Estimation of Peak Roof Displacement of Degrading Structures

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SUMMARY:
To use nonlinear static (pushover) analysis procedure in estimating seismic demands of a multi-story building, a set of invariant lateral force distribution is increasingly applied to the multi-degree-of-freedom (MDF) building structure until the roof displacement reaches a target value. Conventionally, this target (peak) roof displacement is estimated by using response of an equivalent single-of-freedom (SDF) system. This approach has been previously shown to be reasonably accurate where structures can be idealized as bilinear systems. Nevertheless, bilinear SDF systems may not be appropriate for degrading reinforced concrete (RC) structures. Therefore, this study investigates whether the concept of equivalent SDF system can still be usefully applied to estimate the peak roof displacement of a degrading structure. A real eight-story RC frame was subjected to a set of 20 earthquake ground motions. Cyclic pushover analysis was used to determine the resisting force characteristic of the equivalent SDF system to include degradation effects. Then, The peak roof displacement estimated from the response of both degrading and bilinear equivalent SDF systems are compared to the "exact" values obtained from nonlinear response history analysis of the MDF frame. The results show that the accuracy of target roof displacement estimation can be improved when the equivalent SDF systems takes into account the effect of degradation.

Keywords: target roof displacement, pushover analysis, seismic demands, degrading structures

1. INTRODUCTION

Nonlinear static analysis procedure, or static pushover analysis, has become a standard practice in building design and seismic evaluation as it requires much less computing time than the nonlinear dynamic procedure. The seismic demands are determined by increasingly applying lateral loads with an invariant force distribution to the multi-degree-of-freedom (MDF) building structure described by nonlinear material properties until the roof displacement reaches a target value estimated by using response of an equivalent single-of-freedom (SDF) system. Estimation of peak (target) roof displacement of a MDF system by using an equivalent SDF system has been accepted in building code, e.g. FEMA-356 (ASCE 2000), and has been shown to be reasonably accurate by many researchers (Chopra *et al.* 2003) where structures exhibit resisting forces that can be idealized as bilinear systems, such as steel moment resisting frames. Nevertheless, reinforced-concrete (RC) structures, in general, are considered as degrading systems, whose stiffness and strength decreases as the structures undergo many cycles of large deformation.

Degradation of RC structures has been studied by several researchers (Otani 1981, Qi and Moehle 1991, Rahnama and Krawinkler 1993) to understand the behavior and its effects on the performance of RC structures during strong earthquakes. The degradation characteristics of an RC structural member are primarily controlled by the ratio of axial load and corresponding axial capacity, amount of transverse and longitudinal reinforcement, and deformation ductility encountered (Haselton and Deierlein 2007). Extensive researches have been conducted to understand the hysteretic behavior of RC structural components such as flexural and shear behavior of RC columns and RC beam-column joints by experiments on physical models of RC members and the rate of degradation are controlled by damage indices which have been proposed to predict the degradation behavior based on dissipated energy, number of deformation cycles, and deformation ductility (Park and Ang 1985).

Several researchers have studied the influence of degradation on response of SDF systems (Gupta and Krawinkler 1998, Song and Pincheira 2000, Miranda and Ruiz-Garcia 2002, Pekoz and Pincheira 2004, Chenouda and Ayoub 2008) and generally found that deformations of short-period degrading systems are significantly larger than those of non-degrading systems, i.e., bilinear system. Moreover, the effect of strength degradation is more important than stiffness degradation. While it is well aware that effect of degradation can lead to larger displacement of structures, most of the methods for estimating global seismic demands, such as peak (target) roof displacement, are based on using response of non-degrading equivalent SDF system. Those methods may not be suitable for application to degrading RC buildings. Therefore, this research aims to propose a method to determine the parameters of equivalent degrading SDF system and evaluate the accuracy of target roof displacements of a degrading RC frame predicted by using equivalent degrading SDF system.

