SUMMARY:
The probabilistic model was not the best choice for parameters uncertainties analysis in the low seismicity area. When the data and information were scarcely, the convex model provided an alternative path for modeling the uncertainties of $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$ in the seismic hazard analysis. Convex analysis method and China Probabilistic Seismic Hazard Analysis methodology are combined to derive the peak velocity for Ningbo city, China. The peak velocity calculated by Chinese code method is within the range obtained convex models except for several special cases, and the interval of peak velocity is most sensitive to the annual occurrence rate $v$.

Keywords: Peak velocity seismic hazard analysis convex model

1. INTRODUCTION

Peak velocity is an important parameter for earthquake engineering and can be predicted by the seismic hazard analysis procedure. The uncertainties of geological and seismologic information lead to strong uncertainties in each module of seismic hazard analysis, such as the source zones, the seismicity parameters and the attenuation relationship. As a result, the peak velocity of a site calculated by seismic hazard analysis procedure is highly uncertain. The uncertainty in models and parameters of Probabilistic seismic hazard analysis (PSHA) was firstly investigated for seismic hazard assessment of nuclear power plants in middle and eastern United States by McGuire [1], and the aleatory and epistemic uncertainties were modeled by logic tree methodology. The normal distribution function was employed to model the uncertainty of the source zone by Bender [2], and the uncertainty effects in source zone on assessing the seismic hazard were discussed. The use of logic trees for ground-motion prediction equations in seismic hazard analysis was presented by Bommer et al. [3]. The epistemic uncertainty can also be modeled flexibly in the Monte Carlo simulations-based seismic hazard analysis method, and it is also applicable for seismic hazard analysis in a low to moderate seismicity area [4-6].

Mualchin [7] remarked that current understanding of earthquake processes and limited data do not warrant overly complex analyses using logic tree methodology. Such analysis tends to divert attention away from hazard concerns and the investigation costs more in both time and money without necessarily improving seismic safety of structures. The use and misuse of logic trees in probabilistic seismic hazard analysis were also analyzed by Bommer and Scherbaum [8]. The results derived by Grandori et al. [9] and Klügel [10] show that the PSHA based on multiple expert opinions was intrinsically unreliable when the dispersion was very significant, i.e., the mean value may not coincide with the true value in these situations.

The seismicity parameters, such as $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$, are found to be all highly uncertain and have strong effects on seismic hazard assessment. The uncertainties of these seismicity parameters are modeled by the probabilistic model.
when the parameter is consistent with a certain distribution in the current process of PSHA. The probabilistic theory has been proven to be successful in dealing with uncertainty in engineering, and stochastic methods have become powerful tools, which greatly enhance the capability of engineers in making decisions in uncertain situations. Obviously, in a low seismicity region, which is free of the distinct major active faults, the earthquake record is relatively scarce and short term. Because of the sparsity of earthquake occurrence, the modeling of the uncertainty associated with earthquakes using probabilistic concept has been subjective in this situation. Alternatively, the set-theoretic, convex description of uncertainty is more appropriate, which does not need any information about how the data are distributed in its domain. The convex method of modeling uncertainty is well suited when only scarce information is available, as in the case of seismic hazard analysis of the low seismicity region.

In this study, the non-probabilistic convex analysis theory is firstly introduced, and the envelop bound convex model and the ellipsoidal bound convex model are used to deal with the uncertainties of $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M$ in seismic hazard analysis. Finally, the convex set theory and CPSHA (China Probabilistic Seismic Hazard Analysis) are combined to predict the peak velocity for Ningbo city.

