Assessment Of Current Nonlinear Static Procedures For Seismic Evaluation Of Buckling-Restraint Braced Frames

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SUMMARY:
Due to a number of structural performance advantages over conventional braced frames, the BRBF system appears to be gaining in popularity. BRBs show the same load deformation behavior in both compression and tension. Nonlinear static procedures (NSPs) are now standard in engineering practice to estimate seismic demands in the design and evaluation of buildings. This paper aims to investigate comparatively the accuracy of nonlinear pushover analysis in comparison with nonlinear dynamic analyses when they are applied to buckling-restrained braced frame (BRBF) buildings. In this study a series of analyses were carried out for four BRBFs and their results were evaluated. A set of seven code-compliant natural earthquake records was employed to perform inelastic response history analyses. The assessment is based on comparing seismic displacement demands such as target roof displacements, peak floor/roof displacements and inter-story drifts. The NSP estimates are compared to results from nonlinear dynamic analyses, showing good agreements.

Keywords: buckling-restrained brace, nonlinear static analysis, nonlinear dynamic analysis, seismic demands.

1. INTRODUCTION

Nonlinear time history (NTH) analysis is a robust tool for calculating seismic demands, as well as for identifying plastic hinge mechanisms in structures. However, the response of NTH is strongly affected by the modeling parameters and by the characteristics of the earthquake input such as frequency content, intensity, and duration (Kalkan and Kunnath, 2006) . It is therefore necessary to carefully choose a set of representative ground motion records.

Nonlinear static analysis is a simplified analysis procedure that can be useful for estimating seismic demands and providing valuable information about the locations of structural weaknesses and failure mechanisms in the inelastic range (Krawinkler and seneviratna, 1998). Also pushover analysis has the advantage that it is capable of considering a response spectrum of codes as demand diagram to estimate the earthquake induced response of structures (Chopra and Goel, 1999).

Current nonlinear static procedures are Coefficient Method in FEMA-356 (Applied Technology Council, 1996) and Capacity Spectrum Method in ATC-40 (Federal Emergency Management Agency, 2000; Applied Technology Council, (2005). However, NSPs based on invariant load patterns provide accurate seismic demand estimates only for low- and medium-rise moment-frame buildings where the contributions of higher `modes' response are not significant and inadequate to predict inelastic seismic demands in buildings when the higher `modes' contribute to the response (Chintanapakdee and Chopra, 2003; Chopra and Goel, 2002). To overcome these drawbacks, an improved pushover procedure, called modal pushover analysis (MPA), was proposed by Chopra and Goel (2002) to include the contributions of higher `modes'. The MPA procedure has been demonstrated to increase the accuracy of seismic demand estimation in taller moment-frame buildings, e.g., 7- and 9- stories tall, compared to the conventional pushover analysis (Chopra et al, 2004). In spite of including the contribution of higher `modes', MPA is conceptually no more difficult than standard procedures because higher `modes' pushover analyses are similar to the first `mode' pushover analysis. Moreover, MPA procedure considering for the first few (two or three) `modes' contribution are typically sufficient (Chintanapakdee and Chopra, 2003).

Another pushover method is the adaptive pushover procedures, where the load pattern distributions are updated to consider the change in structure during the inelastic phase (Bracci et al, 1997 Fajfar and Fischinger, 1998). In this type of procedure, equivalent seismic loads are calculated at each pushover step using the immediate `mode' shape. Recently, a new adaptive pushover method, called the adaptive modal combination (AMC) procedure, has been developed by Kalkan and Kunnath (2006) where a set of adaptive mode-shape based inertia force patterns is applied to the structure. This procedure has been validated for regular moment frame buildings (Kalkan and Kunnath, 2006). However, it is conceptually complicated and computationally demanding for routine application in structural engineering practice while the MPA method is generally simpler, and thus, more practical than adaptive pushover procedures for seismic design.

