ASSESSMENT OF THE APPLICABILITY OF DRUCKER-PRAGER MODEL WITH CAP TO ADOBE

Luis Quiroz Torres
Associated Researcher, Seismic Department, Japan-Perú Earthquake Engineering and Disaster Mitigation Center (CISMID), Faculty of Civil Engineering, National University of Engineering, Lima, Perú. e-mail: lgquiroz@uni.edu.pe

SUMMARY
In order to adequately describe the behavior of materials, it is necessary to use mathematical constitutive models to represent their response under external loads. The constitutive models are composed of relationship between the stress tensor and the strain tensor. They represent an idealized description of the actual behavior. The two basic models frequently used are the elastic and ideally plastic models. Real materials almost never match the conditions defined by models mentioned before but these were mainly used because of its simplicity, which is essential for professional practice. In recent years, with the development of the theory of plasticity as well as the use of increasingly powerful computers and friendly computer programs, new elasto-plastic models have appeared. These models describe the non-linear characteristics of various materials such as concrete, soil, rock, etc. In this paper, the Drucker-Prager model with cap is used and its applicability to adobe is evaluated using the finite element program ADINA. A comparison of results get from numerical simulations and tests carried out at CISMID was done. It was found that considering Drucker-Prager failure envelope, it is possible to get an acceptable approximation of the stress-strain behavior in terms of initial stiffness and ultimate strength with differences in some cases of 10%, but the model presents a less gradual transition between the elastic and plastic behavior. The model does not reproduce correctly the strength degradation observed in tests.

Keywords: Adobe, constitutive model, plasticity, nonlinearity, Drucker-Prager

1. INTRODUCTION

Many old structures such as Chan-Chan (Figure 1.1) and buildings constructed in the colonial period (Figure 1.2) were built on adobe and / or mud with material available near the structures and local workers as people in charge of construction. Over the years, due to their design or function, these structures are considered historic and they have important cultural significance to society.

Figure 1.1. Chan-Chan, The world's largest clay city. Trujillo, Perú

Figure 1.2. Colonial houses (adobe and quincha.) Lima, Perú

It is known that these structures have high vulnerability (Figure 1.3). In several earthquakes occurred around the world, these structures presented bad behavior and in some cases they collapsed quickly due to earthquakes of moderate magnitude (Figure 1.4) causing economic and cultural losses and also loss of human life.

Figure 1.3. The citadel of Bam after the earthquake of 2003 (6.2 degrees on the Richter scale)

Figure 1.4. Partial collapse of adobe structures. Ica, 1997.

To reduce the vulnerability of adobe structures, it is important to understand the behavior of the material they are made. Adobe is a complex material that due to its differential behavior requires elaborated constitutive models (Blondet et al (2002)). These models are defined by idealized parameters that need to be adjusted or calibrated through test. This calibration is very important if we want to have an acceptable estimation of the results obtained by numerical simulations. The particular objective of the calibration is to optimize the efficiency of finite element models for the prediction of displacements and stress as close as possible to the real, though any model inherently has assumptions and approximations that make a constraint between the real and simulated behavior (Bathe (1996), Zienkiewiz et al (2005)). When the efficiency of the model is achieved, it can be used for other types of analysis with different loading histories.

2. DRUCKER-PRAGER MODEL WITH CAP

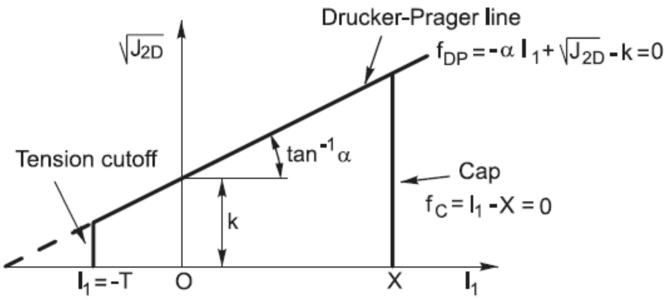


Figure 2.1. Drucker-Prager model with cap

Figure 2.1 shows the Drucker-Prager model with cap (Kojic et al (2005)). It consists of a fixed line defined by equation (2.1) and a cap defined by equation (2.2)

$$f_{DP} = -\alpha I_1 + \sqrt{J_{2D}} - k = 0 \tag{2.1}$$

$$f_C = I_1 - X = 0 \tag{2.2}$$

Where α and k are constants of material, I_1 and J_{2D} are the first invariant of stresses and the second invariant of deviatoric stresses. X is the location of the cap and it depends on the volumetric plastic strain.

