and thus simulate the shaking of the ground during an earthquake. The University of Auckland has a uniaxial shake table excited by a single servo-hydraulic actuator, as presented in Figure 1.



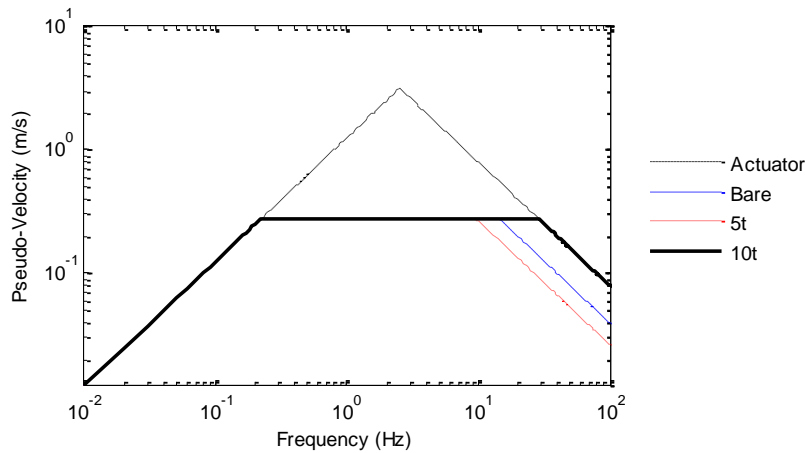
**Figure 1.** The shake table setup

The actuator and hydraulic service manifold are supplied by Shore Western Manufacturing and controlled by proprietary SC6000 software on a computer (Shore Western Mftg 2012). The 913 series double ended actuator has a three stage servo-valve, stroke of  $\pm 200$  mm and shaft area of  $12,664 \text{ mm}^2$ . Hydraulic power is supplied by an MTS506.41 pump at up to 20.7 MPa pressure and 210 litres per minute flow. This combination gives a maximum actuator force of 262 kN and velocity of 0.276 m/s.

The actuator is attached to a concrete strong floor reaction mass via steel bracket. The table platform is 3.6 m long and 2.4 m wide, weighing approximately 5 t. The platform is mounted on linear bearing rails that allow a maximum payload of 10 t acting at 2.5 m above the platform.

### 2.1. Hydraulic Capacity

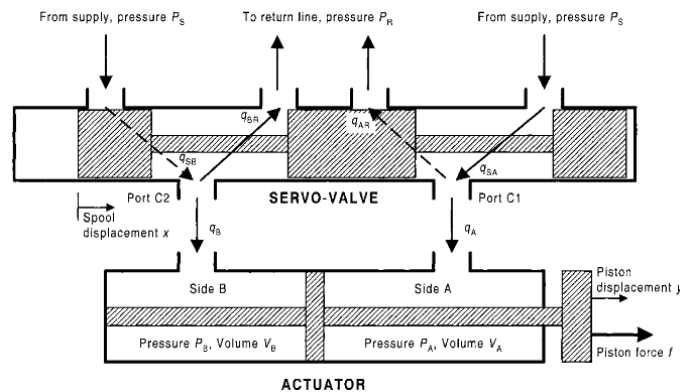
The theoretical frequency response capacity of the table can be expressed as the pseudo-velocity limit of the system under sinusoidal excitation for a given frequency. Figure 2 reveals at frequencies below 0.2 Hz capacity is governed by the stroke of the actuator. From 0.2 Hz to 10-12 Hz (depending on payload mass) the flow rate of the hydraulic supply limits performance. Above this threshold the force capacity of the actuator limits the accelerations that can be applied. It is apparent that the performance of the system is severely limited by the flow capacity of the pump. The tripartite relationship represents ideal response to pure sinusoidal excitation of a unique frequency, but is difficult to extend to multi-frequency excitation with imperfect actuator control.



