SUMMARY:
The main objective of this work is to define an analytical formulation to evaluating the increase in structural factor due to the addition of hysteretic dissipation systems to an existing reinforced concrete framed structure in order to characterize rules to supply q-values. This would enable the use of linear elastic methods in the evaluation of seismic loading also for retrofitted structures.

In this paper, the results of a parametric study of non-linear analyses performed on different types of R/C buildings strengthened by means of Hysteretic Energy Dissipating Bracing (EDB’s), are reported. The selected R/C buildings are representative of seventies/eighties Italian buildings, designed for vertical load only, upgraded considering different design parameter and configuration of the EDB system. A Displacement-Focus Design (DFD) procedure compatible with Italian and European seismic code [NTC08, 2008; EC8-1, 2004] has been considered to evaluate the mechanical characteristics of the dissipating devices, with the purpose of limiting the inter-storey drifts after a target drift.

Keywords: Behaviour factor; Retrofit with dissipation bracing; Metallic hysteretic dampers

1. INTRODUCTION

Several seismic codes have adopted a new generation of simplified linear and nonlinear analysis methods for the design and seismic assessment of buildings. For conventional dissipating structural systems the actual seismic codes allows for evaluating the seismic effects of the seismic actions by using a linear elastic analysis, so referring to an elastic spectrum reduced by a behaviour factor $q>1$. Typically for reinforced concrete buildings different values of q-factor are defined by codes as a function of structural type and regularity criteria [NTC08, 2008]. However in practical application there is still an evident lack of detailed rules and methods of application for structural types differing from new, conventional structures.

In recent years, a series of innovative strategies for controlling the buildings seismic response have been studied and put into practice. One of these methods is the placement of passive control dissipative bracing systems through special devices inserted in the structural frame which have the ability to dissipating large amounts of energy during a seismic event [Soong and Dargush, 1997; Constantinou et al., 2001; Christopoulos and Filiatrault, 2007; Ponzo et al., 2009]. With the aim to increase the use of seismic reinforcement of buildings based on this strategy, the adoption of linear analyses, based on the behaviour factor approach, instead of more complex non-linear analysis, could be very useful. During the design, the engineer only isolates the characteristics of the dissipative braces (elasto-plastic dissipating contributions) as a function of the original structural characteristics, the seismic hazard of the building site, and the predicted targets of the retrofitted structure. For this kind of construction structural factors are not provided by current codes, as for example are provided for a structure with seismic isolation ($q=1.5$). The code instead refers to approved methods which account for the plasticization of both the structure and the dissipative device.

The principle objective of this work was to define an analytical formulation to evaluate the increase in structural factor from the addition of hysteretic dissipation systems to an existing framed concrete structure. This would enable the use of linear elastic methods in the evaluation of seismic loading.
2. ANALYTICAL INVESTIGATION

2.1. Design procedure of dissipative bracing system

A Displacement-Focused Design (FDB) procedure compatible with Italian and European seismic code [NTC08, 2008; EC8-1, 2004] has been considered to evaluate the mechanical characteristics of the dissipating devices, with the purpose of limiting the inter-storey drifts under the target drift. The design of the individual elasto-plastic dissipative brace (initial stiffness \( k_{c,i,s} \) and yield point \( F_{c,i,s} \)) was performed for all buildings based on the method proposed by [Ponzo et al., 2010] which is synthesized in Fig. 1a. The performance objective considered in design was to reach a fixed ductility value of the structure upgraded with different solutions of elasto-plastic dissipative brace under the considered earthquake including specific site amplification.

![Diagram of design procedure](image)

**Figure 1.** a) Proposed procedure for determining the properties of nonlinear hysteretic dampers; b) Step 2

**Figure 2.** Iterative procedure for equal energy criteria (\( T^* < T_C \)) and for equal displacement (\( T^* \geq T_C \))

2.1.1. **STEP ONE:** Evaluation of the existing structure as a single degree of freedom (SDOF) system (\( F_y^*, k^*, d_m^* \)) using pushover analysis.