DiMaggio and Sandler in 1971 proposed a definition for the hardening law of the cap as follow

$$X = -\frac{1}{D}\ln\left(1 - \frac{e_V^P}{W}\right) + {}^0X \qquad (2.3)$$

Where W and D are constants of material, $e^P{}_V$ is volumetric plastic strain and ${}^0X$ represents the initial location of the cap.

3. COMPUTER PROGRAMS BASED ON FINIT ELEMENT METHOD

The finite element method (FEM) is widely applied in engineering practice through a number of modern programs, giving great potential for solving problems of structural analysis. This family of programs can be divided into two groups: general purpose programs and specialized programs.

The general purpose programs (such as ANSYS, NASTRAN, ABACUS, ADINA, PAK, DIANA) can be used to obtain numerical simulations similar to real behavior. The results are then subject to engineering judgment with empirical data of the materials, service specifications and other criteria. Specialized programs in the field of civil engineering (such as SAP2000, STAAD, TOWER, ETABS) are also used to model structures and the results can be transferred to appropriate algorithms for sizing and design of structural elements according to selected codes. Thus, specialized software improve the quantitative performance of the designer during the analysis while the general purpose software changes the point of view of the designer to quality terms (Zoran Bonic Todor Vacev et al (2010a, b)). In order to model the specimens by considering the problem of nonlinearity of the material and the different states of stress and strain, a general purpose program is the most logical choice. The program selected for this purpose was ADINA.

4. OUTLINE OF TESTS

From previous studies conducted at CISMID, it was selected some test results. The tests selected can be classified as follows: axial compression tests on prisms and diagonal compression test on low walls. Triaxial compression tests on cylindrical specimens were carried out in the geotechnical laboratory of CISMID for this investigation.

4.1. Compression tests on adobe prisms

It was selected the results of five compression tests on adobe prisms of the project developed by JICA and CEETyDES (2009). The purpose of these tests was to understand the behavior of adobe to axial loads. In Figure 4.1 it is shown one of the specimens tested. The average dimensions of the specimens were 37 cm side at the base and 47 cm height. Figure 4.2 shows characteristic curves obtained from test. It was found that the average compression strength due to the axial load is about 8.83 kgf/cm^2.

4.2. Diagonal compression tests on adobe low walls

Of the same project mentioned above, it was selected the results of five diagonal compression tests on adobe low walls. The purpose of these tests was to understand the behavior of adobe to axial load and shear. The dimensions of the specimens were 100 cm side and 20 cm thick. The results show greater variability compared to axial compression tests. It was found that the diagonal compression strength is in average 0,266 kgf/cm^2. Figure 4.3 shows one specimen and Figure 4.4 shows typical curves get from diagonal compression tests.