More recently, an improved modal pushover analysis (IMPA) procedure was proposed by Jianmeng et al, (2008) to consider the redistribution of inertia forces after the structure yields. The structural stiffness changes after it yields, so the displacement shape vector also changes. The IMPA procedure uses the product of the time variant floor displacement vector (as the displacement shape vector) and the structural mass matrix as the lateral force distribution at each applied-load step beyond the yield point of the structure. However, to avoid a large computation, only two phase lateral load distribution was recommended. In the first phase, the pushover analysis is performed by using the first few elastic natural `modes' of structure, i.e., similar to the MPA. In the second phase, only for the first `mode' the lateral load distribution is based on assumption that the floor displacement vector at the initial yielding point is the displacement shape vector.

An alternative pushover analysis method to estimate the seismic displacement demands, referred to as the mass proportional pushover (MPP) procedure, was proposed by Kim and Kurama (2008). The main advantage of the MPP is that the effects of higher `modes' on the lateral displacement demands are lumped into a single invariant lateral force distribution that is proportional to the total seismic masses at the floor and roof levels. However, the accuracy of both IMPA and MPP procedures has been verified for a limited number of cases.

With the increase in the number of alternative pushover analysis procedure proposed in recent years, it is useful to assess the accuracy and classify the potential limitations of these methods. An assessment on accuracy of MPA and FEMA pushover analyses for moment resisting frame buildings was investigated by Chopra and Chintanapakdee (2004). Then, an investigation on the accuracy of improved nonlinear static procedures in FEMA-440 was carried out by Akkar and Metin (2007). Meanwhile, the ability of FEMA-356, MPA and AMC in estimating seismic demands of a set of existing steel and reinforced concrete buildings was examined by Kalkan and Kunnath (2006). More recently, an investigation into the effects of nonlinear static analysis procedures which are the Displacement Coefficient Method (DCM) recommended in FEMA 356 and the Capacity Spectrum Method (CSM) recommended in ATC 40 to performance evaluation on low-rise RC buildings was carried out by Irtem and Hasgul (2009).

To assess the ability of current procedures, this paper aims to investigate comparatively the bias and accuracy of MPA, IMPA and MPP procedures when applied to buckling-restrained braced frames (BRBFs), which have become a favorable lateral-force resisting system for earthquake resistant buildings as its hysteretic behavior is non-degrading and much hysteretic energy can be dissipated. BRBF is an innovative structural system that prevents buckling of the braces by using a steel core and an outer casing filled with mortar for the brace. Brace axial forces are resisted only by the steel core and not by the surrounding mortar and steel encasement. The steel core is restrained from buckling by the outer shell and the infill mortar. The BRBF system is considered to have favorable seismic performance over conventional braced frames in that the braces are capable of yielding in both tension and compression instead of buckling, making it an attractive option to structural engineers. BRBF systems are currently used as primary lateral force resisting elements both in new construction and seismic retrofit projects. A more comprehensive background on this system can be found in (Sahoo and Chao, 2010).

2. REVIEW OF CONSIDERED NONLINEAR STATIC PROCEDURES

This section briefly introduces the modal, improved modal pushover analysis (MPA, IMPA) and mass proportional pushover (MPP) procedures in estimating seismic demands for considered building.

2.1. Modal Pushover Analysis (MPA)

The modal pushover analysis (MPA), which has been proposed by Chopra and Goel (2002) is an extension of conventional pushover analysis to include contribution of higher `modes'. A step-by-step

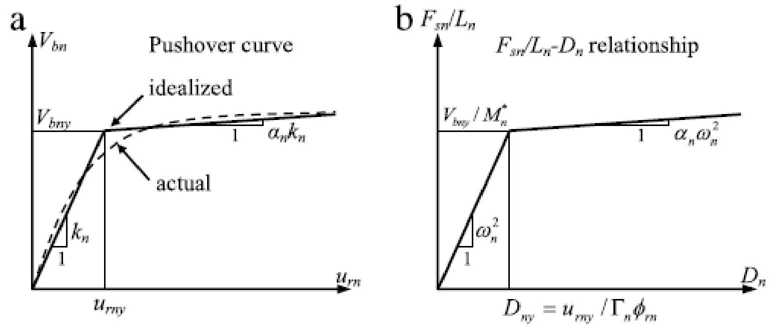


Figure 2.1. (a) Pushover curve and (b) force and deformation relationship of SDF system

summary of the MPA procedure to estimate the seismic demands for building is presented as a sequence of steps:

(1) Compute the natural frequencies, ω_n, and mode shape vectors, ϕ_n, for linearly elastic vibration modes of the building.

