SIMPLIFIED MODEL TO EVALUATE THE SEISMIC ELASTIC RESPONSE OF LARGE REINFORCED CONCRETE DOMES

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SUMMARY:
This paper presents a simplified model to estimate the maximum seismic elastic responses of large reinforced concrete domes, such as the shear forces and the bending moments. This model represents the continuous dome as a vertical cantilever beam (base fixed, and free at the other edge), it has straight shaft and variable section. The result obtained in this study was applied to 10 reinforced concrete domes with variable ratio thickness/mean radius, with the purpose of calibrating the proposed model. The analysis of results allowed verifying that shear deformations are relevant in this type of structure; besides, the model provides satisfactory results considering discretizing in 20 segments. Finally, it was demonstrated that the adopted model, in addition to its simple implementation, allow finding errors in the estimation of seismic responses that do not exceed 15% for all cases studied.

Keywords: Reinforced concrete domes, seismic elastic response, large domes, simplified model

1. INTRODUCTION

Dome structures have been used as the cover of great spans in different modern and ancient architectural works, due to their geometric form, stiffness and self-supporting condition. In recent years, large reinforced concrete domes have been used as innovative forms in the development of silos because of their great height in relation to the span that they cover. Likewise, their structural and architectural advantages have been taken advantage in residential buildings, schools and stadiums. However, the complexity of their design and construction limit their massive use around the world.

There are a number of important analytical investigations about dome-type structures, coming from their static behavior and dynamic behavior in free vibration. However, there also exists a lack of studies to obtain the elastic dynamic response for domes subject to severe seismic loads which allow reasonable designs. For this reason, the present article will show the study which has been carried out to obtain the seismic elastic responses of large reinforced concrete domes by a simplified analysis model.

If a dome structure is modeled and analyzed as a cantilever beam (fixed at the base and free at the opposite end), considering the behavior of the material as linear and elastic, and physical and geometrical properties (area, inertia etc.) variables with height, differential equations of movement will be obtained which will result difficult to resolve in an exact form. The mathematical difficulties presented made the simplification of the problem necessary by way of the discretization of the continuous structure for the obtaining of an approximate solution. The proposed model simplifies the study of the elastic seismic behavior of large reinforced concrete domes which will allow obtain the maximum responses of interest considered in the structural design such as: shear forces and bending moments.

2. BASIC CONCEPTS FOR DISCRETIZING

2.1. Beam elements

The dynamic analysis of a continuous system is obviously complicated due to the resultant inertial forces of the displacements that vary with time and because the mass of the structure is distributed along its length. Because of this the analysis must be formulated in terms of differential equations in partial derivatives because the position along the length of the beam as well as time must be taken as independent variables. If you consider that the mass of the beam is concentrated on discrete points, the analytical problem is simplified considerably as the inertial forces develop only at these points (Clough & Penzien, 2003). On the other hand, if these masses are not fully concentrated, due to they have a finite rotational inertia; it is possible to consider the rotational displacement of such points. In cases where the mass is uniformly distributed, you can suppose that the deflected shape is expressed as a sum of a series of patrons of displacements; these are converted into displacements coordinates of the structure. In this way the expression for the dynamic deflection of a structure is defined in the Eqn. 2.1, where the variable x denotes the position of any point of such structure:

$$v(x) = \sum_n Z_n Y_n(x) \tag{2.1}$$

Where $v(x)$ is the transversal displacement of every point in the perpendicular direction to the local axis of a structural element, $Y_n(x)$ denominate shape functions, Z_n are the generalized coordinates and n is the number of generalized coordinates.

2.2. Shape functions

Shape functions are defined as the form of displacements of a member when a degree of freedom (dof) of the member has a unitary value and the others are equal to zero. With this expression, the shape function of a structural system can be related to the shape functions of its component members by local coordinates (member) and global coordinates (system).

According to the Timoshenko beam theory (Cheng, 2001) the shape functions for a typical element (Fig. 1) considering effects of bending and shear deformations are those shown in Eqns. 2.2 to 2.5.