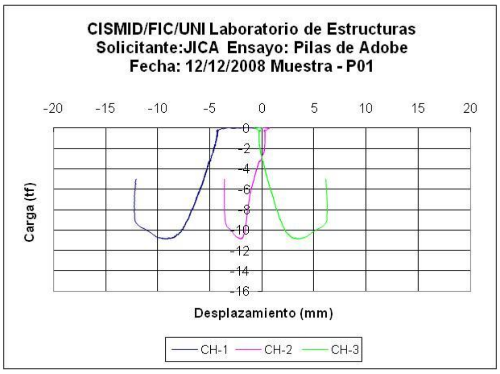


Figure 4.1. Adobe specimen tested to compression

Figure 4.2. Typical curve get from compression test on adobe prisms

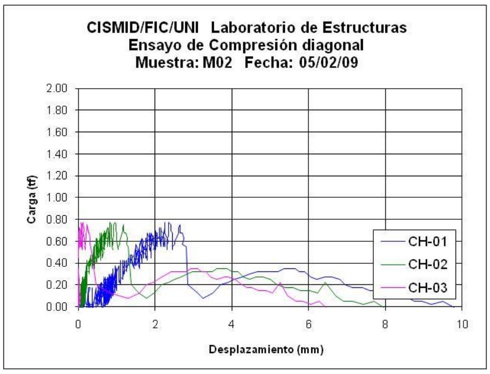
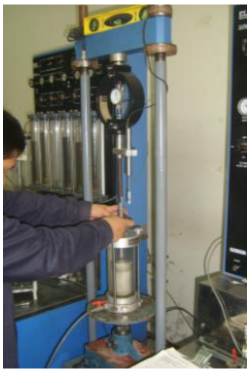


Figure 4.3. Adobe specimen tested to diagonal compression

Figure 4.4. Typical curve get from diagonal compression test on adobe low walls

4.3. Triaxial compression tests on adobe specimens

The material used to manufacture the cylindrical specimens were adobe bricks remaining from tests presented in sections 4.1 and 4.2. The dimensions of the specimens were in average 4.85 cm diameter and 9.5 cm high, due to the requirements of the triaxial testing machine. It was tested two sets of specimens (6 specimens) with different level of confinement: 1 kgf/cm^2, 2 kgf/cm^2 and 4 kgf/cm^2. Figure 4.5 shows the test of one specimen and Figure 4.6 shows typical curves get from triaxial compression test.



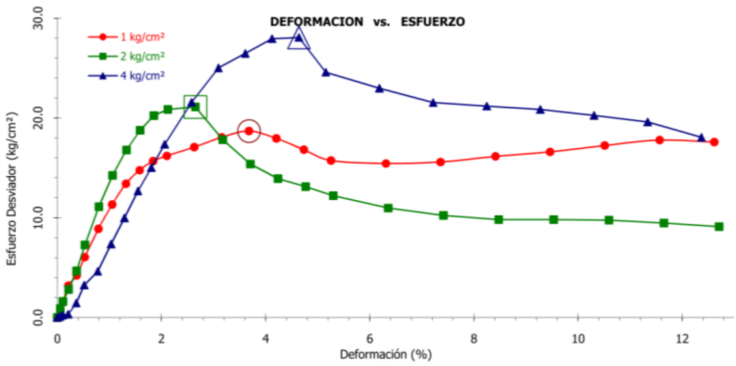


Figure 4.5. Triaxial test of adobe specimen

Figure 4.6. Typical curves get from triaxial compression test

5. ESTIMATION OF PARAMETERS OF DRUCKER-PRAGER MODEL WITH CAP

5.1. Estimation of elastic parameters (E, υ)

The curves obtained from compression tests on prisms were used to estimate the first parameter. Figure 5.1 shows the curves in terms of vertical stress and vertical strain.

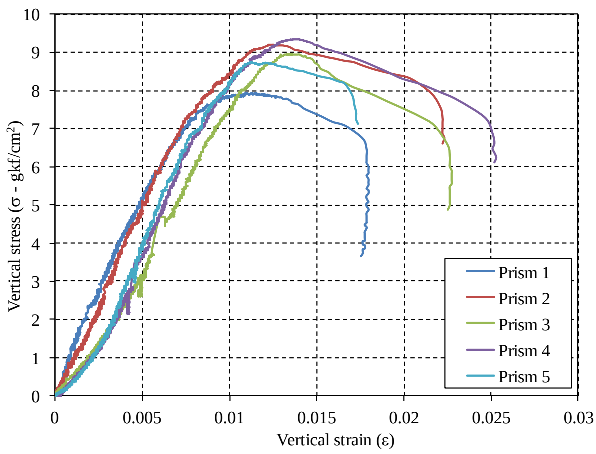


Figure 5.1. Curves obtained from compression tests on adobe prisms

The modulus of elasticity (E) was estimated based on the slope of the curves presented above using two ranges. The first range was set between values of 0% to 25% of maximum compression stress and the second range was set between values of 25% to 50% of maximum compression stress. In case of the first range, it was found a value of 724 kgf/cm^2 for E, while for second range, it was found a value of 820 kgf/cm^2. Based on the results presented before, a value of 800 kgf/cm^2 was set for parameter E. Similar procedure was followed with experimental results from triaxial compression test. It was found that E increased in 15% with a confinement stress of 4 kgf/cm^2.