(2) For the nth-`mode', develop the base shear-roof displacement ($V_{bn} - u_{rn}$) pushover curve by nonlinear static analysis of the building using the force distribution $s_n^* = m\phi_n$ where m is the mass matrix.

(3) Idealize the pushover curve as a bilinear curve (**Figure 2.1**).

(4) Convert the idealized pushover curve to the force-deformation ($F_{sn}/L_n - D_n$) relation of the nth-`mode' inelastic SDF system and determine the elastic modal frequency $\omega_n$, and yield deformation $D_{ny}$. The nth-`mode' inelastic SDF system is defined by the force-deformation curve of **Figure 2.1(b)** (with post-yield stiffness ratio α_n) and damping ratio ξ_n specified for the nth `mode'. Where $M_n^* = \Gamma_n L_n$ is the effective modal mass, $\phi^T m 1$, $\Gamma_n = \dfrac{\phi_n^T m\iota}{\phi_n^T m\phi_n}$, and each element of the influence vector ι is equal to unity.

(5) Compute the peak deformation, $D_n \equiv \max_{\forall t}|D_n(t)|$, of the nth-`mode' inelastic SDF system with the force-deformation relation of **Figure 2.1(b)** due to ground excitation $\ddot{u}_g(t)$ by solving (Eq. 2.1):

$$\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \frac{F_{sn}(D_n, \dot{D}_n)}{L_n} = -\ddot{u}_g(t). \tag{2.1}$$

(6) Calculate the peak roof displacement u_{rno} associated with the nth-`mode' inelastic SDF system from Eq. 2.2.

$$u_{rno} = \Gamma_n \; \phi_{rn} D_n \tag{2.2}$$

(7) Extract other desired responses, r_{no}, from the pushover database when roof displacement equal

to u_{rno}.

(8) Repeat Steps 2-7 for as many `modes' as required for sufficient accuracy; usually the first two or three `modes' will suffice for buildings shorter than 10 stories.

(9) Determine the total response r_{mpa} by combining the peak `modal' responses using appropriate modal combination rule, e.g., Square-Root-of-Sum-of-Squares (SRSS) as shown by Eq. 2.3 or Complete Quadratic Combination (CQC) rule:

$$r_{mpa} = \sqrt{\sum_{n=1}^{j} r_{no}^2} \tag{2.3}$$

where j is the number of `modes' included.
The MPA procedure summarized in this paper is developed for symmetric buildings (Chopra and Goel, 2002).

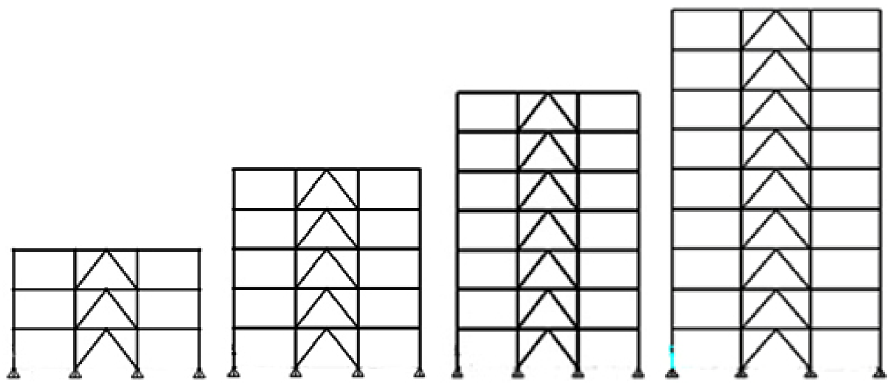


Figure 2.2. Frame elevations of 3-, 5-, 7-, and 9-story BRBF buildings.

2.2. Improved Modal Pushover Analysis (IMPA)

Unlike the MPA procedure where the response is obtained from invariant multi-mode lateral load pattern vectors, the improved modal pushover analysis (IMPA) proposed by Jianmeng et al, 2008) considering the redistribution of inertia forces after the structure yields. The principal improvement of the IMPA is to use deflection shape of structure after yielding as an invariant later load pattern. However, to avoid a large computation, a two-phase lateral load distribution is suggested for the first `mode' while the force patterns for higher `modes' are similar to the MPA approach. The IMPA procedure is summarized by following steps:

(1) Implement the Steps 13 of the MPA procedure described in previous section for first `mode'. The lateral force distribution $s_1^* = m\phi_1$ is considered as the first-phase load pattern.