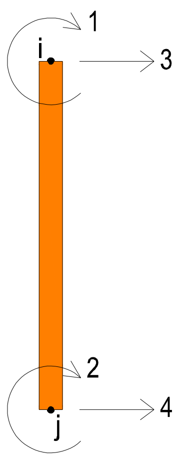


Figure 1. Considered degrees of freedom in a beam element

$$\mathrm{Y}_1 = \frac{\mathrm{L}}{1+\phi}\left[\mathrm{r} - \mathrm{r}^2\left(2 + \frac{1}{2}\phi - \mathrm{r}\right) + \phi\mathrm{r} - \frac{1}{2}\phi\mathrm{r}^*\right] \tag{2.2}$$

$$\mathrm{Y}_2 = \frac{-\mathrm{L}}{1+\phi}\left[\frac{1}{2}\phi + \mathrm{r}^2\left(1 - \frac{1}{2}\phi\right) - \mathrm{r}^3 + \left(\frac{1}{2}\phi\mathrm{r} - \frac{1}{2}\phi\right)^*\right] \tag{2.3}$$

$$Y_3 = \frac{1}{1+\phi}[-r^2(3-2r) + 1 + \phi(1-r)\ ^*] \quad (2.4)$$

$$Y_4 = \frac{1}{1+\phi}[\phi + 3r^2 - 2r^3 + \phi(r-1)\ ^*] \quad (2.5)$$

Where have been considered:

$$\phi = \frac{12EI}{G\mu AL^2} \quad (2.6)$$

$$r = \frac{x}{L} \quad (2.7)$$

Where I and μA are the properties of the transversal section (moment of inertia and effective shear area respectively), L is the length of the considered element, E and G are the module of elasticity and shear modulus of the material, respectively. The terms marked with an asterisk (*) are the resultant shape functions due to the shear deformations.

For a member in space (three-dimensional), the stiffness and mass coefficients can be formulated using its functions through the Energy Theorem (Cheng, 2001), described in Eqns. 2.8 and 2.9:

$$k_{ij} = E\int_V Y''_{bi}\, Y''_{bj} z^2 dV + G\int_V Y'_{si}\, Y'_{sj} dV \quad (2.8)$$

$$m_{ij} = \int_V \rho(x)[Y]^T\,[Y]\,dV \quad (2.9)$$

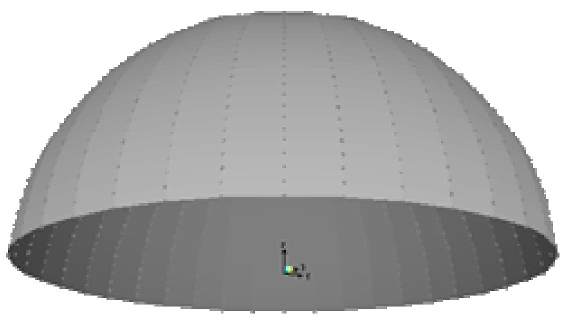
Where Y_b and Y_s are the shape functions due to the bending and shear deformations, respectively; z is the distance from the fiber location to the neutral axis of the cross-section, V is the volume of the structure and ρ is the density of the material.

These formulations apply to various types of finite elements in continuum mechanic.

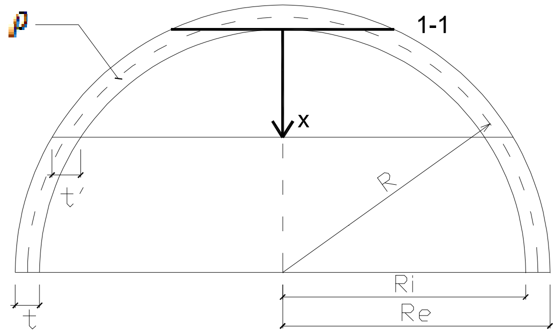
3. DISCRETIZING THE DOME STRUCTURE

Due to the complexity of resolving the differential equations in partial derivatives that result from the movement of a continual element of a dome structure (Fig. 2), it was decided to resolve the problem by discretizing the structure. For this purpose the consistent masses criterion was used to achieve a better representation of the inertial characteristics of the system.