2. STRUCTURAL SYSTEM AND MODELING

A real 8-story RC moment-resisting frame building was considered as an example building in this study. This building is used for classrooms, and offices in Chulalongkorn University, Bangkok, Thailand. The total height is 27.9 m, the total length is 66 m and the total width is 17 m. A typical frame for plane frame analysis is shown in Fig. 2.1, which has a tributary width of 4 m. The cross section of the left-most column has dimensions of 0.40 x 0.40 m throughout the height whereas the dimensions of the middle and right-most columns are 0.40 x 0.60 m. The slab between the middle and right columns is 12 cm thick, whereas the rest is 10 cm thick. The concrete compressive strength is 23.5 MPa and the nominal yield strength of longitudinal steel is 392 MPa. There is no infill-masonry in the typical frame where the space between the columns is used for classroom and corridor. The total mass of this frame is 303 tons. This building is assumed to have Rayleigh damping with 5% damping ratio in the first and second modes. The modal natural periods of vibration are shown in Table 2.1.

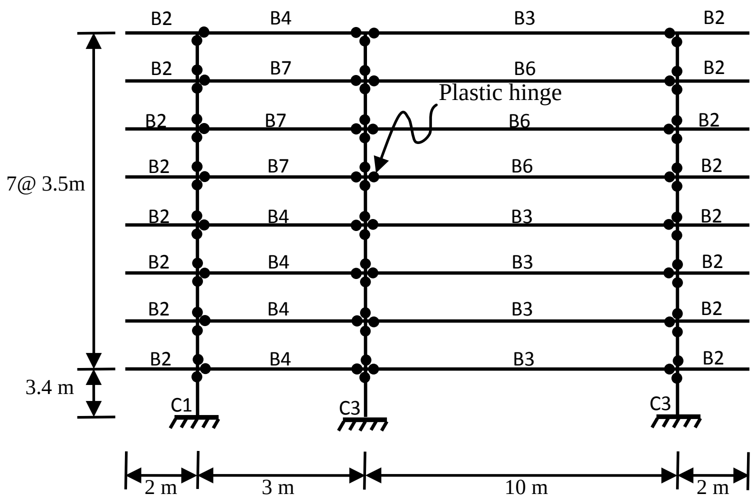
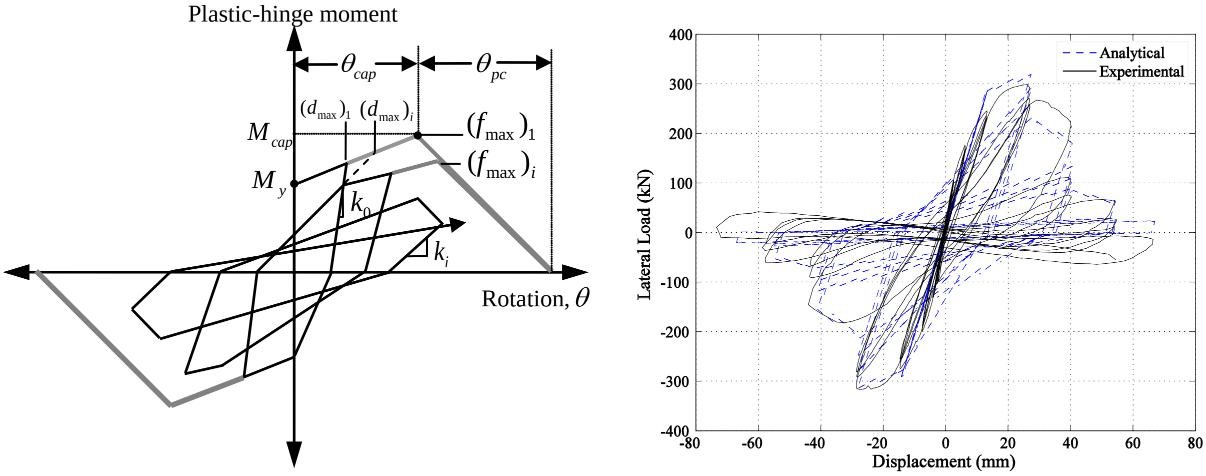


Figure 2.1. Plastic-hinge model of the example eight-story RC frame building

Table 2.1. Modal natural periods of the example eight-story RC frame building

Mode	1	2	3	4	5
Period (sec)	1.511	0.460	0.265	0.178	0.134

Nonlinear plastic-hinge elements were included at both ends of beams and columns to simulate plastic deformation when bending moment exceeds the yield moment of the cross section. Typical moment-rotation hysteretic rule of a plastic hinge in degrading systems used in this study can be shown schematically as in Fig. 2.2(a).