(2) Determine the displacements vector of structure, Ψ_{1y}, at the yielding point with the pushover analysis obtained from Step 1.

(3) Continue pushover analysis from the structure yielding point by applying the load distribution

$s_{1y}^{*} = m\Psi_{1y}$, which is considered as the second-phase lateral load pattern to obtain a new pushover curve. Then, this new pushover curve is used for determining the response of the structure by Steps 47 of MPA procedure described in Section 2.1.

(4) Determine the total response r_{impa} with SRSS or CQC combination rules by combining the response for the first `mode' obtained from Step 3 and the responses due to other higher `modes' obtained from MPA procedure.

2.3. Mass Proportional Pushover Procedure (MPP)

An alternative pushover analysis procedure, called the mass proportional pushover (MPP), was proposed by Kim and Kurama (2008) to estimate the peak seismic lateral displacement demands for buildings. The main advantage of the MPP procedure over other approximate procedures is the use of a single pushover analysis for the structure with no need to conduct a modal analysis to capture the effect of higher `modes'. A summary of the mass proportional pushover procedure, whose details can be found in Kim and Kurama (2008), is as follows:

(1) Determine the multi-degree-of-freedom (MDOF) base shear force versus the roof displacement $(v_b - u_r)$ relationship using the force distribution given by $mg\iota = w\iota$ where *m* is the mass matrix and *w* is weight matrix.

(2) Idealize the pushover curve as a bilinear curve.

(3) Convert the idealized pushover curve to the pseudo-acceleration versus the displacement (AD) relationship of an equivalent SDF system using Eq. 2.4.

$$A = \frac{v_b}{m}; D = \frac{u_r}{\Gamma} \tag{2.4}$$

where M is the total mass and Γ is the participation factor calculated as: $\Gamma = \dfrac{u_e^T ml}{u_e^T mu_e}$; u_e is the lateral floor displacement vector (normalized with respect to the roof) obtained from the linear-elastic response range of the pushover analysis using the $mg\iota = w\iota$ force distribution which is the same as uniform distribution of FEMA-356.

(4) Determine the maximum SDF displacement, $D_{\max}$ by solving Eq. (1) with $F_s/L = A$.

(5) Calculate the maximum MDOF roof and floor displacements of structure as: $u_{\max} = D_{\max}\Gamma u_e$.

3. STRUCTURAL SYSTEMS AND ANALYTICAL MODELS

Four-, 3-, 5-, 7-, and 9-story BRBF buildings, which were designed to meet seismic code criteria, were analyzed to evaluate the bias and accuracy of MPA, IMPA and MPP procedures. The elevation view of all BRBF systems is shown in **Figure 2.2.** Analytical models were created to analyze these BRBF buildings whose details can be found in Chintanapakdee et al, (2009). $P-\Delta$ effect was also considered for this study. Nonlinear static and dynamic analyses were carried out using the computer program SAP2000.

4. GROUND MOTIONS

A set of seven code-compliant natural earthquake records was employed to perform inelastic response history analyses in this study. Table 4.1 provides the information of considered records. To determine the seismic demands of a building due to a set of ground motions, each record was scaled such that the spectral acceleration at the fundamental natural period of the building is equal to the median spectral acceleration for that period. This method of scaling helps reduce the dispersion of results.

Table 4.1. Characteristics of considered earthquake motions

No	Year	Earthquake	Magnitude	Station	PGA (g)
1	1991	SIERRA MADRE	5.61	Los Angeles-Obregon Park	0.221
2	1971	San Fernando	6.61	La Hollywood Stor Lot	0.21
3	1980	Trinidad	7.2	Rio Del Overpass	0.0614
4	1984	Morgan Hill	6.19	Apeel 1E Hayward	0.0406
5	1989	Loma Prieta	7	Halles Valley	0.134
6	1980	Victoria	6.33	Chihuahaua	0.15
7	1994	Northridge	6.69	La-Faring RD	0.231

