Seismic Upgrading of Structures: a Design Procedure for Dissipative Buckling Restrained Braces

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SUMMARY:
Existing reinforced concrete frame buildings with non-ductile detailing suffered severe damage and caused loss of life during earthquakes. Different rehabilitation systems have been developed to upgrade the seismic performance of this kind of structures.
The research discussed in this paper deals with the seismic upgrading of frame structures, in particular for the application presented buckling restrained steel braces (*BRB*) have been selected.
A displacement-based procedure to design dissipative bracing for the seismic protection of frame structures, together with an optimization criteria for the bracing, is proposed and some applications are discussed.
The procedure does not require sophisticated dynamic nonlinear analyses but only common non linear static analyses: it is based on the displacement based design and using the capacity spectrum method.
Two performance objective have been considered developing the procedure: to protect the structure against structural damage or collapse and avoid non-structural damage.This latter is done limiting global displacements and interstorey drifts.
Finally the procedure has been applied to some case studies and existing r.c. frames both 2D and 3D.

Keywords: infilled frames; passive damping; dissipative braces; seismic retrofitting

1. INTRODUCTION

Traditional retrofitting strategies aims to increase structural strength and/or reducing ductility demand, but in the last two decades there has been a large diffusion of new conceptual approaches that can be grouped in two categories: increase of available ductility and reduction of demand.

One way to obtain these latter is increasing energy dissipation using dissipative bracings.

Since bracings have a non linear behaviour, which can modify behaviour of the retrofitted structure, this approach requires the evaluation of nonlinear response e.g. by means of non linear static analysis.

Certainly also nonlinear dynamic analysis is applicable but, with the target of defining a professional designing tool, it appears to be too complex.

Therefore the development of a design procedure based on static non linear pushover can be seen as a useful design tools to check large number of solutions while giving clear indication to move toward efficient design solutions.

In this work a design procedure [*Bergami A.V. & Nuti C.* (2012)] to determine the characteristics of dissipative braces B to retrofit an existing building S, is described and some applications are discussed (the retrofitted structure is *S+B*).

The procedure is based on displacement response control and on the use of the well known non linear static analysis (pushover).

In this paper the procedure, that can be used with any dissipative device, is applied using a widely diffuse and convenient mechanical type of dissipative brace: the buckling restrained brace (*BRB*).

2. DESIGN METHOD

2.1. Relevant parameters

Considering a braced structure (Fig. 1), being its capacity curve represented by the curve *S+B* (Fig. 2), one can assume that this latter is the sum of the capacity curves of the structure (*S*) and of the bracing system (*B*): therefore *B* can be obtained subtracting *S* from *S+B*. In Fig. 2 the capacity curve *S* is approximated as elasto plastic as well as the capacity curve *B*: therefore the curve *S+B* is trilinear.

For a given seismic action expressed in term of response spectrum and for a given capacity curve *S+B*, one can obtain the structural response in term of displacement being known the equivalent viscous damping $\nu_{eq,S+B}$ associated to each point of the curve *S+B*.

It is well known that the force-displacement behaviour of a *BRB* (with *j* the generic device) can be modelled by a simple bilinear law characterized by the elastic axial stiffness $K'_{b,j}$, the yield strength $F'_{by,j}$ and the hardening ratio $\beta_{b,j}$. The parameters of the bracing depend on the geometry of the frame and on the characteristics of the device.

$K'_{b,j}$, $F'_{by,j}$, $D'_{by,j}$ e $\beta_{b,j}$ depend on mechanical properties of the selected devices ($D'_{by,i}$ is the axial displacement at yielding) while the length $l_{b,i}$ and the inclination $\theta_{b,i}$ of each brace can be determined referring to both geometric characteristics of the structure and brace distribution (Fig. 3).