(a) **(b)**

Figure 2.2. (a) Moment-rotation relationship of a plastic hinge in degrading system; (b) Comparison of force-displacement relation from laboratory test of a physical model (Sezen 2002) and numerical model considering stiffness and strength degradation of the plastic hinge

This model includes three damage rules: (1) unloading stiffness degradation, (2) reloading stiffness degradation and (3) strength degradation. The envelope curve, which delineates the upper bound of the moment-rotation relation, is defined by a tri-linear curve governed by four parameters: yield moment (M_y), maximum moment capacity (M_{cap}), plastic rotation capacity (θ_{cap}) and post-capping rotation capacity (θ_{pc}). Panagiotakos and Fardis (2001) have proposed a method to calculate yield moment (M_y), whereas Haselton and Deierlein (2007) have developed predictive equations to calculate M_{cap}, θ_{cap} and θ_{pc}. From the above hysteretic rule, the degree of degradation in each damage rule is controlled by a damage index according to the following equations:

$$k_i = k_0 \cdot (1 - \delta k_i) \tag{2.1}$$

$$(d_{\max})_i = (d_{\max})_0 \cdot (1 + \delta d_i) \tag{2.2}$$

$$(f_{\max})_i = (f_{\max})_0 \cdot (1 - \delta f_i) \tag{2.3}$$

where k_0, $(d_{\max})_0$, and $(f_{\max})_0$ are initial unloading stiffness, maximum historic deformation, initial envelope maximum strength; k_i, $(d_{\max})_i$, and $(f_{\max})_i$ are unloading stiffness, deformation defining the end of the reloading cycle, current envelope maximum strength at time t_i; δk_i, δd_i, and δf_i are damage indices of unloading stiffness, reloading stiffness, and strength degradation as proposed by Park and Ang (1985). Each damage index depends on 4 parameters ($\gamma K1, \gamma K2$, $\gamma K3$, and $\gamma K4$ for δk_i; $\gamma D1$, $\gamma D2$, $\gamma D3$, and $\gamma D4$ for δd_i; and $\gamma F1$, $\gamma F2$, $\gamma F3$, and $\gamma F4$ for δf_i), peak ductility $\left(\bar{d}_{\max} = d_{\max,i} / d_{cap}\right)$ and accumulative dissipated energy (E_i) as the following equations:

$$\delta k_i = \left(\gamma K1 \cdot \left(\bar{d}_{\max}\right)^{\gamma K3} + \gamma K2 \cdot \left(\frac{E_i}{E_{monotonic}} \right)^{\gamma K4} \right) \tag{2.4}$$

$$\delta d_i = \left(\gamma D1 \cdot \left(\bar{d}_{\max} \right)^{\gamma D3} + \gamma D2 \cdot \left(\frac{E_i}{E_{monotonic}} \right)^{\gamma D4} \right) \tag{2.5}$$

$$\delta f_i = \left(\gamma F1 \cdot \left(\bar{d}_{\max} \right)^{\gamma F3} + \gamma F2 \cdot \left(\frac{E_i}{E_{monotonic}} \right)^{\gamma F4} \right) \tag{2.6}$$

where maximum energy dissipation capacity ($E_{monotonic}$) is defined by energy dissipated under monotonic loading multiplied by an additional parameter (γE):

$$E_{monotonic} = \gamma E \left(\int_{\text{monotonic load history}} dE \right) \tag{2.7}$$

This hysteretic rule is available as a material model called "Pinching4" (Lowes *et al.* 2003) in Open System for Earthquake Engineering Simulation (OpenSees 2008) software, which was used as the main structural analysis program for this research.

Currently, there is no method to calculate the value of degradation parameters: $\gamma K1, \gamma K2$, $\gamma K3$, $\gamma K4$, $\gamma D1$, $\gamma D2$, $\gamma D3$, $\gamma D4$, $\gamma F1$, $\gamma F2$, $\gamma F3$, $\gamma F4$, and γE, appropriate for a real structures, so those values used in this study were obtained by calibrating the above hysteretic rule with test results of a physical model of a non-ductile RC column tested by Sezen (2002). The force-displacement relation of the column specimen subjected to cyclic loading is plotted as solid line in Fig. 2.2(b). The analytical model of column specimen was modeled as cantilever column with a degrading plastic hinge at the base.

The degrading parameters were determined by trial and error until the force-displacement relationship in analytical model became consistent with the experimental results. The calibrated degradation parameters of plastic hinge are shown in Table 2.2 and they were used for all plastic hinges in this study.