To estimate seismic demands in the design and evaluation of buildings, the nonlinear static procedures (NSPs) using the lateral force distributions recommended in ATC-40 (1996) and the FEMA-356 (2000) documents are now standard in engineering practice. The nonlinear static procedure in these documents is based on the capacity spectrum method (ATC-40, 1996) and displacement coefficient method (FEMA-356, 2000), and assumes that the lateral force distribution for the pushover analysis and the conversion of the results to the capacity diagram are based on the fundamental vibration mode of the elastic structure. The response of each building to each set of the ground motions was determined by nonlinear response history analysis (NTH), and a nonlinear static procedure (NSP), e.g., MPA, IMPA and MPP. The peak value of inter-story drift (Δ), determined by NTH is denoted by, Δ_{NL-RHA} and from NSP by Δ_{NSP}. From these data for each ground motion, a response ratio was determined from the following equation: $\Delta^*_{NSP} = \Delta_{NSP}/\Delta_{NL-RHA}$. The median values, $\hat{X}$, defined as the geometric mean, of n observed values (x_i) of Δ_{NSP}, Δ_{NL-RHA} and Δ^*_{NSP}; and the dispersion measures δ of Δ^*_{NSP} defined as the standard deviation of logarithm of the n observed values were calculated by Eq. 2.4 and Eq. 2.4.

$$\hat{x} = \exp\left[\frac{\sum_{i=1}^{n} \ln x_i}{n}\right] \tag{4.1}$$

$$\delta = \sqrt{\frac{\sum_{i=1}^{n} (\ln x_i - \ln \hat{x})^2}{n-1}} \tag{4.2}$$

An advantage of using the geometric mean as the estimator of median is that the ratio of the median of Δ_{NSP} to the median of Δ_{NL-RHA} is equal to the median of the ratio Δ^*_{NSP}, i.e., the bias of NSP in estimating the median response is equal to the median of bias in estimating response to individual excitation.

5. EVAUATION OF NONLINEAR STATIC PROCEDURES

The bias and accuracy of the MPA, IMPA and MPP procedures applied to BRBF buildings are evaluated by comparing the target roof displacements, peak floor (or roof) displacements and inter-story drifts compared to more accurate results from nonlinear response history analysis (NTH).

5.1. Target Roof Displacements

Pushover curves, which show the relationship between the base shear force and the roof displacement, for the 3-, 5-, 7- and 9-story BRBF buildings due to the first `mode' load pattern (MPA), variable lateral force distribution (IMPA) and seismic mass (or weight) distribution (MPP). For summary the pushover diagrams are not located in this paper but the target roof displacements are listed in Table 5.1.

Table 5.1. Target roof displacements for BRBF buildings (cm)

Building model	MPP	MPA	IMPA
3 Story	7.4	6.2	6.5
5 Story	11.7	10.2	10.7
7 Story	17.5	15.4	16.2
9 Story	21.3	19.8	20.1

The variable lateral force distribution of IMPA procedure in this study is taken as a three-phase load pattern, which changes at the first and second yielding points of the pushover curve. This results in nearly identical estimates of target roof displacements of both procedures. It implies that the changes of lateral load distribution of IMPA procedure are not significant whereas the $m g \iota = w \iota$ force distribution of MPP leads to different results.
As it seen the target roof displacements noticeably increases when the building height increases.

5.2. Peak Floor/Roof Displacements

The responses of the BRBF buildings studied to the set of ground motions were determined by MPA, IMPA, MPP nonlinear static procedures and by nonlinear response time history analysis (NTH). The combined values of floor displacements and story drifts were computed by using the SRSS modal combination rule. The peak floor/roof displacement demands from the four methods are compared in **Figure 5.1**. The results from modal pushover analysis (MPA) including only the fundamental `mode' lead to the following observations for the BRBF system. The contributions of higher `modes' of MPA and IMPA procedures to floor displacements are not significant. One `mode' pushover analysis, MPA, and IMPA can estimate the peak floor displacements reasonably well with a tendency to slightly overestimate the floor/roof displacement compared to NTH while the MPP tends to significantly overestimate peak floor displacements of lower stories (**Figure 5.1**).
Figure 5.2 shows the median floor displacement ratio $(u^*_{NSP}) = u_{NSP} / u_{NL-RHA}$ due to the set of ground motions. It can be seen that the MPA procedure can accurately estimate floor displacements of the 3-, 5-, 7-, and 9-story BRBF buildings. The IMPA tends to overlap the MPA with slight difference whereas the MPP tends to much overestimate peak floor displacements of lower stories with increasing bias when the building height increases. The bias of MPP is very large for BRBF buildings taller than 5 stories considered in this study.