**Figure 2.** Shake table pseudo-velocity capacity with various payload mass

## 2.2. The Three Stage Servo-Valve

To understand the control process the mechanical and electrical functions of the three stage servo-valve arrangement of the actuator must be considered. A drive current as determined by the controller is sent to a torque motor which culminates in movement of the spool to allow flow in or out of each side of the actuator. The nature of the spool is best demonstrated in Figure 3, where it can be seen that the strategically positioned narrow sections of the spool can change the supply, return and port orifice areas on both sides of the actuator. Thus the position of the spool simultaneously dictates flow into one side of the actuator and out of the other to change the position of the table.



**Figure 3.** Interaction between servo-valve spool and actuator chamber (reproduced from Williams 2001)

This process is controlled in the SC6000 software by two PID loops, as depicted in Figure 4. The inner loop is tuned to optimise the response of the spool to a given drive current. The inner loop control is tuned once for the servo-valve and is not influenced by the varying table, payload and excitation dynamics. A second, outer control loop is implemented to dictate how much current is sent to the servo-valve to achieve the desired displacement. The outer loop response is dependent on the table system dynamics, and must be tuned or optimised for each specific test. In addition to the PID controllers a delta pressure gain is incorporated. The delta pressure transducer measures the pressure difference across the two sides of the actuator. The delta pressure gain ( $dP$ ) adds a fraction of the transducer feedback to the drive signal to compensate for the oscillations in the oil column at its natural frequency.

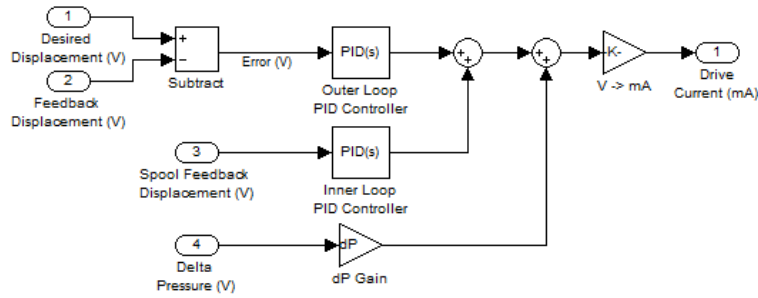


Figure 4. Schematic of the control system

### 2.3. The Outer PID Control Loop

The PID controller responds to the instantaneous difference between the commanded and actual feedback displacements. In theory the Proportional (P) component provides accurate tracking, the Integral (I) component rejects steady state offsets due to load disturbance, and the Derivative (D) component compensates for phase lag (Williams 2001). The basic form of the PID algorithm in the time domain is

$$u(t) = K_p e + K_d T_d \frac{de}{dt} + K_i \frac{1}{T_i} \int e dt \quad (2.2.1)$$

where  $u(t)$  is the output command signal,  $e$  the feedback error equal to the command displacement  $d_c$  minus the table displacement  $d_t$ ,  $K_p$  the proportional gain in decibels,  $T_d$  the derivative gain in seconds, and  $T_i$  the integrator gain in 1/seconds. The  $K_d$  and  $K_i$  terms are introduced to allow independent adjustment of the three gains. A schematic implementation Equation 2.2.1 is shown in Figure 5.

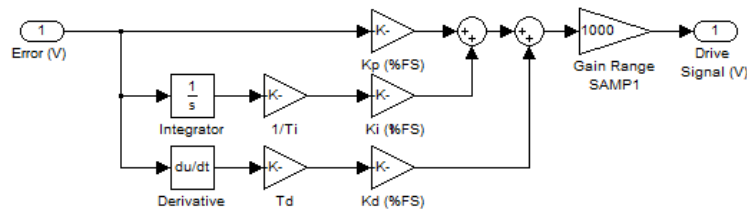


Figure 5. Schematic of outer PID control loop

### 3. TUNING ASSESSMENT METHODOLOGY

Servo-hydraulic equipment manufacturers recommend tuning actuators to a square wave input (Trombetti 1998, Shore Western Mftg 2011). The proportional gain is increased until slight overshoot is achieved. Oscillations about the target displacement can be damped with an increase of the derivative gain and drift compensated for with increased integral gain. Finally the delta pressure gain should be increased to remove any oscillation in the actuator pressures. This process is shown in Figure 6. Unfortunately square wave tuning has been shown to be inadequate for earthquake engineering applications with wide band frequency content – a more scientific and systematic approach is required (Trombetti 1998).