The first step of the method is to determine the idealized elasto-perfectly plastic force-displacement relationship of the existing structure as a SDOF system. This involves the calculation of: the yield force \( F_y^* \), the initial stiffness \( k^* \), and the maximum displacement \( d_m^* \). These values are obtained through the use of a non-linear static analysis (NLSA) in both main directions. This is based on the hypothesis that the structures behavior will be governed by its natural period, consisting of a single significant participating mass [§ 7.3.4.1 NTC08, 2008; § 4.3.3.2.1 EC8-1, 2004].

At this stage of the design method infill panels are accounted for only in terms of their mass [§ 7.2.6 NTC08, 2008; § 4.3.6 EC8-1, 2004]. In the analysis any commercially available software may be used. At least two lateral load distributions should be applied (both uniform and modal), in both the positive and negative direction, at the centre of mass of each floor [§ 7.3.4.1 NTC08,2008; § 4.3.3.4 EC8-1, 2004]. The determination of the equivalent SDOF system is made using the critical capacity curve (i.e. the lower of the two curves). This curve is then idealized to be an elasto-perfectly plastic system reduced by the transformation factor [§C.7.3.5 NTC08, 2008; Eqn. B.3 EC8-1, 2004]. The ultimate displacement \( (d_m^*) \) is assumed to be the minimum between the maximum obtained through analysis and that which is compatible with the structural behavior factor defined by the code [§C.7.3.8 NTC08, 2008; §4.3.4 EC8-1, 2004].

In practice the aim of this first step is the evaluation of the starting value of the iterative procedure contained in STEP 2. The verification of the braced system will be performed in STEP 4.
2.1.2. STEP TWO: Determination of the mechanical characteristics \((F_c, k_c, \mu_c)\) of the single degree of freedom equivalent bracing system through the use of an iterative procedure.

Starting from the idealized elasto-perfectly plastic force-displacement relationship of the existing structure evaluated in STEP 1, the mechanical characteristics \((F_c, k_c, \mu_c)\) of the single degree of freedom equivalent bracing system are determined. The following iterative procedure is applied separately in the principle directions.

2.1. A design objective is established represented as a target displacement value \((d_{s0})\) of the braced structure. One of the following two cases is sought: \(d_{s0} \leq d_{y}^*\) if the aim is for the structure to remain elastic, or \(d_{y}^* < d_{s0} \leq d_{m}^*\) if the aim is to use also the hysteretic capacity of the original structure.

2.2. A design ductility of the equivalent bracing system \((\mu_c)\) is established as a function of the selected dissipation devices. This value refers to the behavior of the dissipative device and the brace itself in series \([\text{§C7A.10.4.1 NTC08, 2008}]\).

2.3. After each \(j^{th}\) step the reference seismic force of the elastic oscillator \((F_{e,j})\) is evaluated. This is done as a function of the period of the complete system made up of the equivalent structure in parallel with the equivalent bracing (corresponding to the elastic part of the bilinear curve, \(k_j^*\), shown in Fig. 1b). The elastic force, \(F_{e,j}\), at each iteration is calculated to be the equivalent mass of the SDOF system multiplied by the acceleration of the elastic spectra \((\xi = 5\%)\) referenced to the ultimate limit state \([\text{§2.6.1 NTC08, 2008}; \text{§2.2.2 EC8-1, 2004}]\). The procedure starts \((j = 0)\) by considering bi-linear curve of the equivalent structure (without bracing): the equivalent period \(T_0^* = T^*\); the elastic stiffness \(k_0^* = k^*\); and the yield strength \(F_{y,0}^* = F_{y}^*\), as prescribed in \([\text{§C7.3.4.1 NTC08, 2008}; \text{Annex B EC8-1, 2004}]\).