5.3. Story Drift Ratio

Figure 5.3 shows the median story drift ratio $\Delta^*_{NSP} = \Delta_{NSP} / \Delta_{NL-RHA}$, due to the set of ground motions. The bias of MPA, IMPA and MPP nonlinear static procedures in estimating seismic demands tends to increase for stronger excitations and the variation of the NSP bias in estimating seismic demands along building height primarily depends on the building height rather than the intensity of ground motions. This is because the MPP significantly overestimates floor displacements for lower stories due to the total seismic mass (or weight) load pattern (**Figure 5.1**).
In general, the dispersion of story-drift ratios of MPA and IMPA increases as the building becomes taller or ground motions become stronger. Meantime, the dispersion of story-drift ratios of MPP for BRBF buildings implies that the accuracy of NSPs in predicting the response due to an individual

ground motion deteriorates when applied to taller BRBF buildings or subjected to stronger ground motions. Among these cases, the dispersion is small, when NSPs are used to estimate the maximum story drift over all stories. In addition, the dispersion of story-drift ratios of MPP is always larger than both MPA's and IMPA's. Therefore, MPA and IMPA can be a useful analysis tool to estimate the peak story drift over all stories in evaluating existing buildings or design of new buildings using BRBFs. Both of these procedures provide practically the same results but MPA is simpler and more practical than IMPA because it involves an invariant load pattern. On the contrary, the MPP method is simple with no need to conduct a modal analysis to capture the effects of higher `modes' but it may be inaccurate in estimating seismic demands for BRBF tall buildings due to strong ground motions.

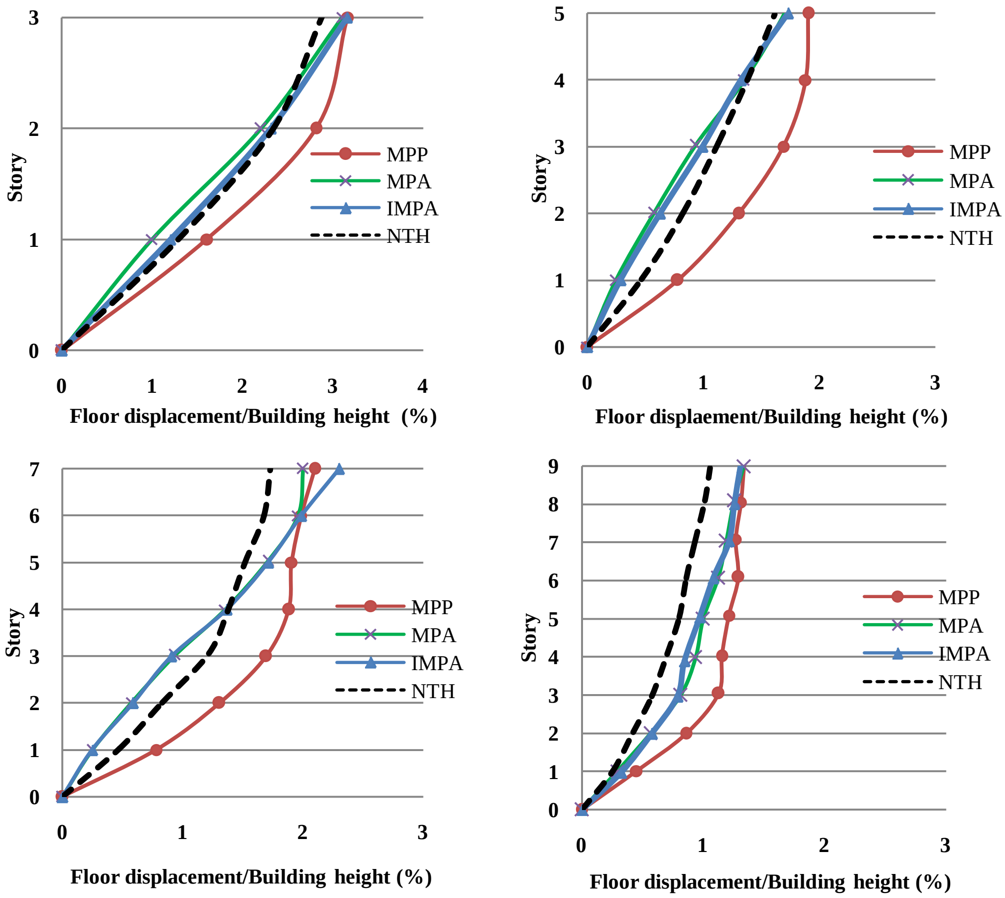
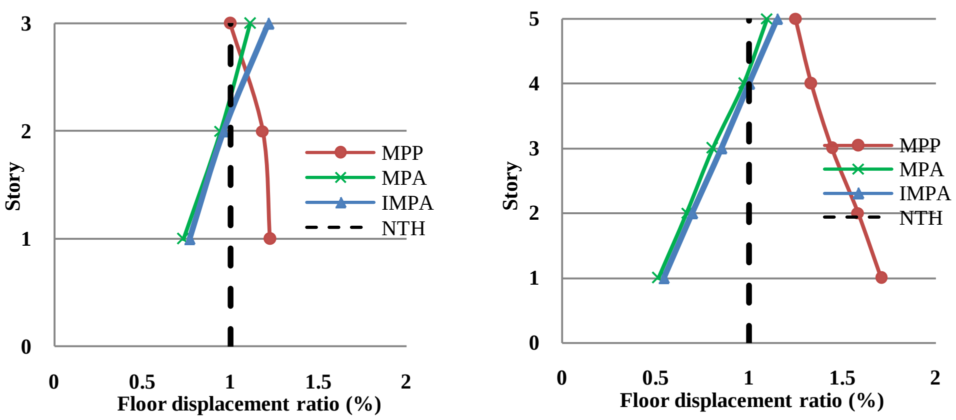


