Assessing the Accuracy of Computational Modeling Of Buildings

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15 WCEE
LISBOA 2012

SUMMARY:
In order to assess the accuracy of computational models of buildings, the results can be compared to forced vibration testing (FVT). The authors are currently assessing results from computational models and full-scale FVT as part of a NSF NEESR project. This paper focuses on a five-story reinforced concrete shear wall university library structure constructed in the 1970's. Characteristics of this structure that make it unique include a large atrium at the center of the building and irregularly placed reinforced concrete shear walls. Comparison of the results from the detailed computational models and the FVT experiments showed that the detailed computational models closely estimated the building frequencies while varying more widely in the mode shape prediction. Variables that led to the differences in the computational and the experimental results include the current concrete strength and stiffness, the estimation of the library mass including the books, and modeling of the flexible diaphragm.

Keywords: forced, vibration, modelling, concrete, shear

1. INTRODUCTION

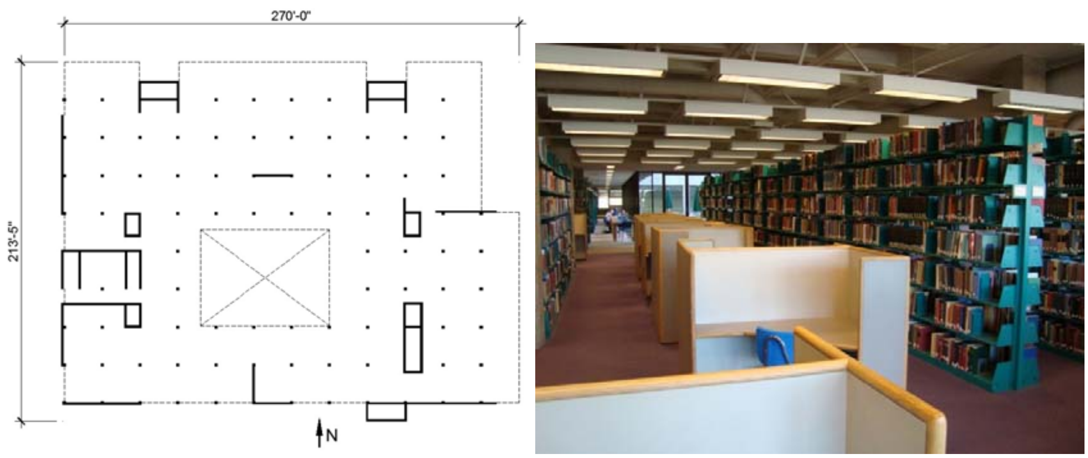
Modal parameters such as natural frequencies, mode shapes, and modal damping ratios are key information in determining the dynamic performance of a building. Typically computational models are created to estimate these parameters. One way to gain reasonable assurance that these computational models are accurate is to compare the models to forced vibration testing (FVT) results (McDaniel and Archer 2009, 2010). The authors are currently assessing results from computational models and full-scale forced vibration tests as part of a NSF NEESR project. This paper critically compares the natural frequencies and mode shapes obtained experimentally with those obtained through computational modeling of the Robert E. Kennedy Library located on the campus of California Polytechnic State University, San Luis Obispo (Fig. 2.1a). The structure is a five-story reinforced concrete shear wall building constructed in 1977. One of the unique characteristics of this building is the large atrium located near the center of the floor plan that runs the entire height of the building, taking up roughly 10-16% of the entire floor area. Additional complexity is added by the irregular arrangement of shear walls, shown in Fig. 2.2a, as well as the unique wall cross sections and discontinuous configurations in a few cases.

2. ANALYTICAL MODELLING

Modeling of the Kennedy Library progressed from a simple hand analysis to a detailed computational model. This progressive modeling started with simplifying assumptions so that the predictions could be calculated by hand. The first model assumed a lumped mass at the center of a rigid diaphragm with three degrees of freedom per floor. This model is illustrated in Fig. 2.3. To estimate the weight of the books, a sample area of bookshelves was weighed resulting in a value of 80 psf. The weight used for table and desk locations was taken as 25% of the design live load of 100 psf. The mass matrix was

(a) (b)

Figure 2.1. (a) Robert E. Kennedy Library; (b) plan view of the Kennedy Library



(a) (b)

Figure 2.2. (a) 2nd floor plan of building; (b) typical arrangement of furniture and books

derived using three degrees-of-freedom at the center of mass of each floor. The shear walls were modeled as line elements forced into double bending by a rigid slab. Originally shear stiffness, torsional stiffness, and out-of-plane wall stiffness were omitted. The resulting first mode frequency of 20.1 Hz was far from the experimentally determined frequency of 3.3 Hz. However, this artificially large stiffness was quickly attributed to shear which provided as much as 90% of the wall flexibility in some cases. The next hand calculation included shear stiffness and left everything else constant. The result of the analysis was a first mode frequency of 5.45 Hz (down from 20.1 Hz) verifying that shear accounts for a significant amount of the flexibility in this structure.