There is no available information from test to quantify a Poisson's ratio. A value of 0.25 was set because of the low variation of reference values reviewed (Yamin et al. (2003), Wang et al. (2001), Roonsson and Boothby (1998), Mroginski et al. (2006), Lopez et al. (2000).)

5.2. Estimation of parameters that define yielding surface (α, κ)

As can be observed in Equation (2.1), the yielding surface of Drucker-Prager is expressed in terms of the first invariant of stresses (I_1), and the second invariant of deviatoric stresses (J_{2D}). Based on the results of compression tests on prisms and triaxial compression tests on cylindrical specimens of adobe, the maximum values of I_1 and J_{2D} are estimated and presented in Table 5.1. σ_c is the stress of confinement of specimens.

Figure 5.2 shows a graphical representation of Table 5.1. It is observed that the points fit to a line. To estimate the equation of the line, it was used a linear regression (Motulsky et. al. (2003)). The fitted line is presented in Equation (5.1) and the values of the parameters that define the yielding surface are α = 0.3342 y κ = 2.3447 kgf/cm^2.

$$\sqrt{(J_{2D})} = 0.3342(I_1) + 2.3447 \qquad (5.1)$$

In Figure 5.2, it is also shown the value of the coefficient of determination, which is close to 1. This would indicate that failures in the specimens are given in the Drucker-Prager line.

Table 5.1. Maximum values of I_1 and $\sqrt{(J_{2D})}$ get from compression tests on prisms and triaxial compression tests on cylindrical specimens of adobe

Test	σ_c (kgf/cm^2)	I_1 (kgf/cm^2)	$\sqrt{(J_{2D})}$ (kgf/cm^2)
Prism 1	0	7.93	4.58
Prism 2	0	9.20	5.31
Prism 3	0	8.94	5.16
Prism 4	0	9.35	5.40
Prism 5	0	8.70	5.02
Set1-M1	1	17.98	8.65
Set1-M2	1	20.69	10.21
Set2-M1	2	25.41	11.21
Set2-M2	2	25.1	11.03
Set3-M1	4	36.4	14.09
Set3-M2	4	36.07	13.90

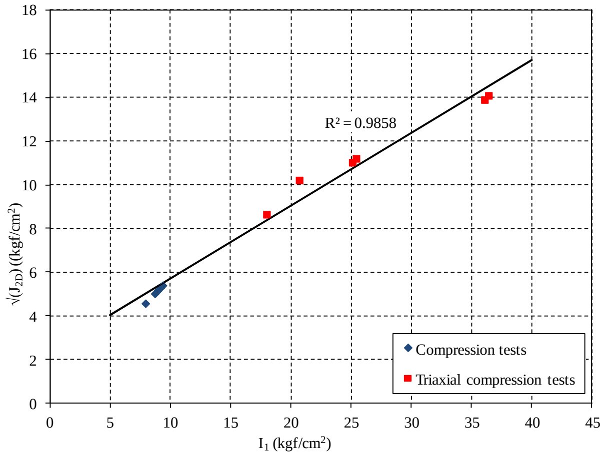


Figure 5.2. Yielding surface of Drucker-Prager obtained from tests

5.3. Estimation of parameters that define the cap (W, D, ^0X)

The values of parameters W and D can be estimated from hydrostatic test results (Desai (1984)). From the results presented in previous section, it is clear that the failure occurs in the yielding surface and the cap is not reached. Based on that, it can be said that the initial location of the cap (^0X) must be larger than 36.4 kgf/cm^2. For the numerical simulations, it was set a value of 40 kgf/cm^2 for parameter ^0X.