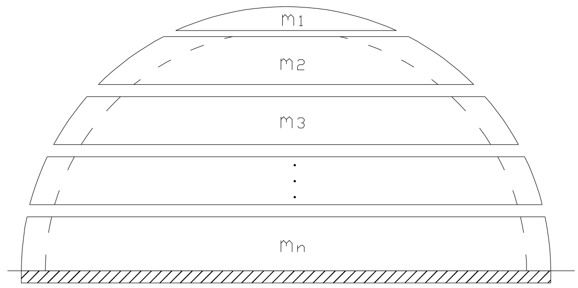
The consistent masses and concentrated masses criteria depends particularly on the stiffness, cross-section of the element, shear coefficient and shear modulus; but the criterion of consistent masses also considers the degree of coupling between the rotational and translational degrees of freedom. Therefore, the consistent masses matrix is a matrix which includes bending and shear effects, and rotational inertia.



a) Structure



b) Geometric characteristics



c) Discretizing

d) Beam model

Figure 2. Simplified model of the dome according to the consistent mass criterion

The dome will be sectioned longitudinally on a determined number of beam elements, which are interconnected in a finite number of nodes. Two dof are considered in these nodes (lateral and rotational displacements). The dome thickness "t" is consistent in the radial direction, but is variable and equal t' in the longitudinal direction (projection to the base).

The model was sectioned in segments of equal height from the axis 1-1 in Fig. 2.b, then the stiffness and mass matrices of the sectioned elements are determined and they are assembled with the respective stiffness and mass matrices of the left element at the top of the dome, with the purpose of obtaining the stiffness and mass matrices of the complete structure.

Therefore, Eqns. 2.8 and 2.9 are reduced to the Eqns. 3.1 and 3.2, which corresponding to stiffness and mass matrices for beam elements of variable cross-section that form the dome structure, *x* being the considered position along the height of the dome.

$$k_{ij} = E \int_0^L Y''_{bi} Y''_{bj} I dx + G \int_0^L Y'_{si} Y'_{sj} \mu A dx \qquad (3.1)$$

$$m_{ij} = \rho \int_0^L Y_{(b+s)i} Y_{(b+s)j} A dx + \rho \int_0^L Y'_{bi} Y'_{bj} I dx \qquad (3.2)$$

Where μ represents the shear coefficient (which varies with respect to the *x* position and influences the formulation of the shape functions). The variation of the shear coefficient for this type of cross-section with respect to the ratio R_i/R_e (internal radius/external radius) is shown in Fig. 3, where the external radius is equal to the sum of the internal radius plus t'.