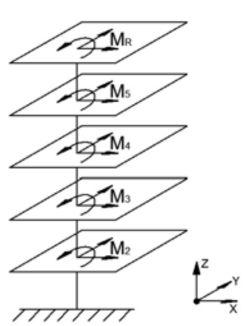


Figure 2.3. Lumped mass model

A computational model was used to adjust the model further. After calibrating the computational model with the previous hand analysis (f=5.45hz), the model was modified to include torsional stiffness and out-of-plane wall stiffness. The first mode frequency increased negligibly to 5.46 Hz.

The next step was to remove the rotational constraints at the floor levels and allow the walls to interact freely with the slab. The floor slabs are a waffle slab/pan joist system. Rather than modeling every joist of the slab, an equivalent thickness was used for the slab. Removing the double bending assumption yielded a fundamental frequency of 2.56 Hz resulting in a model that was too flexible. One issue was the interaction of the wall elements with the slab elements. The interaction of the wall and slab occurs along the length of the wall. Without adding additional constraints to this model, the deformed shape is incompatible as shown in Fig. 2.3. Deformation compatibility was enforced by adding an additional line element, rigid in flexure in the plane of the wall (i.e. R_y is infinite). It was assigned to connect the centerline of the wall/line element to the slab elements along the length of the wall. Adding these constraints to the model raised the fundamental frequency to 2.87 Hz (up from 2.56 Hz).

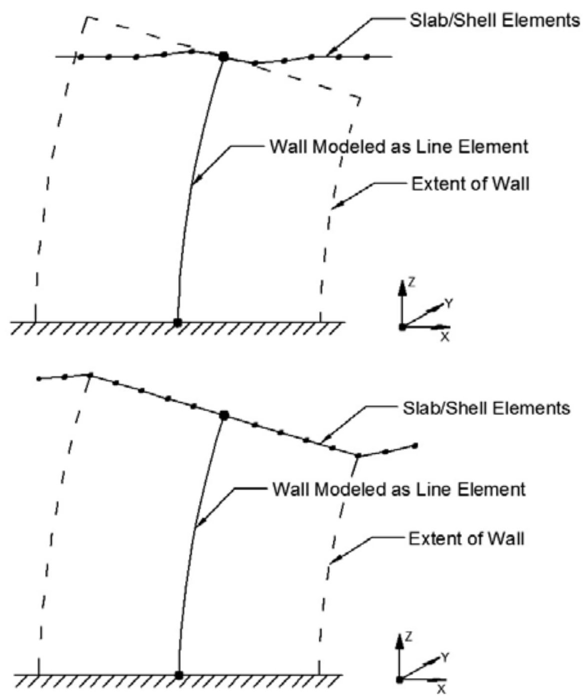


Figure 2.3. Compatibility error at wall/slab connection; rigid constraints at wall/slab connection

The next issue to consider was properly modeling the C, T, L and box shaped walls. The current model treated all wall assemblages as separate, disconnected line elements. Constructing the wall assemblages in this manner led to high underestimation of the flexural and torsional stiffness of the walls. A new model was developed with properties for moments of inertia, shear areas, and torsional constants for the entire wall assemblages. Four node elements could have been used to model the walls as well, however, these elements have been shown to be inadequate for modeling torsion (Wilson 2002), a dominant response of the library due to the irregular shear wall arrangement. To confirm the inadequacy, a 12'-0" tall, 9” thick, 22'-6" long wall was analyzed with a 10,000 kip-in torque applied at its centroid. The wall was analyzed both as a line element and with finely meshed 4 node elements. A mesh density of 32 x 60 elements was used because this density provided convergence to a final solution. Table 2.1 illustrates the inadequate torsional stiffness provided by the plane stress elements.

Table2.1. Comparison between line and wall wlement used for torsion

	Line Element Model	Shell Element Model	% Difference
Rotation about axis of the wall	0.00199 rad	0.00110 rad	81.5%

With the walls properly modeled, the next step was to remove the rigid diaphragm constraints across the floors, thereby modeling the stiffness of the diaphragm. This was necessary due to the diaphragm flexibility created by the large atrium. However, with the rigid diaphragm constraints removed, attention had to be paid to the points where the walls attached to the diaphragm. Previously this was not an issue because the degrees-of-freedom across the diaphragm were locked to each other. To properly model the connection of the walls to the diaphragm, individual rigid diaphragm constraints were placed to ensure that the translational degrees-of-freedom of the slab along the cross section of the wall were locked to the degrees-of-freedom of the point where the line element (geometric centroid of the wall) met the diaphragm, see Fig 2.4.