Table 2.2. Stiffness and strength degradation parameters of plastic hinge model obtained from calibration against experimental results of Sezen (2002)

Unloading Stiffness Degradation		Reloading Stiffness Degradation		Strength Degradation		Energy Dissipation	
$\gamma K1$	0.00	$\gamma D1$	0.50	$\gamma F1$	0.00	γE	4.50
$\gamma K2$	1.00	$\gamma D2$	0.00	$\gamma F2$	1.00		
$\gamma K3$	0.00	$\gamma D3$	1.00	$\gamma F3$	0.00		
$\gamma K4$	1.00	$\gamma D4$	0.00	$\gamma F4$	1.10		

3. PROCEDURE FOR ESTIMATING PEAK ROOF DISPLACEMENT BY EQUIVALENT DEGRADING SDF SYSTEM

Estimation of peak (target) roof displacement of a MDF-system building is generally based on the deformation of an equivalent SDF system as:

$$u_{rn} = \Gamma_n \phi_{rn} D_n \tag{3.1}$$

where $n=1$ representing the fundamental mode, Γ_n is the modal participation factor, ϕ_{rn} is the mode shape ordinate at the roof displacement degree of freedom, and D_n is the peak deformation of the equivalent SDF system. The effect of the n^{th} mode can be considered by using the values corresponding to that mode, if modal roof displacement is to be used, for example, in modal pushover analysis (Chopra and Goel 2002, Bobadilla and Chopra 2008).

When the structure exhibits degrading behavior, e.g., RC frame buildings, the equivalent SDF system should be able to represent the effects of degradation too. It is assumed that the hysteretic rule and damage model discussed in the previous section can be used to represent the degradation behavior of the equivalent SDF system, but degradation parameters of the equivalent SDF system are not necessarily the same as those for a plastic hinge. The properties of the degrading equivalent SDF system should be determined from static pushover analyses of the MDF-system model of the building in 2 stages: (1) monotonic pushover analysis to obtain the envelope curve, and (2) cyclic pushover analysis to obtain the degradation parameters.

3.1. Monotonic Pushover Analysis

The monotonic pushover curve for the example 8-story building is shown as a dashed line in Fig. 3.1. Only the fundamental mode is considered in this paper; thus, the vertical force distribution proportional to the effective modal force in the fundamental mode ($\mathbf{s}_n^* = \mathbf{m}\phi_n$; $n=1$) is used in all pushover analyses in this paper, where $\mathbf{m}$ is the mass matrix and ϕ_n is the n^{th} mode shape vector. The pushover curve was idealized as a tri-linear relationship (solid line in Fig. 3.1) defined by yield roof displacement (u_{ry}), yield base shear (V_{by}), capping roof displacement (u_{cap}), capping base shear (V_{cap}) and post-capping stiffness ratio (α_{cap}).

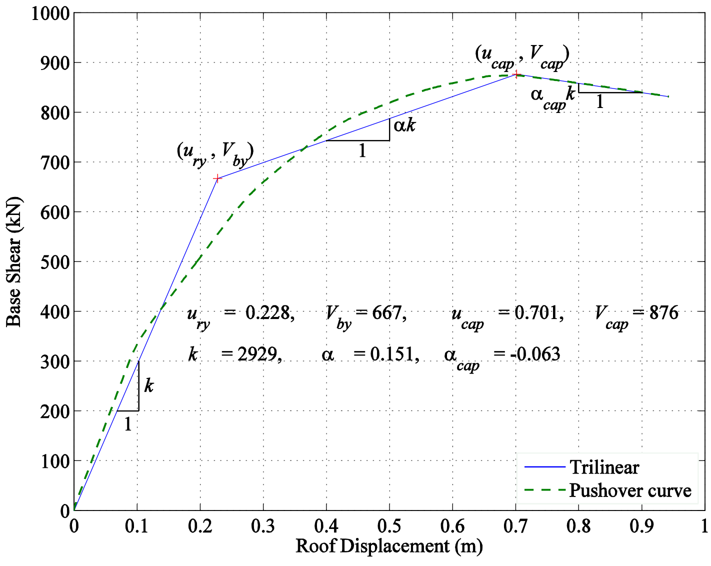


Figure 3.1. Monotonic pushover curve of the example 8-story building idealized as a tri-linear relationship

The base shear and roof displacement relation ($V_b - u_r$) is converted to force and deformation relation of equivalent SDF system (F_s / L_1 - D_1) by

$$F_y / L_1 = V_{by} / M_1^* \quad \text{and} \quad D_y = u_{ry} / \Gamma_1 \phi_{r1} \tag{3.2}$$

$$F_{cap} / L_1 = V_{cap} / M_1^* \quad \text{and} \quad D_{cap} = u_{cap} / \Gamma_1 \phi_{r1} \tag{3.3}$$

where M_1^* is the effective modal mass, $L_1 = \phi_n^T \mathbf{m}\boldsymbol{\iota}$, and $\boldsymbol{\iota}$ is the influence vector of ground motion on effective earthquake force (Chopra 2007).