2.4. The characteristic of the equivalent bracing system are calculated using the equal energy criteria. Imposing an ultimate displacement of the bracing \(d_{c0}\) equal to the target displacement \((d_{s0} = d_{c0})\) the yield displacement of the bracing \((d_{cy})\) is obtained as a function of the design ductility, \(\mu_c\) (step 2.2) as shown in Fig. 1b. The single remaining factor to be estimated is the yield force, \(F_{c,j}\) of the equivalent bracing system for this \(j^{th}\) iteration. Once chosen, the stiffness of the bracing system can be determined by Eqn. 2.1.

\[
d_{cy} = \frac{d_{c0}}{\mu_c}; \quad k_{c,j} = \frac{F_{c,j}}{d_{cy}}
\]  

Summing the two bilinear curves of the initial structure \((S)\) and the equivalent bracing system \((CE)\) the trilinear parallel system curve is found \((S+CE)\). A bilinear approximation of this curve is then made \((\text{EP}(S+CE))\), according to \([\text{NTC08, 2008}]\). Referring to Fig. 1b, the equal energy criteria between the energy accumulated in the infinitely elastic SDOF oscillator and the idealized elasto-plastic response of the braced structure can be expressed as the equality of the area under the oscillator \((E)\) and the elasto-plastic \((\text{EP}(S+CE))\).

From the yield force value of the equivalent bracing system at the \(j^{th}\) step, \(F_{c,j}\), it is possible to determine: a) the bracing stiffness, \(k_{c,j}\); b) the period of the braced structure \(T_j^*\) in correspondence to the stiffness \(k_j^*\); and therefore c) the new value of \(F_{e,j}\). Repeating steps 2.3 and 2.4 the method is considered to have converged when the difference between the value of \(F_{e,j}\) at the \(j^{th}\) step and that of the step before is smaller than an imposed tolerance value: \(|F_{e,j} - F_{e,j-1}| < \varepsilon\).

The initial assumption of equal energy between the elastic and elasto-plastic oscillator valid for short period range \(T_j^* < T_C\), where \(T_C\) is defined in \([\text{§3.2.3.2.1 NTC08, 2008}]\) is assumed due to the stiffening effect of the insertion of bracing into the structural system. In the case of medium and long period range were \(T_j^* \geq T_C\) an equal displacement assumption is made between the elastic and equivalent plastic oscillators, as displayed in Fig. 1b. In this case, beginning as always from the ultimate displacement of the equivalent bracing \(d_{c0}\) and considering the design value of bracing ductility \(\mu_c\), the yield limit \(d_{cy}\) (Eqn. 2.2) is determined and the remaining characteristic values for the equivalent bracing are defined.
2.1.3. STEP THREE: Determination of the characteristics of the single dissipative bracing elements
\((F_{c,i,s}; k_{c,i,s}; \mu_c)\);

The mechanical characteristics of the single bracing system at each floor \((F_{c,i,s}; k_{c,i,s}; \mu_c)\) are calculated following a distribution developed in two phases as described below.

3.1. In the first phase the characteristic values of the equivalent system are distributed up the height of the building thus defining the characteristic value for each floor \((F_{c,i}, k_{c,i}, \mu_c)\). This distribution is calibrated aiming to achieve a uniform inter-storey displacement distribution and thus maximize the efficiency of the dissipation system. This also means that no single floor will suffer an excessive inter-storey displacement which should always be avoided in building, as being linked to damage and collapse due to a soft storey mechanism, a mechanism of collapse frequently seen in existing buildings [§ C7A.10.4.2 NTC08, 2008].

The stiffness of the equivalent bracing at the \(i\)th floor \(k_{c,i}\) is distributed hypothesizing that at the \(i\)th floor the ratio between the stiffness at each floor \(k_i\) and that of the relative bracing \(k_{c,i}\) will be proportional to the ratio \(r_k\) between the stiffness of the equivalent structure \(k^*\) and the equivalent braced system \(k\) (eq. 2.2). The strength of the equivalent bracing at the \(i\)th floor is distributed hypothesising that the ratio between the strength and that of the equivalent bracing \(F_c\) will be equal to the ratio between the strength of the \(i\)th floor of the structure \(F_{y,i}\) and that of the equivalent system \(F^*_{y,0}\) (see Eqn. 2.3)

\[ k_{c,i} = r_k \cdot k_i ; \quad k_i = \frac{1}{\Delta s_i} \cdot \sum \Delta F_i ; \quad r_k = k_c / k^*; \quad (2.2) \]

\[ F_{c,i} = \frac{F_c}{F^*_{y,0}} \cdot F_{y,i} ; \quad F_{y,i} = k_i \cdot d_{y,i} ; \quad d_{y,i} = \frac{\Delta s_i}{s_{TOT}} \cdot d^*_y \quad (2.3) \]