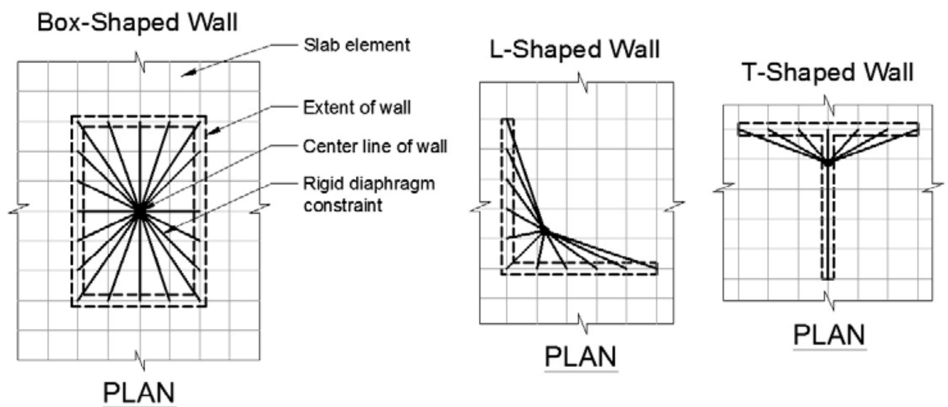
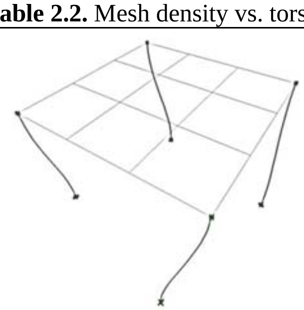
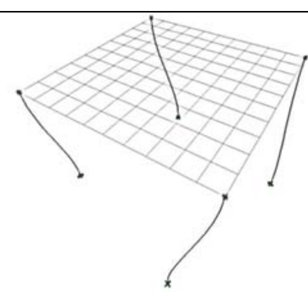


Figure 2.4 Rigid diaphragm constraints at wall/slab intersection

Another consequence of not modeling the diaphragm as rigid is that the mass for each floor could no longer be assigned as point masses at the center of mass. Thus the mass had to be modeled as area masses assigned to each 4 node element. However, modeling the mass as area mass can cause errors in the torsional modes if the mesh of the diaphragm isn't fine enough. Therefore, the relationship between mesh density and the effect on the torsional mode was studied in a series of analyses. Table 2.2 illustrates the results of the analyses. The same mesh density that was used in the test structure to get to within 1% of the theoretical period for the torsional mode (about 2' by 2' square elements) was used in the current model. The full building model is shown in Fig. 2.5. The single bay, single story model had a story height of 12', a bay size of 13.5' x 13.5', a story weight of 1000 kips, 6" square steel columns fixed at the base (shear and torsion of columns neglected), with a rigid diaphragm restrained from rotation about the global x and y axes to enforce the double bending assumption.

Table 2.2. Mesh density vs. torsional mode period

							
Mesh Density	Mode 3 Period		% Diff.	Mesh Density	Mode 3 Period		% Diff.
	Computational	Theory			Computational	Theory	
Area/9	0.91	0.82	10.5	Area/100	0.83	0.82	0.99