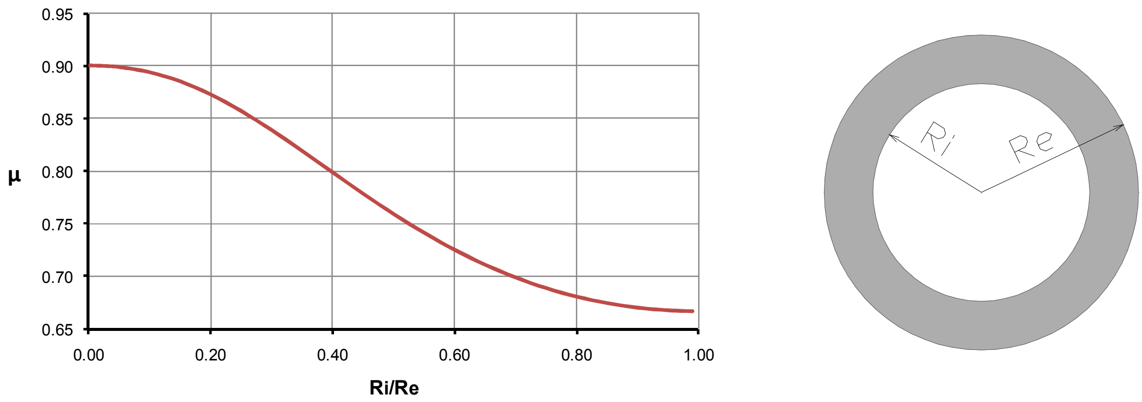


Figure 3. Variation of the shear coefficient in circular hollow sections

4. SENSITIVITY ANALYSIS

If a dome is modeled as a beam with straight shaft and variable cross-section, the ratio h/S (height/Span) will always be approximately equal to 1/2. Therefore, because of the discrete elements have less height than the structure, they will have a h/S ratio less than the said value and for this reason their behavior will be controlled by shear effects (h/S< 2, where h is the height of the element and S is the horizontal distance that they cover). Thus, it is concluded that the effects of bending and shear must be taken into consideration during the analysis of these elements.

The sensitivity analysis was done for responding the question: In how many elements is it necessary to discrete the dome? Table 4.1 shows the dimensions of a real family of ten large reinforced concrete domes and the t/R ratio (thickness/mean radius) of each one.

Table 4.1. Geometrical characteristics of the studied family of domes

N°	Radius (m)	Thickness (m)	t/R
D-1	10.00	0.10	0.01
D-2	6.00	0.20	0.03
D-3	8.00	0.40	0.05
D-4	10.00	1.00	0.10
D-5	8.00	1.20	0.15
D-6	10.00	2.00	0.20
D-7	6.00	1.50	0.25
D-8	8.00	2.40	0.30
D-9	10.00	3.50	0.35
D-10	6.00	2.50	0.42

Each one was analyzed under the following considerations:

- The consistent masses criterion was employed.
- The number of discrete elements was varied in 5, 10, 15 and 20 elements.
- The properties of the concrete were:

-Elastic modulus	=	2 100 000 T/m^2
-Density	=	0.245 $T.s^2/m^4$
-Poisson module	=	0.2
-Shear modulus	=	875 000 T/m^2

With these considerations, the seismic elastic responses of interest (shear forces and bending moments) were evaluated using an own program done in MatLab (Mathworks, 2007) and the resultant responses were compared with the responses obtained by the Finite Elements Method (FEM) considering more complex elements. Such elements correspond to the FRAME, SHELL and SOLID types available in the program SAP2000 (CSI, 2008).

4.1. Seismic parameters

The seismic analysis of each one of the domes was carried out by the modal spectral superposition method and it was considered as design seismic solicitation the pseudo-accelerations response spectrum of the Peruvian standard (SENCICO, 2003). The solicitation was applied in a uni-directional way on the base of the structure and was obtained from the parameters of site shown in Table 4.2.

Table 4.2. Parameters of site used in the design spectrum

ZONE PARAMETER		USE PARAMETER		SOIL PARAMETER		
ZONE	Z	CATEGORY	U	SOIL	S	Tp (s)
3	0.4	B	1.3	S2	1.2	0.6

Where T_p is the period which defines the top floor acceleration of the spectrum for each type of soil. The spectral acceleration is defined by the Eqn. 4.1, where C is the seismic amplification factor, which depends on the period of each participating mode. Due to the reinforced concrete dome is a very rigid structure, the periods of each mode of the structure are inside the range of the spectrum top floor (It has a constant value and unique of C equal to 2.5).

$$S_a = \frac{ZUCS}{(R)} g \tag{4.1}$$

The standard (SENCICO, 2003) allows the reduction of the seismic forces required to maintain the structure in its elastic range of behavior during the design earthquake through the factor (R). Due to the Peruvian standard (SENCICO, 2003) does not give a value of (R) for this type of structure; it was considered the given recommendation by a foreign standard (ASCE, 2010). In effect the value of (R) was taken as equal to 3. This value is common in rigid structures and has little ductility.

The combination of the factors gave as a result a spectral acceleration equal to 0.52g, which was applied in the analysis of the family of domes. The rule used to estimate the corresponding maximum responses was the CQC rule (SENCICO, 2003).

Table 4.3 shows the comparison of the maximum seismic shear forces and maximum seismic bending moments at the base of dome D-6, given by the simplified model and by the different models of refined finite elements.