The strength of each floor of the structure can be calculated, in a simplified manner, from the displacements as the elastic limits of the floor \(d_{y,i}\) determined redistributing the displacement at elastic limit of the original structure \(d^*_y\) as a function of the ratio between the inter-storey displacement \(\Delta s_i\) and the total elastic displacement \(s_{TOT}\) calculated during the previously mentioned static analysis. The stiffness of the \(i\)-th floor \(k_i\) of the original structure can be calculated from the inter-storey displacement \(\Delta s_i\) generated by a distribution of forces \(F_i\) applied to each floor using linear static analysis.

In the case where the distribution of stiffness up the building is irregular (concrete frame plus bracing \(k_{tot,i} = k_i + k_{c,i}\)), the contribution to the total stiffness of the bracing system is modified with the objective to regularize the dynamic performance of the structure. To this end reference is made to the criteria set out in [§7.2.2 NTC08, 2008] for the definition of a building which is regular in height. For a structure of \(n_s\) floors (having a total number of floors grater than 2) see Eqn. 2.4.

\[
\begin{align*}
\text{if } \Delta k^{j}_{tot,i} > 0.3 & \quad k^{j}_{c,i} = 0.7 \cdot k^{j-1}_{tot,i-1} - k_i \\
\text{for } i = n_s, ..., 2; & \\
\text{if } \Delta k^{j}_{tot,i} < -0.1 & \quad k^{j}_{c,i-1} = \frac{k^{j-1}_{tot,i} - k_{i-1}}{1.1} \\
\text{if } -0.1 \leq \Delta k^{j}_{tot,i} \leq 0.3 & \begin{cases} k^{j}_{c,i} = k^{j-1}_{c,i} \\ k^{j}_{c,i-1} = k^{j-1}_{c,i-1} \end{cases}
\end{align*}
\quad (2.4)
\]

where: the number of iterations \(j\) necessary to find convergence of the procedure can be no larger than the number of floors in the structure \(n_s\).

3.2. During phase two the characteristics of a single dissipative bracing element at each \(i\)th floor are determined \((F_{c,i,s}, k_{c,i,s}, \mu_c)\), beginning with the mechanical characteristics of the equivalent bracing
system of the given storey \((F_{c,i}; \ k_{c,i}; \ \mu_c)\) and as a function of the quantity, position and angle of inclination \(\phi\) of the planed bracing for the storey [§C7A.10.4.1 NTC08, 2008]. The stiffness \(k_{c,i}\) and the strength \(F_{c,i}\) of the dissipative brace at the \(i^{th}\) floor is calculated as in Eqn. 2.5. The characteristic values of the single dissipative brace \((F_{c,i,s}; \ k_{c,i,s}; \ \mu_c)\), are determined as the summations of the braces parts in series [§C7A.10.4.1 NTC08, 2008], in which \(k_{d,i,s}, F_{d,i,s}, \mu_d\) are the stiffness, the strength and the ductility of the dissipative devices respectively, while \(k_{a,i,s}, e F_{a,i,s}\) represent the stiffness and the yield force of the devices metal supports, connected with Eqn. 2.6.

\[
k_{c,i,s} = \frac{k_{c,i}}{n_{c,i}} \cdot \frac{1}{\cos^2 \phi_i}; \quad F_{c,i,s} = \frac{F_{c,i}}{n_{c,i}} \cdot \frac{1}{\cos \phi_i};
\]

\[
F_{c,i,s} = F_{d,i,s}; \quad k_{c,i,s} = \frac{k_{d,i,s} \cdot k_{a,i,s}}{k_{d,i,s} + k_{a,i,s}}; \quad \mu_c = \frac{k_{d,i,s} + k_{a,i,s} \cdot \mu_d}{k_{d,i,s} + k_{a,i,s}}
\]

(2.5)

(2.6)

where: \(n_{c,i}\) is the number of braces on the \(i^{th}\) floor and \(\phi_i\) is the angle between the single brace and the horizontal.