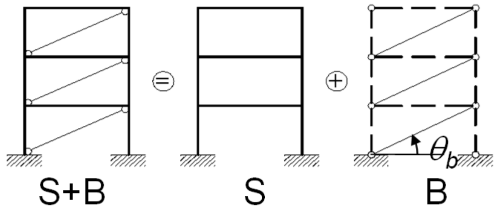

Figure 1. Scheme of the braced structure (*S+B*) as sum of the structure (*S*) and the bracing system (*B*)

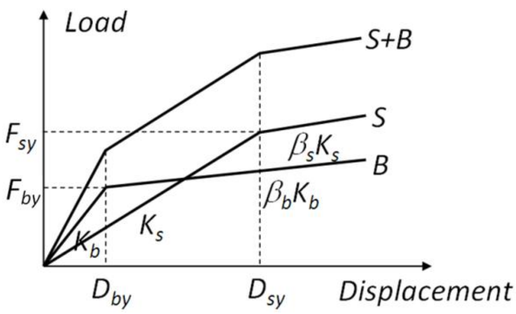

Figure 2. Interaction between the structure (*S*) and the bracing system (*B*) expressed in terms of horizontal components of the force-displacement relationship

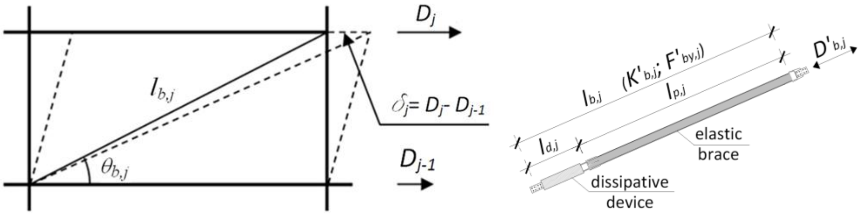

Figure 3. Deformed shape of a generic single part of the braced frame

Being $K_{b,j}$, $F_{by,j}$, $D_{by,j}$ the horizontal components of stiffness, yield strength and displacement at yield of the bracing system *B* respectively, they can be expressed as follow:

$$K_{b,j} = K_{b,j}^{'} \cos^2 \theta_{b,j} \tag{2.1}$$

$$F_{by,j} = F_{by,j}^{'} \cos \theta_{b,j} \tag{2.2}$$

$$D_{by,j} = D_{by,j}^{'} / \cos \theta_{b,j} \tag{2.3}$$

Scope of the design is the definition of the following variables:
1. the plano-altimetric configuration of the bracing system that influences device sizing as it modifies the braced frame deformed configuration both in the linear range as well as beyond the plastic limit;
2. the axial stiffness $K'_{b,j}$ of each brace;
3. the yielding limit of each brace ($D'_{by,j}$, $F'_{by,j}$ in terms of axial components or $D_{by,j}$, $F_{by,j}$ in terms of horizontal components).

The designer can proceed in different manners in order to determine stiffness and strength of the braces to be added at each level. It is evident that if the dissipative system yields before the structure itself ($D_{by}<D_{sy}$) the efficiency of the intervention will increase, therefore this should be a basic assumption.

Moreover the designer, once defined the desired performance for the structure in terms of top displacement, can decide to avoid or admit plastic deformations of the existing structural elements.
If it is accepted that also the structure yields ($D > D_{sy}$), total damping of $S+B$ is the sum of the inherent damping and the damping offered by both the bracing system and the structure itself ($\nu_{tot} = \nu_I + \nu_{eq,B} + \nu_{eq,S}$) while, if the structure remains elastic ($D_{by}<D \le D_{sy}$), total damping is the sum of the inherent plus the one due to braces dissipation ($\nu_{tot} = \nu_I + \nu_{eq,B}$). The former situation is often the case: many existing structures have been designed to resist to vertical loads only or, at most, to very small horizontal forces.

In general yielding of *S* can be accepted for rare earthquakes and excluded for frequent earthquakes in order to limit damage.

It is useful to express each limit state of interest in terms of displacement D^*. The same D^* can be obtained adopting different retrofitting combinations of stiffness, strength and consequently dissipation.

The first parameter to be determined is the stiffness of the braces (additional stiffness).

Different criteria to distribute the additional stiffness are proposed in scientific literature: constant at each story, proportional to story shear, proportional to interstorey drifts of the original structure. In this work the latter is assumed and therefore, given the interstorey drift δ_j, the stiffness $K'_{b,j}$ corresponding to each storey of the bracing system is:

$$K'_{b,j} = K_{global} c_{b,j} \tag{2.4}$$

where:

$$c_{bj} = \frac{\delta_j}{\max_j \{\delta_j\}} \tag{2.5}$$

Each brace is a composite element realized coupling an elastic element (usually a steel profile) with a dissipative device in series. The latter will determines the desired yielding force whereas the former will be designed to assure the desired stiffness of the series.