Table 4.3. Comparison of maximum elastic seismic shear forces and maximum elastic seismic bending moments at the base for dome D-6, employing different numbers of beam elements

SIMPLIFIED MODEL		SAP2000					
		FRAME		SHELL		SOLID	
n	V (Ton)	V (Ton)	% Error	V (Ton)	% Error	V (Ton)	% Error
5	1231.400	1263.346	2.53%	1152.469	-6.85%	1156.628	-6.46%
10	1258.000		0.42%		-9.16%		-8.76%
15	1263.200		0.01%		-9.61%		-9.21%
20	1265.100		-0.14%		-9.77%		-9.38%

SIMPLIFIED MODEL		SAP2000					
		FRAME		SHELL		SOLID	
n	M (Ton-m)	M (Ton-m)	% Error	M (Ton-m)	% Error	M (Ton-m)	% Error
5	7963.000	8097.355	1.66%	7134.754	-11.61%	7269.463	-9.54%
10	8036.100		0.76%		-12.63%		-10.55%
15	8049.500		0.59%		-12.82%		-10.73%
20	8054.200		0.53%		-12.89%		-10.79%

From the analysis, independent of the value of the t/R ratio and the analyzed response, this latest one converges completely when the model has 20 discrete elements plus one that corresponds to the upper disc (upper spherical cap). Therefore, it is recommended that the seismic analysis of the dome be carried out with this number of beam element, as the committed error when it is evaluated shear forces and bending moments with the simplified model does not exceed 15%.

5. MAXIMUM SEISMIC RESPONSES

Figs. 4 and 5 show the variation of the maximum seismic elastic responses along the height of domes D-1 and D-8, respectively. The obtained responses from the simplified model are compared with those of the model of SOLID elements (CSI, 2008) which were considered as an “exact” response. It is recognized that the maximum responses are found on the base and in the same place the maximum errors are also found. The relative errors tend to zero in the upper levels.

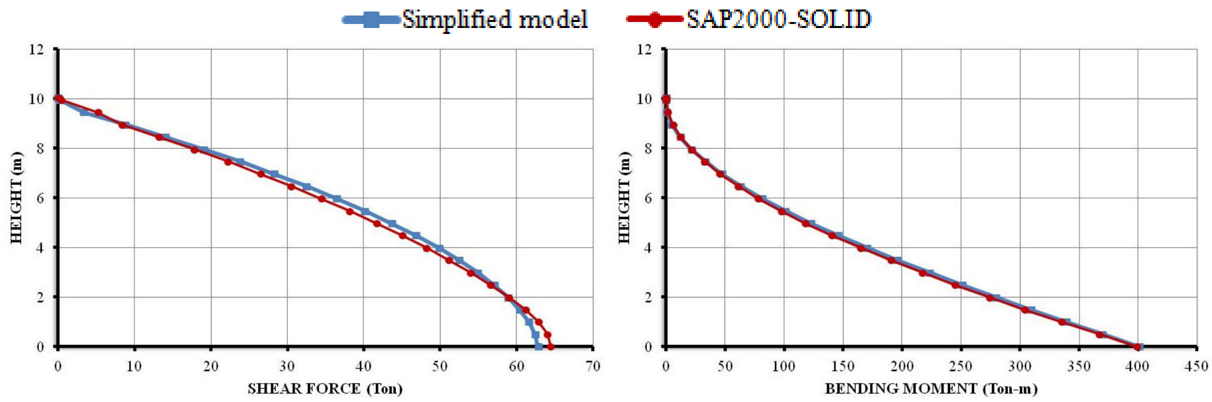


Figure 4. Variation of the shear force and bending moment along the dome D-1’s height