2. BOUND SEISMIC HAZARD ANALYSIS BASED ON CONVEX THEORY

2.1. Convex analysis method

In the convex analysis method, the uncertainties of variables or functions are modeled by some typical convex set models [11]. Among the models, the envelope bound convex model is frequently employed to model the uncertainties of uncorrelated variables, that is

$$\Omega_{EB} = \left\{ \alpha(t) \in R^r : \tilde{\alpha}_j(t) \leq \alpha_j(t) \leq \hat{\alpha}_j(t), j = 1, \cdots, r \right\} \quad (2.1)$$

where $\Omega_{EB}$ is the convex set; $\alpha(t)$ is the vector of uncertain variable; $\tilde{\alpha}_j(t)$ and $\hat{\alpha}_j(t)$ are the lower and upper bounds of the $j$th uncertain variable, and $R^r$ is a real set. Conversely, the ellipsoidal bound convex model may consider the variables correlation, that is

$$\Omega_{ELp} = \left\{ u : (u - u^\circ)^T W (u - u^\circ) \leq \alpha^2 \right\} \quad (2.2)$$

where $\Omega_{ELp}$ is the convex set; $\alpha$ is the radius of the convex set; $u$ is the $n$ dimensional vector of uncertain variable; $u^\circ$ is the nominal value of the vector $u$; $W$ is the positive definite weighting matrix, and $\alpha$ and $W$ describe the uncertain extent of the variable. After a simple mathematical manipulation [12], the ellipsoidal bound convex model described in Eqn. (2.2) can be presented as

$$U(\Delta u, \alpha, u^\circ) = \left\{ u : \sum_{i=1}^{n} \frac{(u_i - u^\circ)^2}{(\alpha \Delta u_i)^2} \leq 1 \right\} \quad (2.3)$$

where $\alpha \Delta u_i = \tau_i$, is the radius of the ellipsoidal Eqn.(2.3).

The determination of the least favorable response (the maximum of response) and the most favorable response (the minimum of response) by convex analysis method is an extreme optimum problem with constraints. Lagrange multiplier method, Kuhn-Tucker method and sequential programming method, and etc., can be used to solve this problem. The process of convex analysis can be described as follows
\[
\text{find}(\vec{X}) \\
\min \text{ or } \max S(\vec{X}) \\
\text{s.t } \vec{X} \in CM
\] (2.4)

where \(\vec{X}\) is the vector of uncertain variable; \(S(\vec{X})\) is the objective function, and \(CM\) is a certain convex model.

### 2.2. Seismic hazard analysis methodology in China

The seismic hazard analysis methodology in Chinese code for seismic safety evaluation of engineering sites [13], which is little different from the PSHA proposed by Cornell [14], considers inhomogeneity of earthquake phenomenon in time and space.

The probability of being in or exceeding a particular intensity is presented as

\[
P(A \geq a) = 1 - \exp\left(-\sum_{i=1}^{N_m} \sum_{j=1}^{N_s} \sum_{i=1}^{N_{x,y}} 2^{\nu_{ij}k} P(A \geq a| M_j, r_{(x,y)_{ij}}) \frac{\exp(-b_i (M_{ij} - M_m))}{1 - \exp(-b_i (M_{ul} - M_m))} \text{Sh}\left(\frac{\beta_i \Delta m}{2}\right) f_{k_m,M_j} f(\theta) d\theta d\phi d\theta
\] (2.5)

where \(A\) is the earthquake intensity; \(a\) is a specific earthquake intensity; \(f(\theta)\) is the directional function; \(\theta\) is the possible primary rupture direction; \(P(A \geq a| M_j, r_{(x,y)_{ij}})\) is the conditional probability of earthquake intensity \(A\) being in, or exceeding, a particular intensity \(a\) when the \(i\)th source zone occurs a given earthquake; \(N_m\) is the number of magnitude interval; \(N_s\) is the number of seismic statistical zone; \(N_{x,y}\) is the number of source zone within the \(k\)th seismic statistical zone; \(v_{ij}\) is the annual occurrence rate of the \(k\)th seismic statistical zone; \(b_i = b_i \ln 10\); \(M_m\) is the minimum magnitude used in statistical analysis; \(M_{ul}\) is the upper bound magnitude of the \(k\)th seismic statistical zone; \(\text{Sh}(\cdot)\) is hyperbolic sine function; \(f_{k_m,M_j}\) is space distribution function, i.e., the conditional probability of being in the \(i\)th source zone, given that a certain \(M_j\) earthquake occurred in the \(k\)th seismic statistical zone.