3. PROPOSED DESIGN PROCEDURE

The procedure is iterative because the addition of dissipative braces modifies the structural response and in particular the capacity curve that has to be updated as long as their characteristics are being defined. The proposed design procedure is based on the well known Capacity Spectrum Method where the total effective damping of a braced structure $\nu_{eq,S+B}$ is expressed in terms of equivalent viscous damping as a linear combination of the equivalent damping of the structure only $\nu_{eq,S}$, the equivalent damping of the braces $\nu_{eq,B}$ and the inherent damping ν_I (usually 5% for r.c. structures and 2% for steel ones).

$$\nu_{tot} = \nu_I + \nu_{eq,S+B} \;;\; \nu_{eq,S+B} = \nu_{eq,S} + \nu_{eq,B} \tag{3.1}$$

Referring to the formula proposed by *A.K. Chopra* (2001), the equivalent viscous damping can be expressed as follows:

$$\nu_{eq,S} = \chi_S \frac{1}{4\pi} \frac{E_{D,S}^{bilinear}}{E_{S,S+B}} ; \nu_{eq,B} = \chi_B \frac{1}{4\pi} \frac{\sum_j E_{D,B,j}^{bilinear}}{E_{S,S+B}} \tag{3.2}$$

with: E_D is the energy dissipated in a single cycle (by the structure *S* or the braces *B*) and E_S is the elastic strain energy. χ_S and χ_B are corrective coefficients for the structure and the braces respectively (the hysteretic cycle of a real structure/device differs from the ideal cycle: χ=1 for the ideal elasto-plastic behaviour.) . In a displacement based design perspective, the performance objective is selected at first as the target displacement to meet a selected limit state for a given seismic action. The required total effective damping to make the maximum displacement less than the target one is then determined and the braces additional damping estimated as the difference between the total damping and the hysteretic damping of the structure only. Dissipative braces characteristics are finally determined to guarantee the required additional damping. Since it usually happens that the performance point of the braced structure is different from the target one, iterations are needed until convergence. The main steps of the procedure follow.

1. **Define the seismic action**: the seismic action is defined in terms of elastic response acceleration spectrum (T-S_a).

2. **Select the target displacement**: the target displacement is selected (for example the top displacement D_t^*) according to the performance desired (limit state).

3. **Define the capacity curve**: the capacity curve of the braced structure *S+B*, in terms of top displacement and base shear (D_t-V_b), is determined via pushover analysis. The pushover analysis can be easily performed using a software for structural analysis: many different force distributions can be adopted selecting the best option for the specific case (*e.g.* modal shape load profile).

If a modal shape load profile has been selected it is important to underline that the modal shape is influenced by the bracing system and consequently, at each iteration, the load profile has to be updated to the modal shape of the current braced structure.

Notice that, at the first iteration, the structure without braces is considered and therefore the capacity curve obtained will be fundamental for the evaluation of the contribution offered by the existing structure to the braced structure of the subsequent iterations.

4. **Define the equivalent bilinear capacity curve**: the capacity curve is approximated by a simpler bilinear curve D_t-F_{s+b} that is completely defined by the yielding point ($D_{s+b,y}$, $F_{s+b,y}$) and the hardening ratio β_{s+b} (at the first iteration the parameters correspond to $D_{s,y}$, $F_{s,y}$, β_s of the existing building).

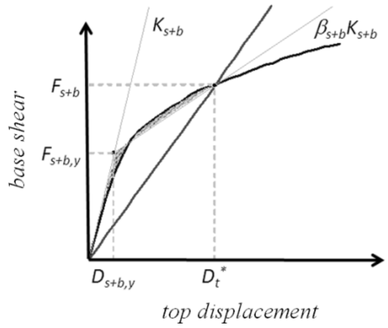

Figure 4. Evaluation of the equivalent bilinear capacity curve

5. **Define equivalent single degree of freedom**: MDOF system is converted in a SDOF system by transforming the capacity curve into the capacity spectrum (S_{dt}-S_{ab})

$$S_{dt} = \frac{D_t}{\Gamma\phi_t}; S_a = \frac{F_{S+B}}{\Gamma \cdot L} \tag{3.3}$$

where Γ is the participation factor of the modal shape ϕ ($\Gamma=(\phi^T MI)/(\phi^T M\phi)$) and $L=\phi^T MI$.
The modal characteristics of the braced structure may change at every iteration due to new brace characteristics. Therefore ϕ, Γ and L have to be updated with the current configuration