The selection of the characteristic values of the single dissipative devices will be within the range provided by the material manufactures while the metal support will be chosen considering \(k_{a,i,s}/k_{d,i,s} \approx 2\) and verified for either buckling in compression or yield in tension under the life ultimate limit state (SLC limit state of collapse) loading [§C7A.10.2 NTC08, 2008].

2.1.4. STEP FOUR: Verification of the braced structure under ultimate limit state (SL life safety) conditions using a non linear static analysis.

The design procedure is completed with a verification of the global soundness of the braced structure under SLV conditions [§C7A.10.6 NTC08, 2008], using a non linear static analysis [§C7A.10.5.2.2 NTC08, 2008; §4.3.3.4.2 EC8-1, 2004], performed considering a numerical model of the structure in which the dissipative elements and structural system can behave nonlinearly [§C7A.10.5.1 NTC08, 2008]. During the non linear static analysis the criteria set out in [§7.3.4.1 NTC08, 2008] are used for the verification of the complete system which includes the dissipating braces. The verification is satisfied when the displacement demand on the structure \(d^*_{\text{max}}\) is less than the design displacement \(d_{\text{S0}}\) [§7.3.3.3 and §C7.3.4.1 NTC08, 2008]. In cases where the displacement demand is greater than the project displacement, therefore the above criteria is not satisfied, it is possible to increase as much as possible the project displacement, or, alternatively modify the design ductility of the bracing \(\mu_c\).

2.2. Cases study

Numerous nonlinear static analyses (NLSA) have been performed on various building types considering different combinations of properties fundamental to dissipative systems design. The results of these analyses have been used in order to evaluate an analytical formula for the definition of a modified structural factor \((q_C)\), relating to concrete frame buildings containing displacement dependant dissipative bracing systems.

Twelve buildings which characterize gravity only design have been used as a base for the parametric analysis. These consisted of four different plans (rectangular, L shaped, and cruciform) with five values of number of stories (3, 4, 5, 6 and 8). C20/C25 concrete and FeB38K steel were used in design. The beam and column dimensions and detailing were keep the same for all four types of structural plan, typical of Italian construction of the 70’s and 80’s.

The flexural and shear stiffness of the models elements are reduced to 50% of the stiffness of the uncracked elements [§ 7.2.6 NTC08,2008] in order to account for the behavior of the brittle materials and the influence of permanent axial loading in the columns [Paulay and Priestley, 1992]. In the case under study it was assumed that the strength of materials were checked and through the use of confidence factors, reduced further in case of brittle mechanism using partial security factors strength values approximately equal to the design values were obtained [§C8.7.2.4 NTC08, 2008].
The numerical modelling was performed using a commercially available finite element program [CDS Win, 2010]. Some examples of numerical models of the bare structures are shown in Fig. 2 for different structural types with 3 storey. The seismic actions relating to different limit states were defined using the elastic spectra shown in Fig. 3. These spectra were developed considering the seismicity of the Potenza area, soil class Type B, and topographic factor T1 [§3.2 NTC08, 2008]. The results of NLSA (step 1 of Fig. 2a) of all initial structures (bare frame) are also compared to the seismic demand in Fig. 3. In Fig. 4 the numerical models of all 3-storey structural types ($n_s$, $3$), with both reinforcing bracing arrangements (V and X) structures, are shown. The regularization of the structure in plan comes from the careful placement of the bracing system on the perimeter frames that leads to a significant increase in the torsional stiffness of the building in plan.

**Figure 2.** Numerical model of the bare structure: a) 3 storey of the different structural types and b) structural type 4 of the different number of storeys.