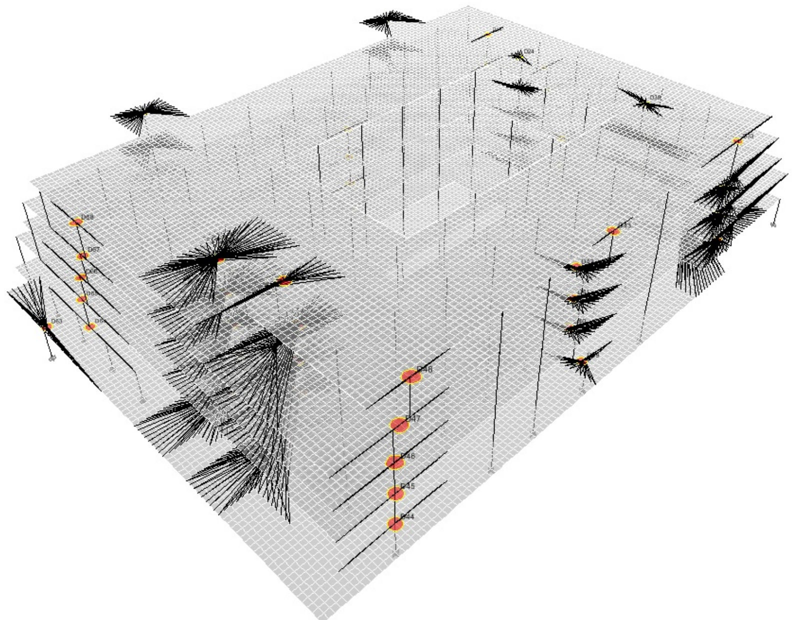


Figure 2.5. Computational model (CSI 2005)

Table 2.3 shows that the difference between the experimental natural frequencies and those obtained from the computational model are less than 20%.

Table 2.3 Computational model vs. experimental results

	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Computational Model Frequency	3.97	4.45	5.83
Experimental Frequency	3.30	3.65	4.56
% Difference	16.8 %	17.9 %	18.4%

Concrete cracking in the shear walls has the effect of reducing the initial stiffness of each wall. In order to account for cracked concrete, an effective stiffness of $0.7 I_g E_c$ was used for each wall (ACI, 2008); where I_g is the gross moment of inertia of the wall and E_c is Young's modulus for concrete ($E_c = 3605$ ksi). A comparison between the natural frequencies determined experimentally and those of the line wall element model with a reduced effective stiffness are presented in Table 2.4.

Table 2.4. Cracked concrete computational model vs. experimental results

	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Computational Model Frequency	3.54	4.05	5.16
Experimental Frequency	3.30	3.65	4.56
% Difference	6.7%	9.8%	11.6%

These results show a close correlation between the experimental and analytical results. Given the estimation of the effective live load and the library books, a 10% difference in natural frequencies is reasonable. In order to understand what a 10% difference in natural frequencies amounts to, two additional analyses were run. The first one included an additional 3" of 150 pcf concrete across each floor. Three inches of normal weight concrete amounts to about 14% of the estimated floor weight.

The results are shown in Table 2.5. The second analysis reduced the stiffness of the walls to 60% of the gross stiffness. Since using 70% of the gross stiffness of the shear walls is a rough approximation in ACI 318, a further reduction was explored here to study the sensitivity of the results to this value. The results are presented in Table 2.6.

Table 2.5. Experimental results vs. cracked concrete computational model (additional 3" concrete)

	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Model Frequency	3.17	3.64	4.64
Experimental Frequency	3.30	3.65	4.56
% Difference	4.3 %	0.3 %	1.8%

Table 2.6. Experimental results vs. cracked concrete computational model (60% gross stiffness)

	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Model Frequency	3.39	3.91	5.01
Experimental Frequency	3.30	3.65	4.56
% Difference	2.8 %	6.9 %	9.9 %

The results in these tables suggest that the mass of the building was likely underestimated since adding mass rather than reducing stiffness resulted in better correlation between the experimental natural frequencies and those from the computational model. A summary of all the models and their resulting natural frequencies is presented in Table 2.7.

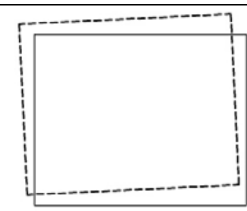
Table 2.7. Summary of computational models

Model Assumptions	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Walls modeled as disconnected rectangular line elements in double bending Neglect shear and torsional stiffness Neglect out-of-plane stiffness Rigid diaphragm assumption	20.08	33.00	51.28
Include wall shear stiffness	5.45	6.57	7.97
Include torsional and out-of-plane wall stiffness	5.46	6.57	7.97
Remove double bending constraint	2.56	3.64	4.92
Rigid constraints to enforce plane sections remaining plane at wall/slab intersection	2.87	3.90	5.11
Model C,T and L shape wall assemblage Semi-rigid diaphragm	3.97	4.45	5.83
Account for concrete cracking (0.7 I_g)	3.54	4.05	5.16
14% increase in mass	3.17	3.64	4.64
Experimental Data	3.30	3.65	4.56

3. COMPARISON OF ANALYTICAL TO EXPERIMENTAL MODE SHAPES

In addition to the natural frequencies, the experimental and computational mode shapes were compared to further assess the accuracy of the computational models. The experimental mode shapes for the first two frequencies are shown in Table 3.1. In spite of the large opening created by the atrium, the diaphragm behaved in a rigid manner for the first two modes. Preliminary investigation into the higher modes showed a flexible response of the diaphragm.