6. **Evaluate the required equivalent viscous damping**: the equivalent viscous damping $\nu^*_{eq,S+B}$ of the braced structure to meet the displacement of the equivalent SDOF system and the target spectral displacement $S_{dt}^*=D_t^*/(\Gamma\phi^T)$ is determined.
According to the Capacity Spectrum Method the demand spectrum is obtained reducing the 5% damping response spectrum by multiplying for the damping correction factor h that is function of ν_{tot}

$$\eta = \sqrt{\frac{10}{5+\nu_{tot}\cdot 100}} = \frac{S_{\nu_{eff}}}{S_{5\%}} \tag{3.4}$$

From Eq. (3.4) one obtain ν^*_{tot} the damping needed to reduce displacement up to the target S_{dt}^*.

$$\nu^*_{tot} = 0.1\left(\frac{S_{5\%}}{S^*_{dt}}\right)^2 - 0.05 \tag{3.5}$$

7. **Evaluate the equivalent viscous damping contribution due to the naked structure**: the contribute to damping of the structure $\nu^*_{eq,S}(D^*_t)$ can be determined from Eq. (3.2) being D_t^* the top

displacement corresponding to $E_{D,S}^{bilinear}$ and $E_{S,S+B}$ that are the energy dissipated by S and the elastic strain energy of $S+B$ ($E_{D,S}^{bilinear}$ and $E_{S,S+B}$ are determined from the capacity curve of S and $S+B$ respectively).

8. **Evaluate the additional equivalent viscous damping contribution due to braces**: given v_{tot}^{*} from Eq. (3.5) the equivalent viscous damping needed to be supplied by the braces $v^{*}_{eq,B}(D_t^{*})$ is evaluated from Eq. (3.1) and Eq. (3.2) as follows:

$$v^{*}_{eq,B}(D_t^{*}) = v_{tot}^{*}(D_t^{*}) - v_{eq,S}^{*}(D_t^{*}) - v_I \quad (3.6)$$

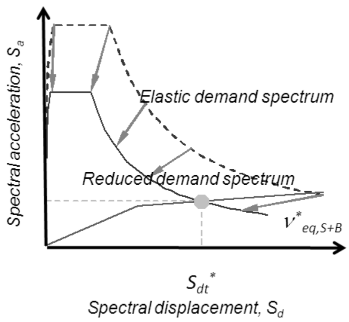

Figure 5. Evaluation of the equivalent viscous damping needed to achieve the target performance point

9. **Dimensioning of the braces**: once the required equivalent viscous damping $v^{*}_{eq,B}(D_t^{*})$ has been evaluated from Eq. (3.6), axial stiffness and yielding strength required to achieve the desired additional damping can be determined with the same procedure previously adopted for the structure (step 7). The energy dissipated by the braces inserted at each j_{th} level can be expressed as:

$$E_{D,B}^{bilinear} = \sum_{j=1}^{n} 4\left(F_{by}^{'}\delta_j^{'} - \delta_{y,j}^{'}F_{b,j}^{'}(\delta_j^{'})\right) \quad (3.7)$$

being δ'_j the component of the interstory drift δ_j at j_{th} of the n floors along the axe of the brace ($\delta'_{y,j}$ is the axial displacement corresponding to yielding of the device).

The axial displacement of the damping brace at the j_{th}-floor δ'_{bj} can be determined from its inclination angle $\theta_{b,j}$ and interstorey drift $\delta_j = D_j - D_{j-1}$: therefore $\delta'_{b,j} = \delta_j \cos\theta_{bj}$.

The dissipative brace is usually constituted by a dissipative device (*e.g.* the *BRB*) assembled in series with an extension element (*e.g.* realized with a steel profile) in order to connect the opposite corners of a frame (Fig. 6).