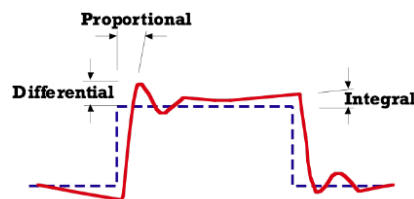
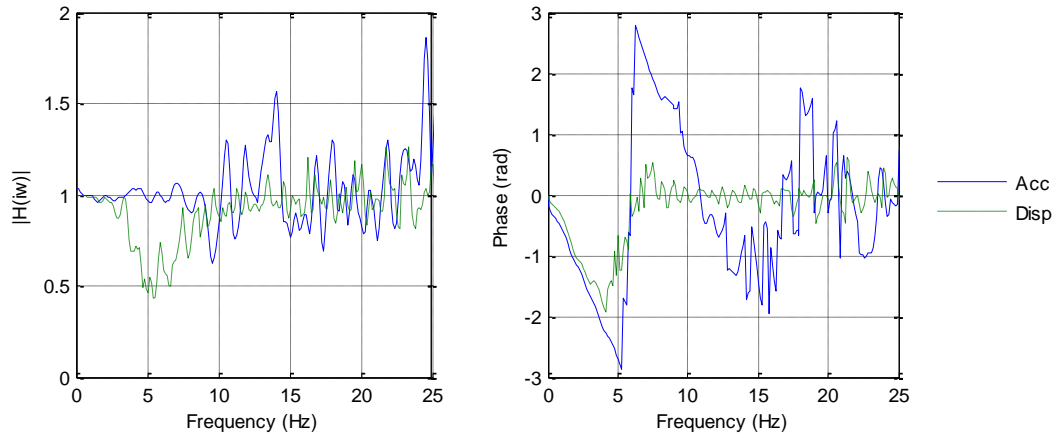


Figure 6. Manufacturer recommended PID tuning process (reproduced from Shore Western Mftg 2012)

Both the theoretical pseudo-velocity tripartite plot and square wave response are inadequate in describing the performance of the servo-hydraulic actuator and table system. The frequency domain transfer function and corresponding phase plot are more suitable. The transfer function quantifies the ratio of measured table response to desired command across the frequency range of interest. A transfer function of constant magnitude of unity is ideal. The phase plot refers to the phase lag at each frequency and is quantified from the ratio between real and imaginary components of the transfer function. A phase lag of zero across all frequencies is ideal. The experimentally determined displacement and acceleration transfer function and phase estimates at optimal bare table tuning are presented in Figure 8.

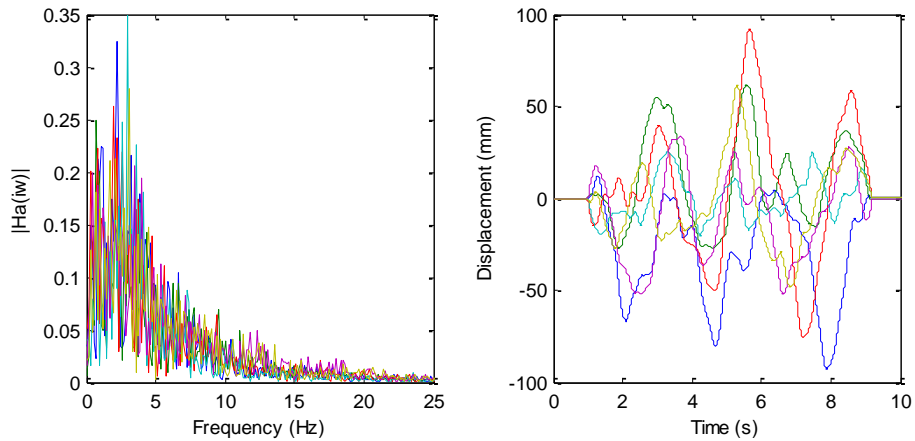


**Figure 8.** Optimal experimentally determined displacement and acceleration table transfer functions and corresponding phase plots