3.2 Cyclic Pushover Analysis

To capture the degradation behavior of the structure, a cyclic load with an invariant vertical distribution of lateral forces was applied to the buildings, while the roof displacement was being monitored and controlled. The roof displacement history (protocol) can be chosen in many possible ways. Referring to displacement history protocols used in physical testing of RC specimens (Krawinkler 2009), this study uses a protocol modified from the ISO protocol by reducing the number of repeated cycles from three to two to reduce the computing time (Table 3.1).

Table 3.1. Displacement history of the modified-ISO protocol

No. of cycles	1	2	2	2	2	2	2	2
Displacement	$0.05u_{cap}$	$0.10u_{cap}$	$0.20u_{cap}$	$0.40u_{cap}$	$0.60u_{cap}$	$0.80u_{cap}$	$1.0u_{cap}$	$1.25u_{cap}$

The degradation parameters as appeared in Eqn. 2.4 to Eqn. 2.6 can be obtained by optimization minimizing the sum of squares of differences between the force-deformation relationships obtained from cyclic pushover curve and the equivalent SDF system as shown in Fig. 3.2. The values of those parameters obtained are shown in Table 3.2. Other displacement protocols were also tried and they resulted in similar values for these parameters. Conversely, these parameters are not very sensitive to the choice of protocols.

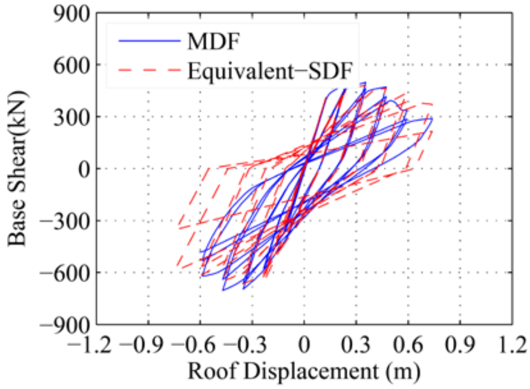


Figure 3.2. Comparisons of cyclic pushover curves and hysteresis loop of equivalent-SDF system

Table 3.2. Stiffness and strength degradation parameters for the example eight-story building

Unloading Stiffness Degradation		Reloading Stiffness Degradation		Strength Degradation		Energy Dissipation	
$\gamma K1$	1.22	$\gamma D1$	0.00	$\gamma F1$	0.42	γE	2.84
$\gamma K2$	0.30	$\gamma D2$	1.78	$\gamma F2$	0.72		
$\gamma K3$	0.82	$\gamma D3$	1.21	$\gamma F3$	1.08		
$\gamma K4$	0.90	$\gamma D4$	1.02	$\gamma F4$	0.65		

After the properties of degrading equivalent SDF system are obtained, the peak deformation D_n can be determined by solving the governing equation of motions of the equivalent SDF system. In this study, the nonlinear response history analysis (NL-RHA) of SDF system will be used to determine D_n and the peak (target) roof displacement is estimated according to Eqn. 3.1. The accuracy of the proposed procedure to estimate peak roof displacement of degrading RC building will be examined next by applying the method to the 8-story building subjected to a set of 20 ground motions and compare the results to the 'reference' value determined by NL-RHA of MDF-system model of the building.

4. GROUND MOTIONS AND RESPONSE STATISTICS

A set of 20 Large-Magnitude-Small-distance (LMSR) records used in this study were selected from California earthquake records of magnitude ranging from 6.6 to 6.9 recorded at distances of 13 to 30 km on firm soil (Chintanapakdee and Chopra 2003). These ground motions were scaled to three different intensity levels such that the spectral acceleration at the fundamental period of the building $A(T_1)$ equal to 0.208g, 0.50g, and 0.70g to investigate deterioration of the method as the structure experiences more yielding and damage. The value of $A(T_1) = 0.208g$ corresponds to the elastic design spectrum for Chiang Mai province in the northern part of Thailand, which has the highest seismic risk in Thailand. Fig. 4.1(a) shows the median spectrum of the scaled ground motions. Fig. 4.1(b) shows the pseudo-acceleration spectra of scaled ground motions to have $A(T_1) = 0.5g$.