### 2.3. Convex model of seismicity parameters

Considering the fact that the samples of earthquake are lack in the low seismicity area, the envelope bound convex model as presented in Eqn.(2.6) is employed to describe the uncertainties of \(b\) value, the annual occurrence rate \(v\) and the upper bound magnitude \(M_u\),

\[
\Omega = \left\{ b, v, M_u : \begin{cases} b_l \leq b \leq b_p \\ v_l \leq v \leq v_p \\ M_l \leq M_u \leq M_p \end{cases} \right\}
\] (2.6)

where \(b_p\) and \(b_l\) are the upper and lower bounds of \(b\) value; \(v_p\) and \(v_l\) are those of the annual occurrence rate; \(M_p\) and \(M_l\) are those of the upper bound magnitude.
As for the $k$th seismic statistical zone, let $g(b,v,M_u) = \frac{v \exp(-\beta(M_u - M_o))}{1 - \exp(-\beta(M_u - M_o))} \text{Sh}(\frac{\beta \Delta m}{2})$ be the uncertain parts of Eqn.(2.5), and then the uncertainty of peak velocity caused by $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$ can be derived by the uncertainty of $g(b,v,M_u)$.

As the Hessian matrix of $g(b,v,M_u)$,

$$
\nabla^2 g = \begin{bmatrix}
\frac{\partial^2 g(b,v,M_u)}{\partial^2 b^2} & \frac{\partial^2 g(b,v,M_u)}{\partial^2 b v} & \frac{\partial^2 g(b,v,M_u)}{\partial^2 b M_u} \\
\frac{\partial^2 g(b,v,M_u)}{\partial^2 v b} & \frac{\partial^2 g(b,v,M_u)}{\partial^2 v v} & \frac{\partial^2 g(b,v,M_u)}{\partial^2 v M_u} \\
\frac{\partial^2 g(b,v,M_u)}{\partial^2 M_u b} & \frac{\partial^2 g(b,v,M_u)}{\partial^2 M_u v} & \frac{\partial^2 g(b,v,M_u)}{\partial^2 M_u M_u}
\end{bmatrix}
$$

is negative definite, then $g(b,v,M_u)$ is not a convex function [15]. Solving the extreme of $g(b,v,M_u)$ turns to be a nonlinear programming, i.e., searching the minimum or the maximum of $g(b,v,M_u)$ when $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$ vary in the envelope bound convex model by Eqn.(2.6). The programming problem is described as follows

$$
\text{find}(b,v,M_u) \quad \text{max or min}(-g(b,v,M_u)) \\
\text{s.t.} \quad \begin{cases}
b_i - b & \leq 0 \\
-b - b_p & \leq 0 \\
v_i - v & \leq 0 \\
v - v_p & \leq 0 \\
M_i - M & \leq 0 \\
M - M_p & \leq 0
\end{cases}
$$

(2.8)

The envelope bound convex model may approximately model the bound uncertainties of $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$, but the correlation of these three variables is ignored.

To investigate the effects of correlation of these three variables on the seismic hazard assessment, the ellipsoidal bound convex model described in Eqn.(2.8) is employed to model the uncertainties of $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$,

$$
U(\Delta u, r_u) = \left\{ u : \frac{(b-b_i)^2}{(r_b)^2} + \frac{(v-v_i)^2}{(r_v)^2} + \frac{(M_u-M)^2}{(r_M)^2} \leq 1 \right\}
$$

(2.9)

Similar procedure as the envelope bound convex model is repeated, and $g(b,v,M_u)$ is not a convex function. The programming problem are also described as
The sequential quadratic programming method is used for the nonlinear programming problem with inequality constraint as described in Eqns.(2.8) and (2.10).