The difference between displacement and acceleration transfer functions can be attributed to the flexibility of the actuator support. Support movement allows the actuator ends to achieve target displacements relative to each other while curtailing absolute accelerations. For this reason the table transfer function is derived from accelerations.

The table transfer function response can be estimated from the ratio of response to command amplitude for a series of discrete frequency sinusoidal commands. However Trombetti (1998) showed that the same curve can be generated from a single excitation derived from white noise. Luco et al (2010) recommend using twice integrated white noise accelerations as tuning displacement histories scaled to the actual test amplitude. Initial attempts to tune at 0.25 g peak white noise accelerations were limited by the force capacity of the actuator in the frequency range above 25 Hz. This was alleviated by scaling the accelerations in the frequency domain to have a Fourier amplitude distribution similar to a real earthquake. Six different command histories were generated and are shown in Figure 9. The transfer function for the table's response to each excitation was calculated and then averaged across the six records to give the transfer function for that tuning.

The optimal tuning combination was derived as that with the transfer function closest to unity. A single Root Mean Square (RMS) of the transfer function error as recommended by Luco (2010) is inadequate due to fluctuations within the seismic frequency range of 0-30 Hz. Instead the RMS errors for banded frequency ranges are compared as demonstrated in Figure 11 and the best gain combination intuitively chosen.



**Figure 9.** Tuning excitation histories – a) Fourier amplitude of scaled white noise accelerations, and b) desired displacement time histories.