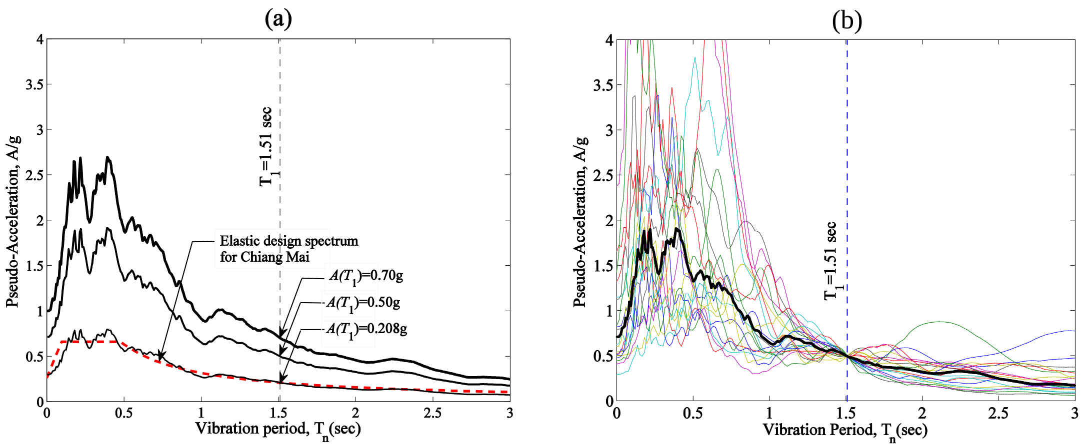


Figure 4.1. (a) Median spectra of scaled ground motions such that $A(T_1)$ =0.208g, 0.50g, and 0.70g; and (b) Pseudo-acceleration spectra of individual records and their median value when scaled to have $A(T_1)$ =0.50g; damping ratio, ζ =5%

The peak roof displacement due to each ground motion estimated by using the degrading equivalent SDF system is denoted by $u_{r,SDF}$ and the 'reference' value determined by NL-RHA of the MDF system by $u_{r,MDF}$. The ratio of the two values $\left(u_r^*\right)_{SDF} = u_{r,SDF} \div u_{r,MDF}$ closed to unity represents good accuracy of proposed procedure. Assuming that the distribution of the data is lognormal, the median of 20 response values is calculated as the geometric mean and the dispersion measure is calculated as the standard deviation of logarithm of data (Chintanapakdee and Chopra 2003).

5. ACCURACY OF THE PROPOSED PROCEDURE TO ESTIMATE PEAK ROOF DISPLACEMENT

Fig. 5.1 plots the peak roof displacements of the 8-story building estimated by NL-RHA of equivalent SDF systems ($u_{r,SDF}$) versus the value determined by NL-RHA of the MDF-system model ($u_{r,MDF}$). The median and dispersion of the ratios $\left(u_r^*\right)_{SDF}$ are also noted. There are 20 data point corresponding to the responses due to the 20 ground motions. Data points located near the diagonal line indicates accurate estimation. The upper row of plots shows the estimates from using a non-degrading (bilinear) equivalent SDF system, whereas the lower row shows the estimates from using a degrading equivalent SDF system.

Three columns of the plots correspond to the three intensity levels of ground motions. From Fig. 3.1, the yield base shear of idealized pushover curve is 667 kN corresponding to base shear coefficient of 0.224g. Thus, the three intensity levels $A(T_1)$=0.208g, 0.5g, and 0.7g would correspond to R=0.93, 2.23, and 3.12, respectively, if R is defined as the elastic demand divided by global yield strength. It should be kept in mind that the first yielding occurs at much lower base shear force; and the R values would be larger than the above, if it is defined by ratio of elastic demand and strength at first yielding, instead.