Figure 5.1. Median floor displacements of 3-, 5-, 7- and 9-story BRBF buildings determined by one `mode' pushover analysis, MPA, IMPA, MPP and NTH



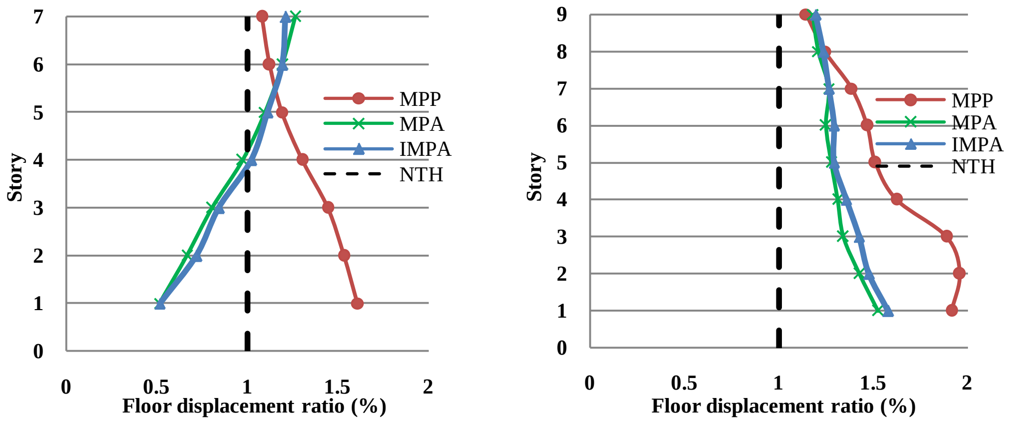


Figure 5.2. Floor displacement ratio of 3-, 5-, 7- and 9-story BRBF buildings

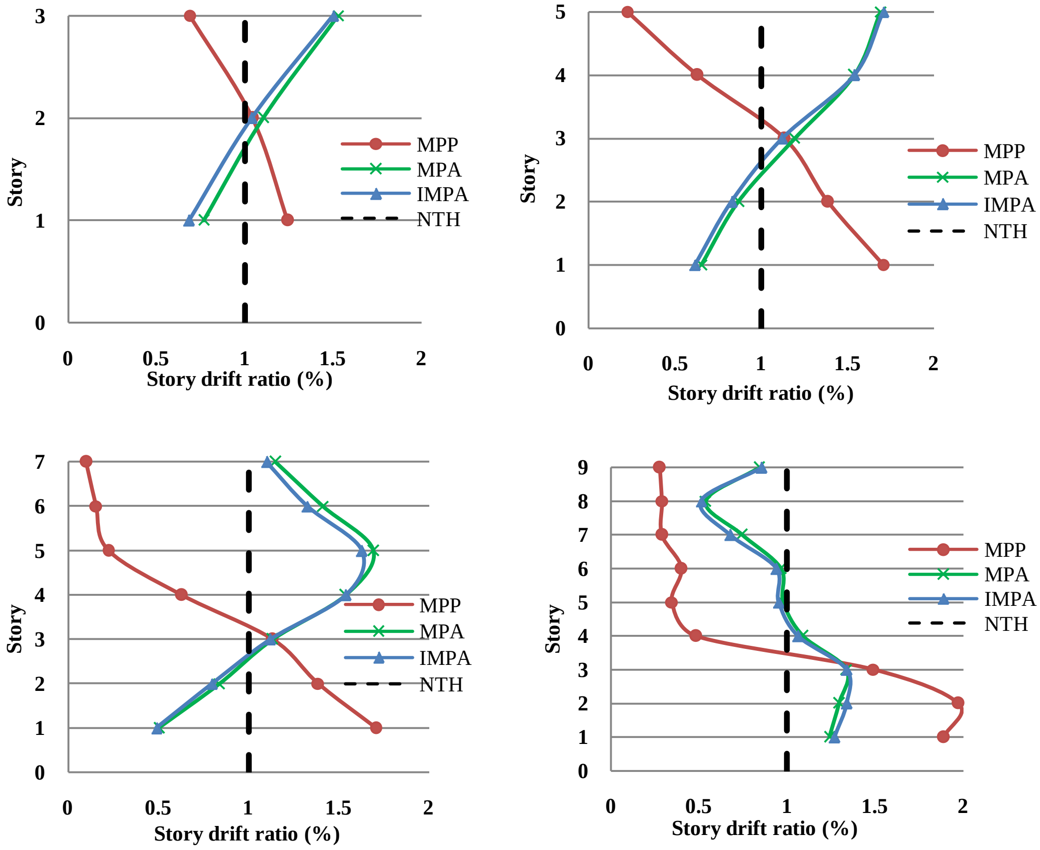


Figure 5.3. Story drift ratio of 3-, 5-, 7- and 9-story BRBF buildings

6. CONCLUSIONS

The following summary and conclusions can be drawn based on the research findings of this study. These conclusions are based on a comparison of NSP estimates of seismic demands and the corresponding values determined by NTH for 3-, 5-, 7-, and 9-story BRBF buildings which were designed to meet seismic code criteria.

(1) The IMPA tends to predict the median and dispersion of target roof displacements better than MPA. However, the difference is not significant while the MPP tends to estimate the maximum roof displacements slightly less accurately than both MPA and IMPA.

(2) The bias of MPA and IMPA procedures in estimating the maximum story drift over all stories is generally small. However, the bias of these procedures in estimating peak story drift at an individual story can be considerable for certain cases. Both of these procedures develop practically similar results whereas MPA is more practical and slightly simpler than IMPA as it includes an invariant load pattern. In opposition, the bias in estimating maximum story drifts over all stories of MPP can be large.

(3) The story drift demands predicted by MPA and IMPA are able to follow the NTH results. However, the higher `modes' contributions of these procedures in the response of low-rise (3-, and 5-story) BRBF buildings are generally not noticeable, so the first `mode' alone may be sufficient.

(4) The MPP tends to noticeably overestimate seismic demands for lower stories but underestimates story drifts for upper stories with increasing bias when the building height increases. In addition, the story drifts predicted by the MPP procedure seem to be uniform in upper stories, especially for 7- and 9-story BRBF buildings considered in this study.

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