3. SEISMIC HAZARD ANALYSIS OF NINGBO

Ningbo city is an important port city and economical center of south China. The seismicity of this area is relatively low and the magnitude of earthquake is very small. In this study, the seismic statistical zone is categorized into twenty-five source zones [16]. The major source zones model is shown in Fig.3.1, and the source zones considered in this paper include Hangzhou (6.0), Zhoushan (6.5), Ningbo (6.0), Background 16(5.0), and Background 25 (5.0).

Figure3.1. Major source zones around Ningbo city
3.1. Statistical analysis of seismicity parameters

The earthquake sample in Ningbo seismic statistical zone is analyzed by six cases as follows:

(1) The direct statistical analysis of all the earthquake samples from 228 to 2008, i.e., the earthquake activity is analyzed from the first ground motion record, but some earthquake records are omitted in this period.

(2) The direct statistical analysis of the earthquake samples from 1467 to 2008, i.e., the earthquake activity is considered with the complete ground motion record.

(3) The comprehensive statistical analysis of the earthquake samples with various amplitudes over various periods from 228 to 2008, because the minor earthquake are neglected from 228 to 1969.

(4) The comprehensive statistical analysis of the earthquake samples with various magnitudes over various periods from 1467 to 2008, because the minor earthquake are neglected from 1467 to 1969.

(5) The calibration of seismicity parameters according to the earthquake samples from 228 to 2008 through comparison of the calculated results with the actual annual occurrence rate.

(6) The calibration of seismicity parameters according to the earthquake samples from 1467 to 2008 through comparison of the calculated results with the actual annual occurrence rate.

The statistical results of \( b \) value and the annual occurrence rate \( v \) are scheduled in Table 3.1.

<table>
<thead>
<tr>
<th>Case</th>
<th>( b )</th>
<th>( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Direct analysis of 288-2000</td>
<td>0.7620</td>
<td>0.0582</td>
</tr>
<tr>
<td>(2) Direct analysis of 1467-2000</td>
<td>0.7189</td>
<td>0.1712</td>
</tr>
<tr>
<td>(3) Comprehensive analysis of 288-2000</td>
<td>1.2481</td>
<td>0.5758</td>
</tr>
<tr>
<td>(4) Comprehensive analysis of 1467-2000</td>
<td>0.9857</td>
<td>0.6020</td>
</tr>
<tr>
<td>(5) Calibration of 288-2000</td>
<td>0.7620</td>
<td>0.1019</td>
</tr>
<tr>
<td>(6) Calibration of 1467-2000</td>
<td>0.7189</td>
<td>0.3151</td>
</tr>
</tbody>
</table>

Mean (m) \( 0.8659 \) \( 0.3040 \)
Standard deviation \( (\sigma) \) \( 0.2122 \) \( 0.2374 \)
Upper bound \( (m+1.0\sigma) \) \( 1.0782 \) \( 0.5414 \)
Lower bound \( (m-1.0\sigma) \) \( 0.6537 \) \( 0.0667 \)
Upper bound \( (m+0.75\sigma) \) \( 1.0251 \) \( 0.4821 \)
Lower bound \( (m-0.75\sigma) \) \( 0.7068 \) \( 0.1260 \)
Upper bound \( (m+1.25\sigma) \) \( 1.1312 \) \( 0.6007 \)
Lower bound \( (m-1.25\sigma) \) \( 0.6006 \) \( 0.0073 \)

Variation in seismogenic information and statistical method lead to the variation of the seismicity parameters. The \( b \) value is proposed to be within 0.8-1.2 by some individual researchers [17], and
then the upper and lower bound of the envelope bound convex model are suggested as $m \pm 0.75\sigma$, $m \pm 1.0\sigma$ and $m \pm 1.25\sigma$. The nominal value of the ellipsoidal bound convex model is proposed as the mean value, and the radius of the ellipsoidal is $0.75\sigma$, $1.0\sigma$ and $1.25\sigma$, respectively.