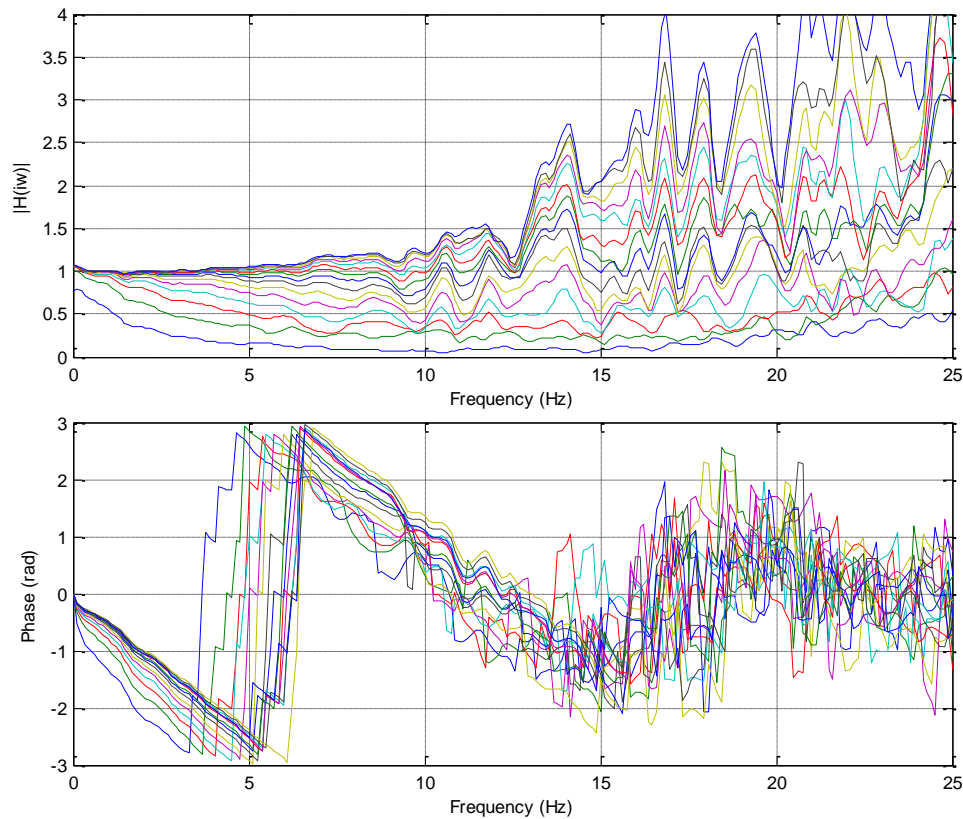
### 3. SYSTEMATIC EXPLORATION OF GAIN PARAMETERS

A systematic investigation on the effects of the four gain parameters was undertaken to obtain a better understanding of the nature and interaction of the gains, and to determine the optimal bare table tuning for use as the default setting.

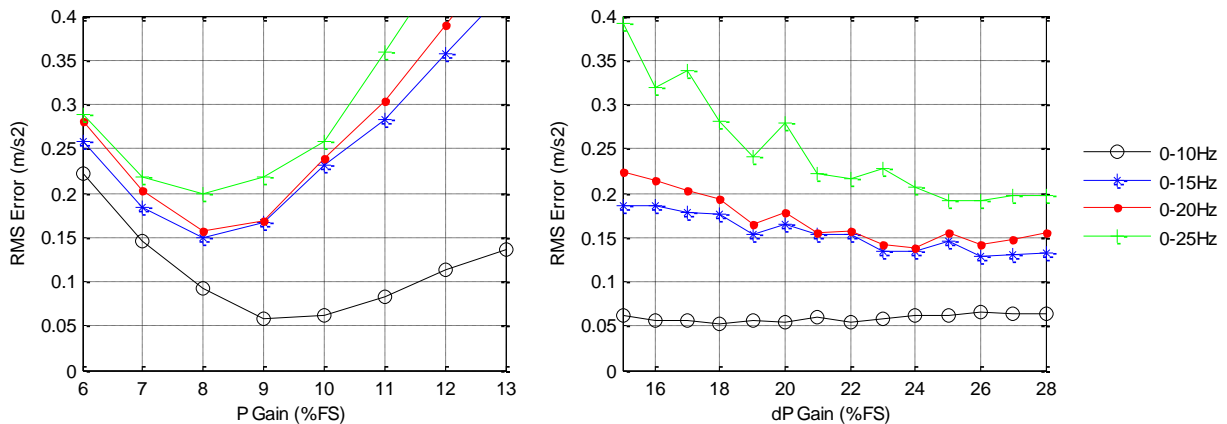
#### 3.1. Proportional Gain

An increase in the proportional gain increases the response of the actuator to the magnitude of the displacement error. Figure 10 shows the impact of increasing proportional gain on the frequency response. The lowest magnitude transfer function plotted corresponds to a proportional gain of 1% full scale, and the largest to 15%. Intermediate transfer functions correspond to proportional gains of incremental 1% increases. In the phase plot, 1% P gain has the fastest rate of increase in phase lag with frequency and 15% the slowest.

Figure 10 shows that an increase in the proportional gain increases the frequency response across the entire frequency range of interest. As the proportional gain is increased the transfer function increases approximately radially about a centre point at 0 Hz. An increase in proportional gain reduces the rate of phase difference increase in the 0-5 Hz range. Figure 11a shows the effects of varying proportional gain on the RMS error over the 0-10, 0-15, 0-20, and 0-25 Hz frequency bands.