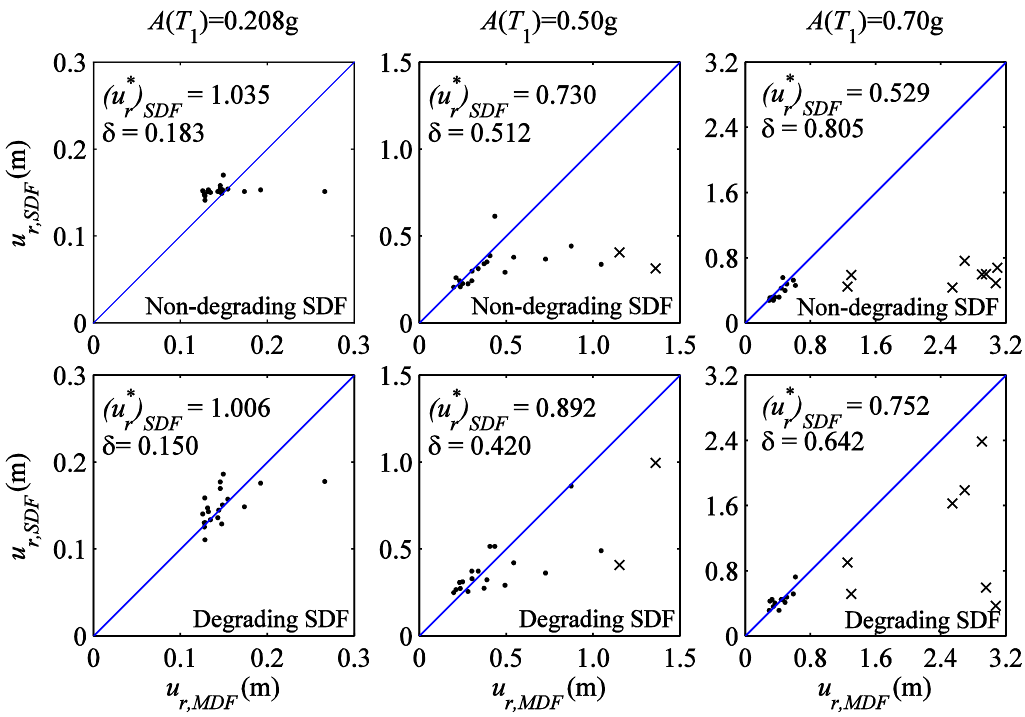


Figure 5.1. Plots of peak roof displacement estimates using equivalent SDF systems versus the 'reference' values from NL-RHA of MDF-system model of the 8-story building ('x' data point denotes collapse indicated by numerical instability)

Because NL-RHA of MDF system also considers P-Δ effects, the building collapses are encountered in some cases when the ground motions are strong. Those collapse cases are marked by 'x' data point. In such cases, the peak displacements shown are the last values before numerical instability occurs and the statistics of peak roof displacements are based on these values. It can be observed that the accuracy of the proposed procedure deteriorates as the ground motions become stronger, or as inelastic deformations become larger. The use of degrading equivalent SDF system as shown in the bottom row of Fig. 5.1 can provide significantly more accurate estimation of peak roof displacement. To demonstrate this superiority, Fig. 5.2 shows the roof displacement response history of the 8-story building when subjected to the Agnews State Hospital ground motion record from 1989 Loma Prieta earthquake determined by three methods: (1) NL-RHA of MDF system, (2) NL-RHA of degrading equivalent SDF system, and (3) NL-RHA of non-degrading (bilinear) equivalent SDF system.

From Fig. 5.2, it can be seen that the result from using degrading equivalent SDF system can follow the result of NL-RHA of MDF system surprisingly well, whereas the result from using non-degrading equivalent SDF system can not. However, we do not always achieve such excellent accuracy; the estimation could be inaccurate in some cases as shown in Fig. 5.1. Although the median displacement ratio shows that the bias is small, if dispersion is large, then there could be inaccurate estimation for individual ground motions.