The upper bound magnitude $M_u$ is 7.0 according to Fig.3.1, and the perturbation interval of the upper bound magnitude is presented as 0.5 [18]. In order to coordinate with the bounds of $b$ value and $v$, the lower and upper bounds of the upper bound magnitude $M_u$ is adopted as 6.7-7.3, 6.6-7.4 and 6.5-7.5.

### 3.2. Bounds of seismic hazard analysis

The attenuation relationship provided by Wang et al. [19] is adopted herein. The peak velocity attenuation relationship of east China is

**Major axis:**

$$\log V_a = 0.013 + 0.793M - 2.212\log[(R + 2.789\exp(0.451M))] \quad \sigma = 0.327$$  \hspace{1cm} (3.1)

**Minor axis:**

$$\log V_b = -0.943 + 0.655M - 1.506\log[(R + 1.046\exp(0.451M))] \quad \sigma = 0.327$$  \hspace{1cm} (3.2)

where $M$ is the earthquake magnitude; $R$ is epicentral distance; $\sigma$ is the standard deviation.

In order to ascertain the position of seismic hazard analysis with Chinese code for seismic safety evaluation of engineering sites [13] in the interval of convex analysis, a reliability index is proposed as

$$R_b = \left(1 - \frac{PL_u - PL_l}{PL_u - PL_i}\right)$$  \hspace{1cm} (3.3)

where $PL_c$ is peak velocity from Chinese code, the parameters associated with the calculation of $PL_c$ can be obtained in Chinese code for seismic safety evaluation of engineering sites [13]; $PL_u$ and $PL_l$ are the upper and lower bounds of peak velocity in convex set theory; $R_b$ approaches to unity when $PL_c$ is near to $PL_u$, and to zero when $PL_c$ to $PL_l$.

#### 3.2.1. Modeling the uncertainties of the three seismicity parameters

The uncertainties of $b$ value, the annual occurrence rate $v$ and the upper bound magnitude $M_u$ are described by the envelope bound convex model and the ellipsoidal bound convex model. The bounds of these parameters have been given in Table 3.2. The peak velocity of various exceedance probabilities and the reliability index $R_b$ are presented in Table 3.2. EB and ELP are the abbreviation of the envelope bound convex model and the ellipsoidal bound convex model, and LB and UB are that of the lower and the upper bounds, respectively.

As shown in Table 3.2, the peak velocity interval of a certain exceedance probability calculated using the envelope bound convex model is little different from that using the ellipsoidal bound convex model. The uncertainties of parameters have strong effects on the lower bound of peak velocity, while the influence on the upper bound is relatively small. The peak velocity calculated by Chinese code method is within the range obtained using convex model except for several special cases, which usually locates the lower half interval of convex model.
### Table 3.2. Peak velocity of various exceedance probabilities with three uncertain parameters

<table>
<thead>
<tr>
<th>Method</th>
<th>Peak velocity</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>63%</td>
<td>10%</td>
<td>2%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chinese code</td>
<td>0.22</td>
<td>2.54</td>
<td>5.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Convex model</td>
<td>LB UB</td>
<td>$R_b$</td>
<td>LB UB</td>
<td>$R_b$</td>
<td>LB UB</td>
<td>$R_b$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EB</td>
<td>0.75σ</td>
<td>0.22</td>
<td>1.20</td>
<td>0.00</td>
<td>2.43</td>
<td>5.03</td>
<td>4.23</td>
<td>5.10</td>
<td>6.71</td>
</tr>
<tr>
<td></td>
<td>1.0σ</td>
<td>0.04</td>
<td>1.46</td>
<td>12.68</td>
<td>1.11</td>
<td>5.26</td>
<td>34.46</td>
<td>3.93</td>
<td>6.80</td>
</tr>
<tr>
<td></td>
<td>1.25σ</td>
<td>0.01</td>
<td>1.80</td>
<td>11.73</td>
<td>0.04</td>
<td>5.45</td>
<td>46.21</td>
<td>0.70</td>
<td>6.87</td>
</tr>
<tr>
<td>ELP</td>
<td>0.75σ</td>
<td>0.28</td>
<td>1.07</td>
<td>-7.59</td>
<td>2.70</td>
<td>4.86</td>
<td>-7.41</td>
<td>5.32</td>
<td>6.65</td>
</tr>
<tr>
<td></td>
<td>1.0σ</td>
<td>0.04</td>
<td>1.22</td>
<td>15.25</td>
<td>1.39</td>
<td>5.06</td>
<td>31.34</td>
<td>4.56</td>
<td>6.73</td>
</tr>
<tr>
<td></td>
<td>1.25σ</td>
<td>0.01</td>
<td>1.41</td>
<td>15.00</td>
<td>0.04</td>
<td>5.23</td>
<td>48.17</td>
<td>0.71</td>
<td>6.79</td>
</tr>
</tbody>
</table>