Table 3.1. Experimental mode shapes (EMS)

EMS #	Frequency (Hz)	EMS
1	3.3	
2	3.65	

The test for orthogonality of two modes is called the modal assurance criterion (MAC) (Allemang, 2003). In this experiment a mass-weighted modal assurance criterion was used (McDaniel and Archer 2010). The MAC compares two modal vectors and results in a number between 0 and 1. Two orthogonal modes have a MAC number of 0 and two identical modes have a MAC number of 1. The mass-weighted MAC formula is provided in Equation 1.

$$MAC = \frac{\left(\phi_i^T M \phi_j\right)^2}{\left(\phi_i^T M \phi_i\right)\left(\phi_j^T M \phi_j\right)} \tag{3.1}$$

Where ϕ_i and ϕ_j represent the two mode shapes being compared, and

M is the mass matrix of the structure.

Table 3.2 below shows the mass weighted MAC numbers that compare the experimental mode shapes to the mode shapes from the computational model.

Table 3.2 MAC numbers comparing the computational model and experimental mode shapes

	Mode 1	Mode 2
Experimental mode shapes	0.726	0.866

Although reasonably accurate, the MAC numbers could be improved by refining the computational model, particularly in regards to the modeling of the effective live load and the library book mass, concrete wall effective stiffness and the flexibility of the semi-rigid diaphragm.

4. CONCLUSIONS

Typically computational models are created to estimate key information in determining the dynamic response of a building such as natural frequencies, mode shapes, and modal damping ratios. One way to gain reasonable assurance that these computational models are accurate is to compare the models to forced vibration testing (FVT) results. The structure focused on for this research was the Kennedy Library, a 5-story reinforced concrete shear wall building on the California Polytechnic State University, San Luis Obispo campus. Characteristics of the library that make it unique and challenging to model include a large atrium at the center of the building and irregularly placed reinforced concrete shear walls. Increasingly complex computational models of the library building were created to determine the most influential modeling variables.

Models of the library that could be easily analyzed by hand were first explored. This consisted of modeling the building with three degrees-of-freedom per floor. This first model neglected shear deformation, out-of-plane stiffness, and torsional stiffness of the walls while treating all the walls as disconnected rectangular sections. The slab was treated as rigid and the mass was assigned as point masses at the center of mass. This analysis resulted in a fundamental frequency of 20.08 Hz, far larger than the experimental fundamental frequency of 3.30 Hz. The next hand analysis included the concrete wall shear stiffness, resulting in a fundamental frequency of 5.45 Hz. Such a dramatic difference shows the significance of accounting for shear stiffness of the walls. Removing the rotational constraints at the ends of each wall (removing the double bending assumption) dropped the fundamental frequency from 5.46 Hz to 2.56 Hz. As far as influencing the dynamic response, removing the double bending assumption was 2nd only to including the shear stiffness of the walls due to the cantilever behavior of the shear walls.

Next, improving the modeling of the C, T, L and box shaped walls was addressed. Treating the wall assemblages as separate, disconnected line elements led to high underestimation of the flexural and torsional stiffness of the walls. A new model was developed with properties for moments of inertia, shear areas, and torsional constants for the entire wall assemblages. To model the flexibility of the diaphragm, the diaphragm could no longer be modeled as rigid. Individual rigid diaphragm constraints were then necessary so that the translational degrees-of-freedom of the slab along the cross section of the wall were locked with the point where the line element (geometric centroid of the wall) met the diaphragm. These revisions shifted the fundamental frequency from 2.87 Hz to 3.97 Hz. In order to account for the effective concrete stiffness due to cracking, seventy percent of the gross stiffness for bending was used in the next model, resulting in a fundamental frequency of 3.54 Hz. This brought the computational model to within 7% of the experimental frequency, a close estimate considering the multiple variables influencing the dynamic response of the building.

The computational model mode shapes were also compared with the experimental mode shapes; mass weighted MAC numbers of 0.726 and 0.866 were calculated for the first and second modes, respectively. Although reasonably accurate, the MAC numbers could be improved by refining the computational models. Both mass and stiffness were varied to determine where refinement of the computational model should be focused. Increasing the mass of the building was found to be more effective than decreasing the building stiffness. Variables that could have led to the differences in the computational and the experimental results include the current concrete stiffness, the estimation of the library mass including the books and the effective live load, and modeling of the flexible diaphragm including the atrium.

ACKNOWLEDGEMENT

This project was supported in part by the George E. Brown, Jr. Network for Engineering Simulation Research (NEESR) program of the National Science Foundation under Award Number CMMI-1135037.

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