**Figure 3.** Pushover curves of the base structures compared with the ultimate state (SLV) seismic action

**Figure 4.** Numerical models of 3 storey of different structural types braced with two configurations (V and X).
The design of the bracing systems was performed two times for both directions of each base structure. This was done considering two diverse bracing arrangements: V inverted (V) and diagonal systems (X), in total 24 different cases were considered. In each case the procedure was used considering different design targets: (i) four values of structural ductility ($\mu_s$) 1.0, 1.15, 1.3, 1.5) and (ii) three values of bracing ductility ($\mu_C$) 4, 8, 12), summing to 960 cases. All of these cases did not consider any specific intervention to the structure elements (beam and columns) where the bracing was applied. In the finite element software considered in this study [CDS Win, 2010] a particular element, used to simulate the performance of the elasto-plastic bracing, is enabled.

3. EVALUATION OF THE STRUCTURAL FACTOR

All of simplified nonlinear analysis methods for the design and seismic assessment of buildings combine the pushover analysis of a multi-degree-of-freedom (MDOF) model with the response spectrum analysis of an equivalent single-degree-of-freedom (SDOF) system, to provide an estimation of the global displacement response of structures that exhibit nonlinear behaviour under strong earthquakes. The main methods based on Nonlinear Static Analysis (NLSA) are: (i) the Capacity Spectrum Method as adopted by [ATC 40, 1996]; (ii) the Displacement Coefficient Method, considered by [FEMA 356, 2000]; and (iii) the N2 Method which has been recently implemented in Eurocode 8 [EC8-1, 2004] and adopted also by Italian seismic code [NTC08, 2008], developed through the adornment [Circ. 617, 2009].

For conventional concrete structures the structural factor $q$ used in linear static analysis is generally defined in the design codes [NTC08, 2008; EC8-1, 2004] as $q = q_0 \cdot K_R$ where: $q_0$ is the maximum possible structural factor which depends on structural type and the ratio $\alpha_u/\alpha_1$ defined as the ratio between the collapse multiplier ($\alpha_u$), and the multiplier for flexural yield ($\alpha_1$), and $K_R$ is a reduction factor which represents the regularity up the structure. For concrete structures with energy dissipation bracing system the structural factor $q_c$ for linear static analysis is not defined in the codes [NTC08, 2008; EC8-1, 2004]. The factor $q_c$ of the braced structure is a function of the specific building and the system of dissipation used and will be independent of the seismic actions of the building site being examined. The analytical formula proposed for the calculation of the structural factor for braced buildings uses a coefficient $C$ to augment building initial value of $q$ as expressed in Eqn. 3.1 [Di Cesare et al., 2011].

\[
q_c = q(T^*, \mu_s) \cdot C(\mu_s, \mu_C, n_s, T_f^*/T_i^*, F_C/F_y^*)
\]

This coefficient $C$ is a function of parameters easily evaluated through linear analysis: structural ductility $\mu_s$; the ductility demand on the bracing $\mu_C$; the number of floors $n_s$; the ratio between the bilinear equivalent period of the braced structure at the end of the final step (step 4, see Fig. 1a) and that of the original structure $T_f^*/T_i^*$; the ratio between the yield point of the bracing and the resistance of the original structure $F_C/F_y^*$ (referring to Fig. 2b). The results of the verification of the braced structures, using the NLSA after each iteration (step 4 of Fig. 2a) for all considered structures have shown at least a verified case for both direction; some exception required specific intervention to the structure elements (beam and columns) where the bracing is applied.