From Kojic, Milos; Bathe, Klaus-Jurgen (2005), it was found that the value of W varies between 0.066 and 0.18 while the value of parameter D varies between 0.00953 (kgf/cm^2)$^{-1}$ and 0.711 (kgf/cm^2)$^{-1}$. In the present study, it was considered the value of 0.18 for parameter W and the value of 0.711 (kgf/cm^2)$^{-1}$ for parameter D. As it was mentioned before, the cap is not reached in the case of the tests presented in section 4.

5.4. Estimation of tension cutoff (T)

In order to model the material failure due to the effects of tension, it was set a limit (T). For the present study, the value of T was set to zero because adobe does not support tension stresses.

6. NUMERICAL SIMULATIONS

6.1. Numerical simulation of compression tests

Three-dimensional solid isoparametric elements of eight nodes were used to simulate the compression tests. As can be observed in Figure 6.1, a quarter of the specimen was modeled. In the same figure, it is shown the boundary conditions and loads (vertical displacements (δ) at top face) used in the model.

6.2. Numerical simulation of triaxial compression tests

In this case, two dimensional isoparametric elements of eight nodes based on displacement were used. The assumption of axisymmetric element was considered. Figure 13 shows a scheme of the model with its boundary conditions and loads applied (vertical displacement (δ) at top and stress of confinement (σ)).

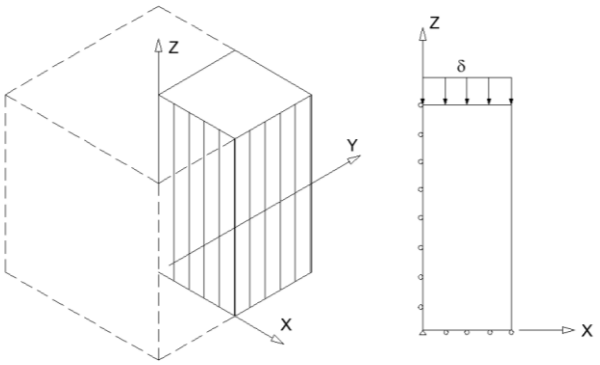


Figure 6.1. Schema of model for compression tests

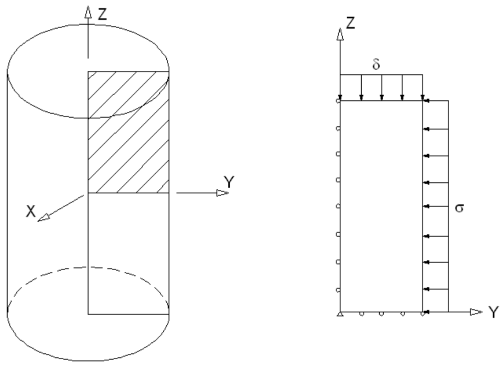


Figure 6.2. Schema of model for triaxial compression tests

6.3 Numerical simulation of diagonal compression tests

In this case, as in the case of the numerical simulation of compression test, it was used three-dimensional solid isoparametric elements of eight nodes. Figure 6.3 shows the numerical model for diagonal compression test. In the same Figure is shown the boundary conditions and load (prescribed displacement) applied.

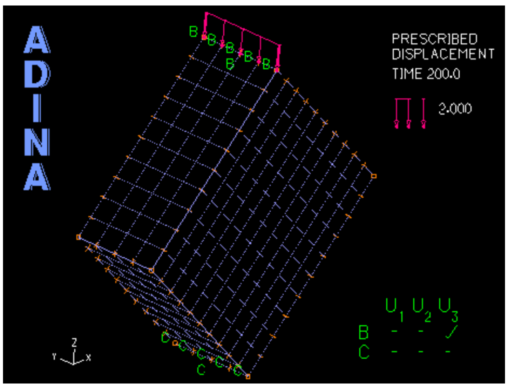


Figure 6.3. Numerical model for diagonal compression tests

7. ANALYSIS OF RESULTS

7.1. From compression tests on prisms

Figure 7.1 shows the comparison between tests and numerical simulations in terms of “vertical stress – vertical strain”. It is observed that there is an acceptable agreement between test results and numerical simulation for the maximum strength. It is also observed a less gradual transition from elastic to plastic behavior in the numerical simulation.