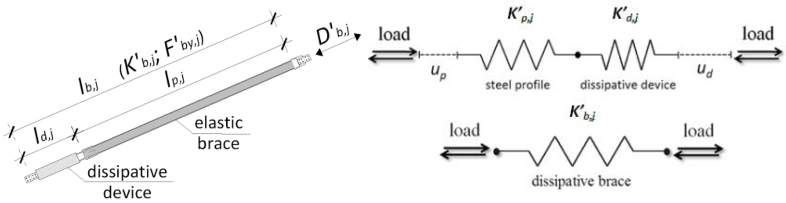

Figure 6. Dissipative device "*j*" assembled in series with an extension element (*e.g.* a steel profile): equivalent model of springs in series ($K'_{d,j}$; $K'_{p,j}$) and equivalent single spring model ($K'_{b,j}$)

Therefore, being $K'_{b,j}$ and $K'_{by,j}$ the equivalent stiffness of the spring series in the elastic and plastic range respectively, a= $K_{p,j}{}'/K_{d,j}{}'$ the ratio between elastic stiffness of the steel profile and of the device and $\beta_{d,j}$ the ratio between stiffness after and before yielding of the dissipative device, the following expression can be derived:

$$K'_{b,j} = \frac{K'_{d,j}}{\frac{1}{\alpha_j}+1}\,;\; K'_{by,j} = \frac{\beta_{b,j}K'_{d,j}}{\frac{\beta_{b,j}}{\alpha_j}+1}\,;\; \alpha_j = \frac{K'_{p,j}}{K'_{d,j}} \tag{3.8}$$

Therefore:

$$F'_{b,j} = F'_{by,j} + (\delta'_j - \delta'_{y,j})\frac{\beta_{b,j}K'_{d,j}}{\frac{\beta_{b,j}}{\alpha_j}+1} \tag{3.9}$$

$$\delta'_{y,j} = \frac{F'_{by,j}}{K'_{b,j}} = \frac{F'_{by,j}}{K'_{d,j}}\left(\frac{1}{\alpha_j}+1\right) \tag{3.10}$$

Consequently, if there is one brace per direction and per floor, substituting Eq. (3.9) into Eq. (3.7), $v^*_{eq,B}(D^*_t)$ can be expressed in the following way:

$$v^*_{eq,B}(D^*_t) = \chi_B \frac{2}{\pi}\frac{\sum_{j=1}^{n}\left\{F'_{by,j}\delta'_j - \delta'_{y,j}\cdot\left[F'_{by,j} + (\delta'_j - \delta'_{y,j})\frac{\beta_{d,j}K'_{d,j}}{\frac{\beta_{d,j}}{\alpha_j}+1}\right]\right\}}{F_{S,S+B}(D^*_t)\cdot D^*_{S,S+B}} \tag{3.11}$$

δ'_j are determined from the pushover analysis for the top displacement D_t and $\delta'_{y,j}$, that is the yielding displacement of devices, can be reasonable assumed as $\delta'_{y,j} \le \delta'_j/4$. $F'_{y,j}$ is, for each direction, the yielding force of the floor brace: once $\delta'_{y,j}$ has been defined $F'_{y,j}$ is consequently determined Eq. (3.10). Thus, remembering Eq. (2.4) and according to (3.8), $K'_{d,j}$ can be expressed as follows:

$$K'_{d,j} = K_{global}\cdot c_{b,j}\cdot\left(\frac{1}{\alpha_j}+1\right) \tag{3.12}$$

Therefore substituting Eq. (3.12) into Eq. (3.11), K_{global} can be determined as follows:

$$K_{global} = \frac{\pi\cdot v^*_{eq,B}(D^*_t)\cdot F_{S,S+B}(D^*_t)\cdot D^*_{S,S+B}}{2\cdot\chi_B\cdot C_1} \tag{3.13}$$

with:

$$C_1 = \sum_{j=1}^{n} c_{b,j}\left\{\delta'_{y,j}\cdot\delta'_j - \delta'_{y,j}\left[\delta'_{y,j} + (\delta'_j - \delta'_{y,j})\frac{\beta_{b,j}\left(\frac{1}{\alpha_j}+1\right)}{\frac{\beta_{b,j}}{\alpha_j}+1}\right]\right\} \tag{3.14}$$