3.1 Linear Correlation Analyses

The correlation between $C$ and each variable of the proposed analytical formulation is shown in Fig. 5-7 considering the results after each iteration be it verified or not. It can be seen that coefficient $C$: i) grows with growth in the ratio $F_C/F_y^*$ (in the range of 0.3 – 1.4), ii) grows with decrease in the initial ductility $\mu_s$ (in the range of 1.0 – 1.5), iii) grows with reduction in the ratio $T_f^*/T_i^*$ (in the range of 0.2 – 0.8), iv) seems almost constant with the value $\mu_c$ and v) does not have any strong correlation to the value of $n_s$ for the cases studied.
Figure 5. Correlation between C and base structural ductility $\mu_s$

Figure 6. Correlation between C and the ductility demand on the bracing $\mu_c$ and the number of floors $n_x$

Figure 7. Correlation between C and the ratio between period $T_f^*/T_i^*$ and the ratio between resistance $F_C/F_y^*$
3.2 Linear Regression Analyses

The best correlation between the values of $C$ evaluated by NLSA of braced structures and that calculated $C_{cal}$ considering different combinations of the five proposed independent variables ($\mu_s; T_f^*/T_i^*; F_C/F_y^*$; $\mu_C; n_s$) is obtained through a linear regression performed on the results of all iterations written in the formulation of Eqn. 3.2.

$$C_{cal} = m_1 \cdot \mu_s + m_2 \cdot \frac{T_f^*}{T_i^*} + m_3 \cdot \frac{F_C}{F_y^*} + m_4 \cdot n_s + m_5 \cdot \mu_C$$  \hspace{1cm} (3.2)

where: $m_1, ..., m_5$ are the coefficients of the considered independent variables.

The values of the coefficients $m_1, ..., m_5$ are shown in Table 3.1 together with the correlation degree $R^2$ obtained comparing $C$ and $C_{cal}$, as shown in Fig. 8. In Fig. 8 the single coefficient’s weight in the linear correlation law is also shown. As you can see the linear regression analysis was repeated twice considering a combination of only three independent variables ($\mu_s; T_f^*/T_i^*; F_C/F_y^*$).

<table>
<thead>
<tr>
<th>Direction</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
<th>$m_5$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
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<td>2.43</td>
<td>0.01</td>
<td>0.05</td>
<td>0.97</td>
</tr>
<tr>
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<td>2.33</td>
<td>0.02</td>
<td>0.02</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 3.1. Linear regression coefficients

Figure 8. Diagram of coefficient $C$ evaluated by mean of NLSA and $C_{cal}$ calculated by the proposed analytical formulation) and weight of considered variables

4 DISCUSSION AND CONCLUSION

The results of the verification displayed the efficiency of the design method used [Ponzo et al., 2010] in providing at the end of the procedure at least one solution which satisfies the Italian code [NTC08, 2008]; in the other cases does not provide a verified solution. In the last cases, the failure of the
solutions can be attributed to the limited strength of the structure compared to the reference seismic action, these would therefore require local reinforcement to adjust the minimum levels of resistance thus leading to convergence of the procedure.

The increase of the structural factor, defined by the coefficient $C$, obtained through the insertion of hysteretic type dissipative bracing systems into existing R/C frame structures for the design parameters used in the cases studied varied from $1 < C < 4$, depending on the combinations of design parameters. The best correlation between the values of $C$ evaluated using NLSA and that calculated $C_{cal}$ is obtained through a linear regression considering only three main parameters easily valuable through linear analysis.

Among the beneficial effects of the insertion of bracing is the ability to make the structural regular in both elevation and plan in all cases.

In all case investigated a good agreement between the capacity curve of the braced system having 1 DOF with that calculated using NLSA and the MDOF was observed. In particular In many cases did the agreement improve if the strength of the bracing was increased by a factor of $1.2 – 1.5$. From the analyses performed in order to avoid the overloading of the elements of the original structure it is recommended that: i) the yield force of the equivalent bracing $F_C$ is not too high in with respect to the yield force, $F_y^*$ of the original structure (i.e. $F_C / F_y^* < 1.3$), and ii) the stiffness of the braced structure is not too high with respect to the original structure (ie. $T_C^* / T^* > 0.2$). Alternatively it is possible to increase the number of braced bays or increase the strength of the original structure through the use of various classical retrofit methods localized on the elements near the bracing systems.

The comparison between the different bracing forms used (X and inverted V) showed that the regularization is more significant in the case of bracing form X. This configuration is better in resisting the axial deformation of the column when compared to the form inverted V.

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