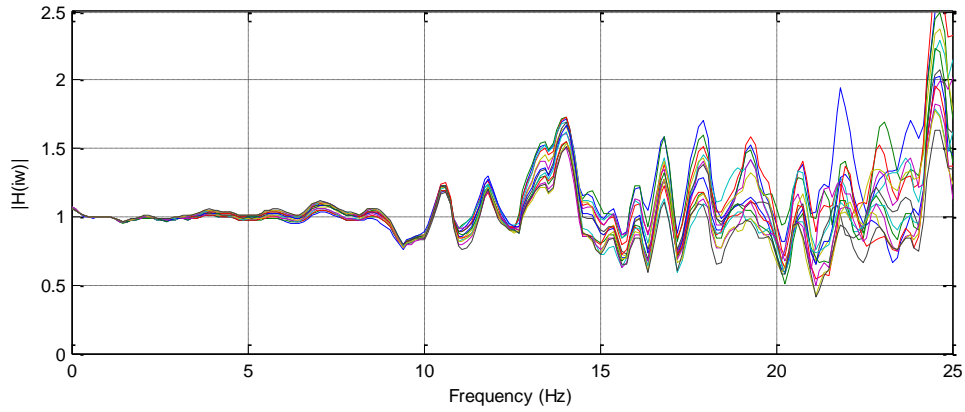
**Figure 10.** Effect of proportional gain on table transfer function and phase – gain increased in integer increments from 1 to 15% full scale



**Figure 11.** Variation in root mean square error of acceleration transfer function over various frequency bands – a) varied proportional gain, and b) varied delta pressure gain

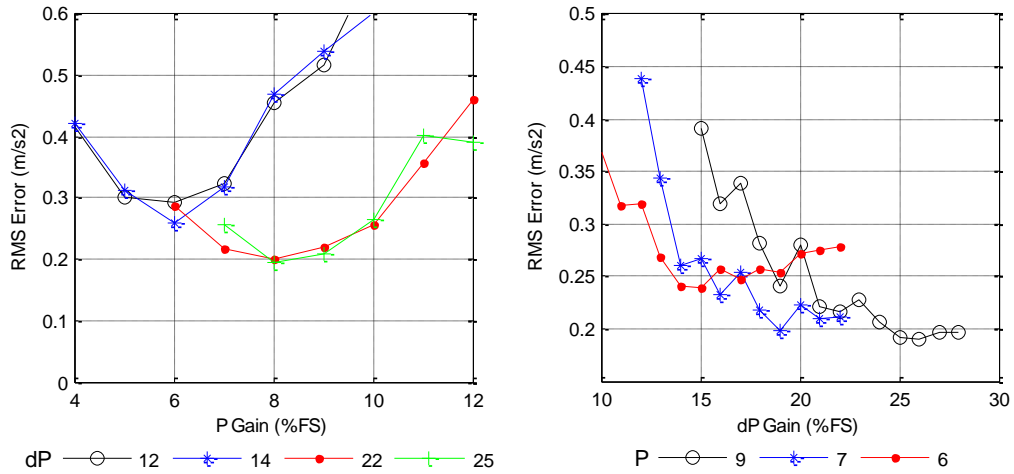
### 3.2. Delta Pressure Gain

An increase in delta pressure gain decreases the magnitude of the transfer function, as shown in Figure 12. There is no effect below 10 Hz but above this threshold the reduction of the transfer function increases with frequency. Delta pressure gain does not affect the phase lag.



**Figure 12.** Effect of delta pressure gain on table transfer function - gain increased in 1% increments from 15 to 28% full scale

The delta pressure and proportional gains do not alter the transfer function independently. An increase or decrease of the proportional gain needs to be offset by an increase or decrease of the delta pressure gain. This interaction is demonstrated in Figure 13, where the RMS acceleration transfer function errors are plotted for different proportional and delta pressure combinations.