A value of $a_j>3$ is usual in applications, therefore $K'_{b,j}>3/4K'_{d,j}$, while the steel profile must be stronger (neither yielding nor buckling) than the device: for a given interstorey drift the larger is a_j the larger are device displacements and hysteretic cycles. At this point all terms of Eq. (3.13) are known so, from Eq. (3.12) and Eq. (3.8), the floor brace stiffnesses $K^{'}_{b,j}$ can be defined (the yielding force $F^{'}_{by,j}$ can be directly derived since the stiffness $K^{'}_{b,j}$ and the yielding displacement $\delta^{'}_{y,j}$ have been defined). Though in this paper the procedure is discussed referring to Eq. (3.11) it is important to underline that, in a general case, one can have *m* different braces for each level *j*. In fact, at the same level, each brace *i* can be characterized by its specific properties as a consequence, for example, of the geometry of the bays of the structural frame. Consequently Eq. (3.11) can be generalized as follows.

$$v^{*}_{eq,B}(D^{*}_{t}) = \frac{2}{\pi}\frac{\sum_{j=1}^{n}\sum_{i=1}^{m}\chi_{B,i}\left\{F^{'}_{by,j,i}\delta^{'}_{j} - \delta^{'}_{y,j,i}\cdot\left[F^{'}_{by,j,i} + (\delta^{'}_{j} - \delta^{'}_{y,j,i})\dfrac{\beta_{d,j,i}K^{'}_{d,j,i}}{\dfrac{\beta_{d,j,i}}{\alpha_{j,i}}+1}\right]\right\}}{F_{S,S+B}(D^{*}_{t})\cdot D^{*}_{S,S+B}} \quad (3.15)$$

A simplified approach of this step is presented in Appendix A: this simplified procedure is useful to get a first dimension of the bracing system.

10. **Check convergence**: one must repeat steps from 3 to 9 until the performance point of the braced structure converges to the target displacement with adequate accuracy.

11. **Possible optimization**: the calibration of the mechanical characteristics between the extension element (stiffness) and the device (stiffness and strength) allows to optimize the dissipative brace.

4. APPLICATION OF THE PROPOSED PROCEDURE

4.1. Case study of a R.C. plane frame

The proposed design procedure has been applied to retrofit an existing r.c. frame structure designed to resist vertical loads only. The structure is a 2D r.c. regular frames (Fig. 7) with three bays (5.00 m long) and six stories (2.85 m interstorey height). According to the proposed approach, pushover analyses have been carried out to define the capacity curves and to evaluate the structural response of both existing and braced frames. First mode proportional load profiles have been applied but it is worth noticing that different kinds of pushover methods (e.g. multimodal or adaptive) could be used as well if considered more suitable for the purpose without any changes in the procedure. The retrofitted structure has been analyzed also adopting a mass proportional load profiles.

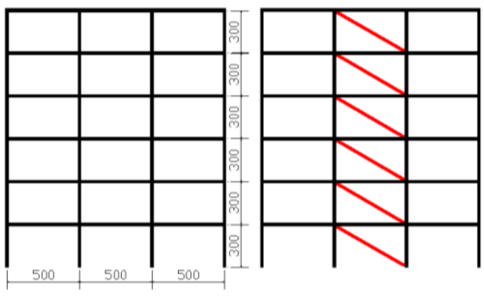

Figure 7. (left) 3x6 r.c. existing frame [cm]; (right) BRB distribution adopted

The Capacity Spectrum Method is used to predict the performance point of the structure and then to evaluate its corresponding seismic response in terms of horizontal story displacements and damage distribution. The performance point of the existing structures in terms of base shear and top displacement are V_S=227 kN and $D_{t,S}$=83 mm. Then, the performance objective is to reduce maximum displacements so as to avoid structural damage *($D_{t,S,targ}$=46 mm)* in case of a 0.30 g seismic event. As shown in Fig. 8a, BRB addition modifies the capacity curve of the existing frame and thus its performance point. The iterative procedure converges quickly to the target displacement (the tolerance can be defined by the designer) as depicted in Fig. 8b, the procedure can be interrupted as the convergence or a conservative result has been reached. The performance point of the braced frame with *BRB* designed at step 6 is defined by a base shear V_{S+B} =353 kN and a top displacement $D_{t,S+B}$ =41 mm (practically coincident with the target one).