#### 3.2.2. Modeling the uncertainty of single seismicity parameter

Here, only one of the three parameters is regarded as uncertainty parameter, and other two parameters are deterministic. That is the uncertainties of $b$ value, the annual occurrence rate $\nu$ and the upper bound magnitude $M_u$ are respectively modeled by the envelope bound convex model. The deterministic values of parameters listed in Table 3.1 (case (6)) are adopted. The lower and upper bounds of peak velocity with considering the uncertainty of single parameter are calculated and listed in Table 3.3.

The upper bound and the lower bound of peak velocity are almost the same when only the uncertainties of the upper bound magnitude $M_u$ is considered, which is almost identical to the peak velocity calculated by Chinese code method. The observation above implies that the peak velocity interval is independent from bound of $M_u$. In addition, the peak velocity is very sensitive to the uncertainty of the annual occurrence rate $\nu$ and the interval of peak velocity increases when the bound of the annual occurrence rate $\nu$ varies from $0.75\sigma$ to $1.25\sigma$. The $b$ value also has some impact on the peak velocity.
Table 3.3. Peak velocity for various exceedance probabilities with single uncertain parameter

<table>
<thead>
<tr>
<th>Method</th>
<th>Peak velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>63%</td>
</tr>
<tr>
<td>Chinese code</td>
<td>0.22</td>
</tr>
<tr>
<td>EB</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>$0.75\sigma$</td>
</tr>
<tr>
<td></td>
<td>$1.0\sigma$</td>
</tr>
<tr>
<td></td>
<td>$1.25\sigma$</td>
</tr>
<tr>
<td>$v$</td>
<td>$0.75\sigma$</td>
</tr>
<tr>
<td></td>
<td>$1.0\sigma$</td>
</tr>
<tr>
<td></td>
<td>$1.25\sigma$</td>
</tr>
<tr>
<td>$M_u$</td>
<td>$0.75\sigma$</td>
</tr>
<tr>
<td></td>
<td>$1.0\sigma$</td>
</tr>
<tr>
<td></td>
<td>$1.25\sigma$</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

This paper proposes an approach of predicting peak velocity by integrating convex set theory with CPSHA, which is well suited for the low seismicity region. The following conclusions can be drawn from this study:

The peak velocity calculated by Chinese code method is within the range obtained convex models except for several special cases, and locates the lower half interval of convex model. The top contributor to uncertainty in peak velocity is the annual occurrence rate $v$. The interval of peak velocity is moderately sensitive to the $b$ value and is not sensitive to the upper bound magnitude $M_u$. The different convex models have little effect on the interval of peak velocity.

As shown in this paper, the upper bound or the lower bound is far from the center of the convex set, and the result of convex analysis method is more conservative or adventurous. This phenomenon will be improved with the increase of seismic and seismogenic information in the study area, however, the results of seismic hazard analysis based on probabilistic model are unreliable in this situation. Simultaneously, it is still a top issue to be addressed about how to employ the bound results of convex analysis to make decisions.

AKCNOWLEDGEMENTS

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