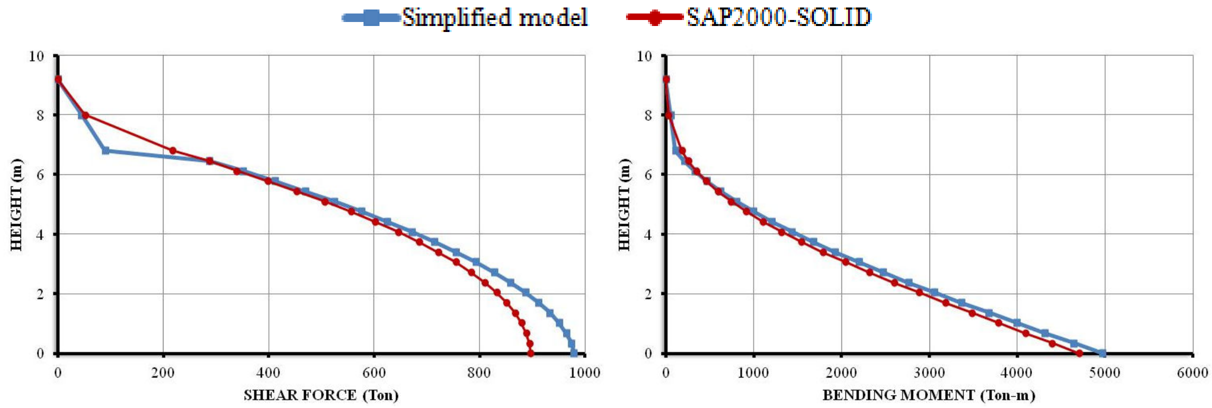


Figure 5. Variation of the shear force and bending moment along the dome D-8’s height

The analysis carried out for all the dome family is resumed in Table 5.1, where the relative errors are shown among the maximum seismic elastic responses obtained by the simplified discretized model in 21 elements and denominated "exact" responses obtained by the model of finite elements using the SOLID element (CSI, 2008).

Table 5.1. **S**implified model validation

		D-1	D-2	D-3	D-4	D-5	D-6	D-7	D-8	D-9	D-10
V_{max} (Ton)	Real	64.48	44.95	156.28	589.59	446.15	1156.63	313.16	897.03	2067.36	539.00
	Simpl. Model	62.83	45.17	160.91	629.45	484.44	1265.10	342.76	978.95	2241.10	580.38
	% Error	**2.56%**	**-0.47%**	**-2.96%**	**-6.76%**	**-8.58%**	**-9.38%**	**-9.45%**	**-9.13%**	**-8.40%**	**-7.68%**
M_{max} (Ton-m)	Real	399.04	164.65	760.58	3600.73	2197.23	7269.46	1208.20	4700.53	13756.29	2209.23
	Simpl. Model	400.97	173.86	822.92	4019.00	2471.00	8054.20	1307.20	4970.10	14200.00	2201.80
	% Error	**-0.48%**	**-5.60%**	**-8.20%**	**-11.62%**	**-12.46%**	**-10.79%**	**-8.19%**	**-5.73%**	**-3.23%**	**0.34%**

The proposed simplified model does not exceed 15 % in the estimation of the maximum seismic elastic responses, which is considered as an acceptable error for engineering purposes. Thus, the obtained results are useful to take into account in the structural design of this special type of structure. It shows that the adopted model, apart from its simple implementation, offers an adequate precision in the mentioned seismic responses of interest.

6. CONCLUSIONS

1. Since the shear deformations are relevant, a correct determination of the shear coefficient is necessary, in this case the circular hollow sections. This coefficient varies from 0.667 to 0.90 for full circular sections.

2. The seismic responses given by the simplified model converge rapidly as the number of discrete elements increases. Therefore, it is noted that the same relative errors are obtained in the responses for 5 discrete elements as for 20 elements, where the precision found after dividing the structure in more elements is minimal.

3. The maximum relative error in the shear forces and bending moments given by the simplified model and the real responses is found in the base of the dome. Such an error is reduced as we move further away from the base.

4. Good results are obtained in the estimation of seismic elastic bending moments and shear forces of large reinforced concrete domes modeling the structure with these considerations: consistent masses criterion, effects of shear deformations and discretization in 20 elements.

5. The simplified model provides relative errors in the seismic responses that do not exceed 15% for all cases of the family.

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