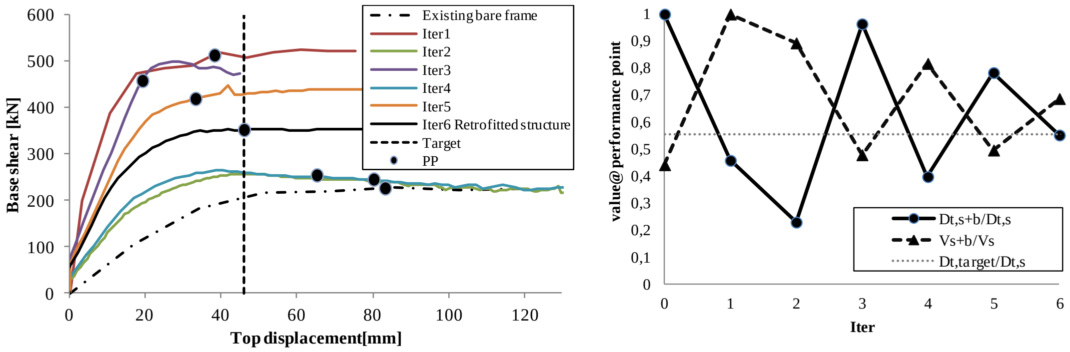

Figure 8. 2D frame: a) *BRB* effect on capacity curves, b) convergence of the procedure

4.2. Design of a BRB system for an existing structure

The proposed design procedure has been applied to retrofit an existing structure built in Avezzano (province of L'Aquila, Italy) and designed on 1964. The structure is a regular five storey r.c. frame with one basement; the procedure has been applied in order to prevent damage on both structure and infills in case of a severe seismic event with a p.g.a.=0.284 (Italian technical code D.M. 2008).

According to the proposed approach, pushover analyses have been carried out to define the capacity curves and to evaluate the structural response for both longitudinal and transverse directions (first and second mode proportional load profiles have been applied). The adopted *BRB* distribution together with one of the *BRB*-frame connections designed is shown in Fig. 9; in order to guarantee a uniform load redistribution on foundation beams the basement has been stiffened and reinforced by means of r.c. shear walls. The selected target displacement D^*, adopted in the *BRB* design procedure, corresponds with the achievement, at whichever level, of an interstorey drift of 0.005 ($D_{0.005}$) and it's however lower than the collapse displacement $D_{s,u}$ (D^*=$D_{0.005}$ = 65 mm). The procedure has been interrupted at the third iteration, The fourth one corresponds to the final check of the selected commercial *BRB* available in the Italian market (commercial BRB were chosen with the aim of obtain a bracing system as closer as possible to what was previously obtained at iter 3).

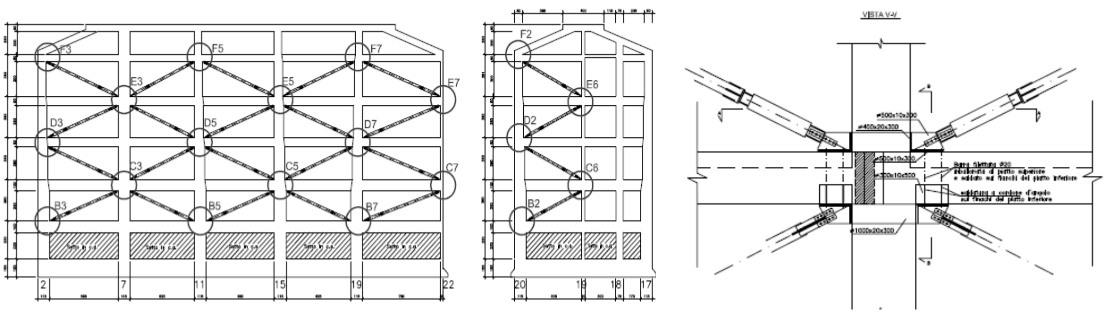

Figure 9. Existing structure: *BRB* distribution inside the frame (a), detail of BRB connection (b).