The average maximum vertical stress obtained from test is about 8.77 kgf/cm^2 and the maximum vertical stress obtained from numerical simulation is 9.64 kgf/cm^2. The model estimates the maximum strength (σ_{axial}) with a relative error of 10%. Similar values are also observed in Díaz and Ríos (2005.) The model does not reproduce the strenght degradation observed in the tests.

7.2. From triaxial compression tests

Figure 7.2 shows the comparison between the tests and numerical simulation for triaxial compression tests considering a confinement stress of 4 kgf/cm^2. The dashed curve is the average from the curves obtained from tests. As in the case of compression tests, it is observed that there is a good approximation of maximum strength but the model does not describe appropriately the behavior of material after maximum strength.

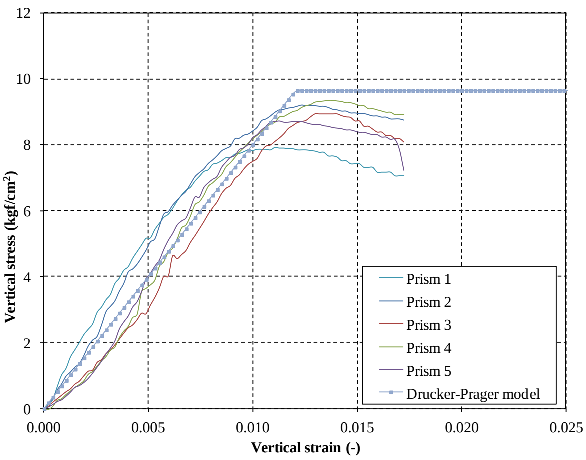


Figure 7.1. Comparison between tests and numerical simulation for compression tests on prisms.

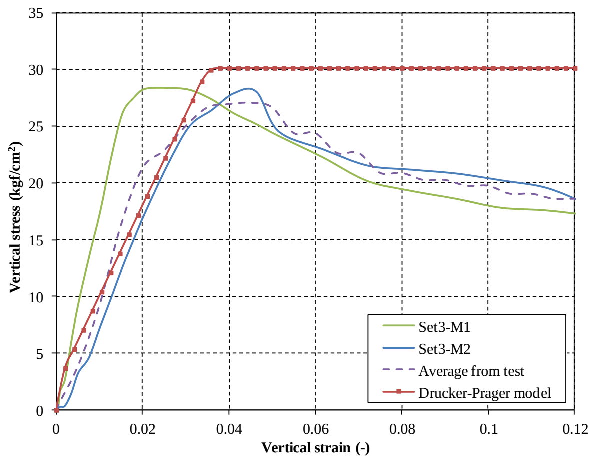


Figure 7.2. Comparison between tests and numerical simulation for triaxial compression tests (Confinement stress of 4 kgf/cm^2)

Table 7.1 shows the maximum strength obtained from test and numerical simulation for all triaxial compression tests. As can be observed, the relative error is about 10%.

Table 7.1. Maximum strength obtained from tests and numerical simulation for triaxial compression tests

Test	Maximum σ_{axial} (kgf/cm^2)	Average σ_{axial} (kgf/cm^2)	σ_{axial} [1] (kgf/cm^2)	Relative error (%)
Set1-M1	15.98	16.53	14.77	-10.64
Set1-M2	18.69			
Set2-M1	21.41	19.86	19.89	0.15
Set2-M2	21.1			
Set3-M1	28.4	27.045	30.14	11.43
Set3-M2	28.07			

[1] Maximum strength obtained from numerical simulation.