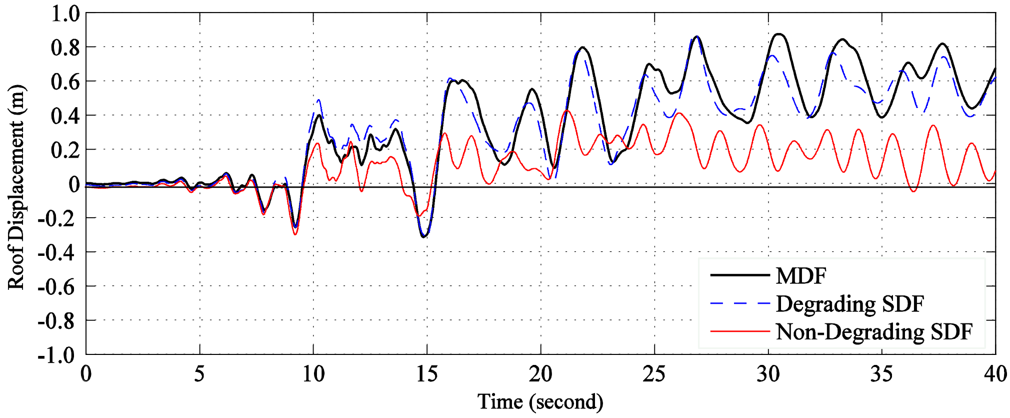


Figure 5.2. Roof displacement response history of the 8-story building from NL-RHA of (a) MDF-system model, (b) degrading equivalent SDF system, and (c) non-degrading (bilinear) equivalent SDOF system when subjected to Agnews State Hospital ground motion from 1989 Loma Prieta earthquake (scaled to $A(T_1)$ =0.5g)

When the base-shear force is plotted versus roof displacement as a hysteresis loop for each of those three methods in Fig. 5.3, we can observe that the result from using degrading equivalent SDF system are quite similar to the result of NL-RHA of MDF system, whereas using non-degrading equivalent SDF system resulted in a different shape. Therefore, using a degrading equivalent SDF system should be more appropriate than non-degrading SDF system in the estimation of target roof displacements of degrading RC buildings.

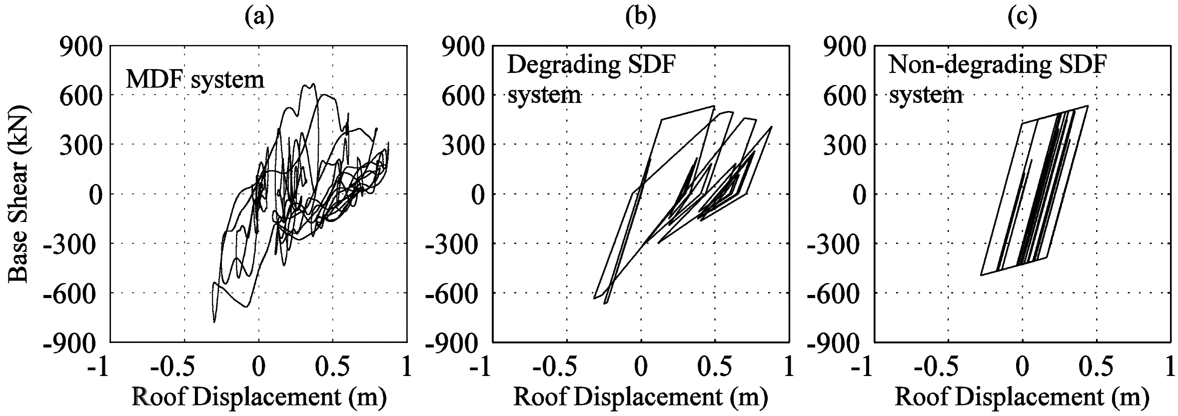


Figure 5.3. Base-shear force versus roof displacement hysteresis loop of the 8-story building calculated by NL-RHA of (a) MDF-system model, (b) degrading equivalent SDF system, and (c) non-degrading equivalent SDF system subjected to Agnews State Hospital ground motion from 1989 Loma Prieta earthquake (scaled to $A(T_1)$ = 0.5g)

6. CONCLUSIONS

The procedure to estimate the peak (target) roof displacement of a degrading RC frame building by using deformation of a degrading equivalent SDF system has been presented. The force-deformation relation of the degrading equivalent SDF system can be determined by monotonic and cyclic pushover analysis and its parameters are not very sensitive to the displacement history used in the cyclic pushover analysis. Investigation of the accuracy of the proposed procedure led to the following conclusions:

1. Using degrading equivalent SDF systems in estimation of peak (target) roof displacement of degrading RC buildings provides more accurate estimates than using non-degrading SDF systems.

2. The accuracy of the proposed procedure for estimating target roof displacement deteriorates when the ground motion intensity increases and the structure experiences significant inelastic deformation.

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