As shown in Fig. 10 the performance point of the existing structure is identified by a top displacement $D_{S,pp}$ = 100 mm (the collapse limit for the existing structure is achieved at $D_{s,u}$ = 70 mm) and the base shear $V_{S,pp}$ = 4299 kN; instead for the retrofitted structure (iter 3) the performance point corresponds to $D_{S+B,pp,3}$ = 61 mm and $V_{S+B,pp,3}$ = 3857 kN. Is therefore clear that dissipative braces are able to provide supplemental damping (the equivalent viscous damping for the existing and the retrofitted structure are respectively ν_S=0,21 and $\nu_{S+b,3}$=0,43) without a considerable increase of shear action at the foundation level. At iter 4 the theoretical mechanical parameters of the designed bracing system have been switched with what available in the Italian domestic market. The performance point of the executive project is: $D_{S+B,pp,4}$ = 67.0 mm, $V_{S+B,pp,4}$ = 4590 kN $\nu_{S+b,4}$=0,32.

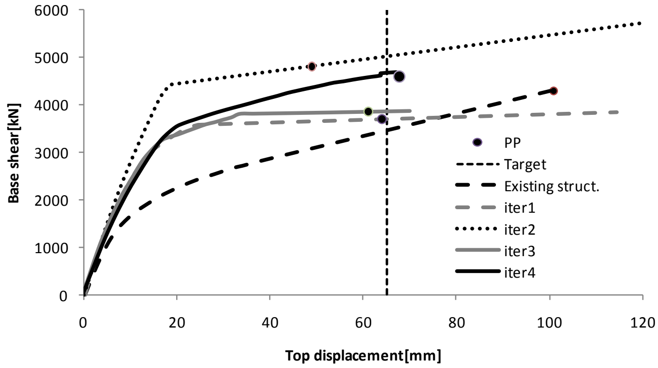

Figure 10. Existing structure: capacity curves and performance top displacement at each step.

5. CONCLUSIONS

A displacement based design procedure to design *BRB* for the seismic rehabilitation of existing r.c. frames has been presented; the procedure is simple and it can be used for professional applications. In this paper applications on one case study and on one real project, in order to assess the effectiveness of the proposed procedure, have been presented.

The procedure, which determines stiffness and yielding force of the *BRBs*, although relatively simple as it is based on static (non linear) analysis, proves to be effective and efficient requiring few iterations to converge.

Moreover it is adaptable to different situations that can be found working with existing structures: irregularity in plane and elevation, low plastic limit and other negative characteristics.

The procedure represent a substantial improvement of displacement based design for retrofitting using dissipative braces. It proves to be simple and allow to determine stiffness and strength of all braces to be added to the structure.

REFERENCES

Applied Technology Council. (1996). Seismic evaluation and retrofit of concrete buildings. *Report ATC-40*, Redwood City, California.

Bergami, A.V. and Nuti, C., (2012). A design procedure of dissipative braces for seismic upgrating structures. accepted for publication in *Earthquake and Structures*, **Vol. 3**, No. 6.

Bergami, A.V. and Nuti, C. (2011). Seismic retrofit of r.c. structures with dissipative braces, design and sustainability. *fib Symposium 2011*, Prague, Czech Republic.

Chopra, A. K. (2001). Dynamics of Structures: Theory and Applications to Earthquake. Engineering, 2nd ed., Prentice Hall, Upper Saddle River, N.J.

FEMA-274 (1997). “NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings”. *Federal Emergency Management Agency Publication*, U.S.A., 274.

FEMA – ASCE 356 (2000). “Prestandard and Commentary for the Seismic Rehabilitation of Buildings”. Washington, DC.

Filiatrault, A. and Cherry, S., (1988). “A simplified seismic design procedure for friction damped structures”. *Procedings of Fourth U.S. National Conference on Earthquake Engineering*, 3, Palm Springs, U.S.A..

Filiatrault, A. and Cherry, S. (1990). “Seismic design spectra for friction damped structures”. *Journal of Structural Engineering*, **116**(5), 1334-1355.

Kim, J. and Choi, H. (2004). Behavior and design of structures with buckling-restrained braces. *Engineering Structures* **26**, 693-706.

Zhang, R.H. and Soong, T.T. (1992). Seismic design of viscoelastic dampers for structural applications, *ASC, J. of Structural Engineering* **118: 5**, 1375-1392.