7.3 From diagonal compression tests

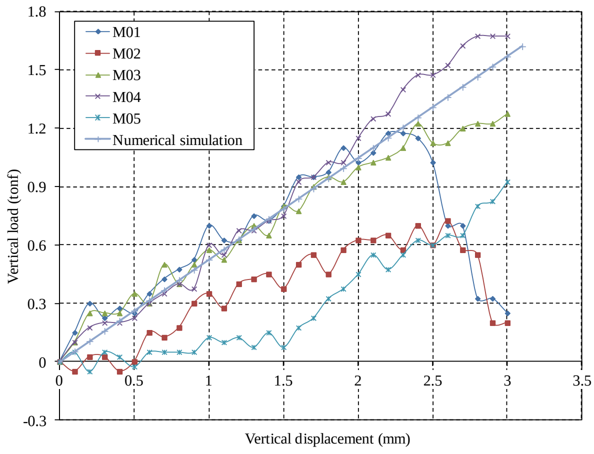


Figure 7.3. Comparison between tests and numerical simulation for diagonal compression tests

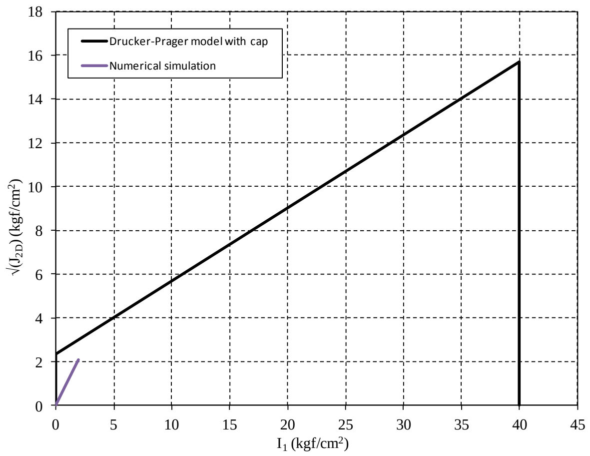


Figure 7.4. Numerical simulation of history of stresses for diagonal compression tests.

Figure 7.3 shows the results obtained from tests and numerical simulation in terms of vertical load and vertical displacement. It is observed that the experimental results show large variability but they tend to present a linear behavior. From the numerical simulation, it is observed that the low walls keep in the elastic range. To verify this last observation, it was traced the history of stresses in the plane $\sqrt{(J_{2D})}$ – I_1. From Figure 7.4, it is observed that the points remained in the elastic zone until the failure.

CONCLUSIONS

In general, all test results have variability; especially those get from diagonal compression tests. In axial compression test we could observe that the behavior is approximately linear until the maximum stress. For small deformations, it is proposed a modulus of elasticity of 800 kgf/cm^2. In triaxial compression tests performed in specimens made of the same homogeneous material of the bricks, it wasn't observed considerable variation in the modulus of elasticity. There is no available information from test to quantify a Poisson's ratio. For small deformations, it is assumed that values around 0.20 to 0.25 might be appropriate according to the reported by other authors. The maximum strengths obtained in uniaxial compression tests are in the range of 8 to 10 kgf/cm^2.

To simulate the behavior of adobe, it was used the Drucker-Prager model with cap. A procedure was presented to estimate the parameters of the model based on results of axial compression tests and triaxial compression tests with different confining pressures. It was found that the maximum stresses fit good to a line defined by the Drucker-Prager parameters α = 0.3342 and κ = 2.3447 kgf/cm^2. However, assuming a straight failure envelope, the resistance observed in diagonal compression test is overestimated. In no one of the tests was observed the necessity of consider a cap, even in triaxial compression tests with confining stress of 4 kgf/cm^2. Note that in adobe structures are expected lower confining stresses.

In numerical simulations performed with ADINA program, it was got an acceptable approximation of the stress-strain behavior in terms of initial stiffness and ultimate strength, with differences in some cases in the order of 10%. Numerical simulations show a less gradual transition between elastic and plastic behavior. This gradual transition observed in the tests may be the result of non-uniform distribution of stresses in the specimens.

Given the limited experimental data available and the large variability observed in the results, it is necessary to develop a more extensive testing program. It is particularly recommended to perform more triaxial compression test with low confining stresses.

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