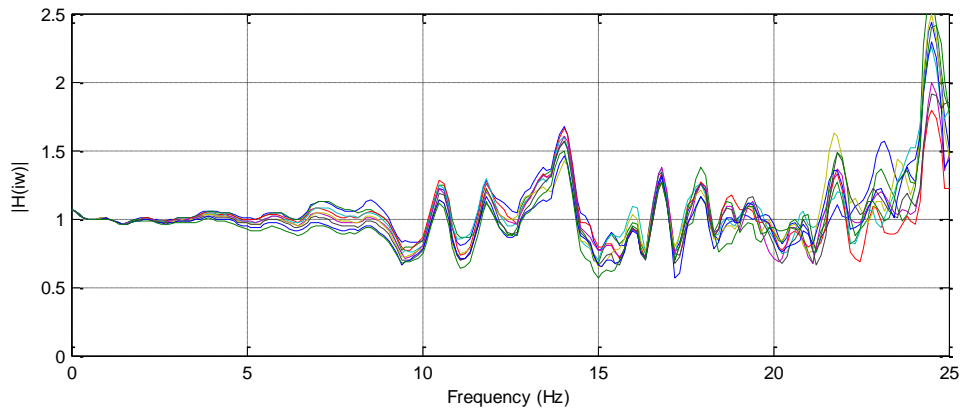


**Figure 13.** Interaction between proportional and delta pressure gains on RMS acceleration transfer function error over the 0-25 Hz range

### 3.3. Integral and Derivative Gains

The integral and derivative gains have less effect on the accuracy of the transfer function. In fact the integral gain prompted no discernible change to the transfer function over the range of interest. An increase in the derivative gain decreases the magnitude of the transfer function for frequencies above 2.5 Hz as shown in Figure 14. Neither integral or derivative gain alters the phase lag.





**Figure 14.** Effect of derivative gain on table transfer function - gain increased in 1% increments from 0 to 9% full scale

### 3.4. Recommended sequential tuning process

The tuning process begins with both integral and derivative gains set to zero, and the delta pressure gain at a mid-range value. First the proportional gain should be increased to achieve unity in the transfer function below 5 Hz, and slightly greater than unity between 5 and 10 Hz. Second, the derivative gain should be increased to achieve unity in the 5-10 Hz range. Third, the delta pressure gain should be increased to decrease the magnitude at frequencies above 10 Hz to unity. If the delta pressure gain is increased significantly in the third step the process should be repeated to check the effects of the interaction between the proportional and delta pressure gains.

The sequential tuning process produced the optimal gain settings of 8.5% proportional, 0% integral, 4% derivative, and 25% delta pressure. This combination will remain the default tuning setting for the table.

Researchers are concerned by the effect of tracking error on the response of the mounted structural model rather than the transfer function. The optimal transfer function still has regions not equal to unity, which influences the SDOF spectral accelerations. The accuracy of the spectral accelerations can be improved by iteratively adjusting the input excitations using an updated inverse of the table transfer function (Luco et al 2010).

## 4. FURTHER WORK

This paper summarises the initial setup work for the uniaxial shake table at the University of Auckland. On-going work includes numerical modelling of the system in Simulink (The Mathworks 2011) to facilitate offline tuning of the table. Different adaptive controllers will be investigated numerically to select the most suitable for this uniaxial system. The work will culminate with the control knowledge and software tools being applied to the University's hybrid simulation hardware.

## 5. CONCLUSIONS

An understanding of the function of servo-hydraulic actuators and the nature PID tuning are essential to effectively operate a shake table and achieve the desired acceleration output. Wide band white noise accelerations scaled to a Fourier amplitude distribution similar to measured earthquakes are most suitable for the non-specific tuning of uniaxial shake tables.

Tuning should follow the presented sequence as variation of the proportional, derivative and delta pressure gains change the magnitude of the transfer function in different frequency ranges. The technician must be wary of the interaction between the proportional and delta pressure gains when selecting the optimal tuning combination. The integral gain has no influence on the transfer function so should remain set to zero.

Selecting the perceived optimal gains may not be sufficient to ensure adequate table response. Response should be checked against that desired via the respective SDOF acceleration spectra. If the level of accuracy is insufficient the input displacement can be iteratively adjusted in the frequency domain until the acceleration spectrum error is minimised.

Further work on numerically modelling of the shake table system and exploration of more advanced controllers is required to ensure the potential of the facility is maximised.

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