Drifts in Simple Buildings Subjected to Near Field Fault-Parallel and Fault-Normal Displacements

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SUMMARY:
Dependence of maximum linear and non-linear drifts in multi-story buildings excited by fault-parallel (permanent) displacement and fault-normal pulses is investigated. A system of nonlinear equations of motion of the model is solved by the fourth-order Runge-Kutta method. For linear system response, the differential-ground-motion is important only for the first story, and, depending on the time delay and earthquake magnitude, it can be 2 to 3 times larger than the drift computed for synchronous horizontal ground motion. The drift of the top stories is not amplified and is relatively insensitive to the differential motion effects. Because of the large initial velocity present in the ground motion near earthquake faults the story drifts quickly exceed the typical design levels and fault-normal pulses produce more intense drift demands relative to those for fault-parallel displacements. For nonlinear systems, the effect of vertical and rocking differential ground motions contribute more to top-story drifts. In the nonlinear response range, simultaneous action of horizontal, vertical, and rocking differential ground motions can amplify the drifts by more than 2 and 3 times relative to the drifts computed in common analyses, which consider only uniform horizontal ground motion.

Keywords: Near-fault ground motion, earthquake response, differential strong ground motion.

1. INTRODUCTION

In the near field of large earthquakes, and especially close to surface faults, the strong ground motion can be dominated by the permanent displacements (typically parallel to the fault surface) and by large pulses (often perpendicular to the fault). Traces of these large displacements and pulses may not always be obvious in the processed records of the recorded motions because of band-pass filtering, designed to eliminate digitisation and processing noise (Trifunac and Lee 1979).

When the distances between the multiple support points are large (e.g., bridges, dams, tunnels, and long buildings), the effects of differential motions become important and should be considered in dynamic analyses. Spatial and temporal representations of strong earthquake motion required for such analyses have been investigated in numerous papers (Zerva 2009). Their consequences have been studied for the response of beams, bridges, simple models of three-dimensional structures, long buildings (Todorovska and Trifunac 1989, 1990a,b), and dams (Kojić and Trifunac 1988, 1991a,b). Simple analyses of two-dimensional models of long buildings suggest that when \( a/\lambda < 10^{-4} \), where \( a \) is wave amplitude and \( \lambda \) is the corresponding wave length, the wave propagation effects on the response of simple structures can be disregarded. For shorter waves, but still longer than the characteristic dimensions of the structure, Trifunac and Todorovska (1997) and Trifunac and Gičev (2006) showed that the common response spectrum method for synchronous ground motion can be extended to be applicable for earthquake response analyses of extended structures experiencing differential in-plane and out-of-plane ground motion. Jalali et al. (2007) and Jalali and Trifunac (2007, 2008, 2009) found strong dependence of the R-factor on the magnitude of an earthquake for the response of a one-story system to in-plane motions close to an earthquake source.
The purpose of this study is to investigate the variation of maximum linear and nonlinear drift in the three-story buildings subjected to fault-normal pulses and fault-parallel displacements.

2. DYNAMIC MODEL

As can be seen from Fig. 1, the model we consider is a three-story building consisting of three rigid floors with masses \( m_i \), polar mass moments of inertia \( I_i \), and length \( L \), supported by six rigid mass-less columns connected at two ends by circular springs. The stiffness of the springs is assumed to be bilinear, as shown in Fig. 2. The mass-less columns are also connected at two ends by circular dashpots providing a fraction of critical damping. Rotation of the columns is assumed not to be small, which leads us to consider the geometric nonlinearity. The masses are acted upon by the acceleration of gravity, \( g \), and are excited by differential ground motions at two piers. The deformed shape of the structural model is shown in Fig. 3. We define the parameters of the model as follows:

- \( k_{si} \) = Initial rotational stiffness of column springs of \( i \)-th story;
- \( c_{si} \) = Linear rotational damping coefficient of columns of \( i \)-th story;
- \( m_i \) = Mass of rigid beam of \( i \)-th story
- \( L_i = \frac{1}{12} m_i L^2 \) = Polar moment of inertia of rigid beam of \( i \)-th story;
- \( h_i \) = Height of \( i \)-th story;
- \( \psi_{si} \) = Relative rocking angle of \( i \)-th column of the first story;
- \( \phi_{si} = \theta_{si} + \psi_{si} \) = Absolute rocking angle of \( i \)-th column of the first story;
- \( \phi_{ji} \) = Relative rocking angle of \( i \)-th column of \( j \)-th story;
- \( u_{gi}, v_{gi}, \theta_{gi} \) = The free-field horizontal, vertical, and rocking motions of ground surface at the base of \( i \)-th column \((i = 1, 2)\); and
- \( U_{gi}, V_{gi}, \Theta_{gi} \) = Absolute horizontal, vertical, and rotational motions of the centre of gravity of \( i \)-th rigid beam.

Because of the assumed rigidity of the columns and beams, we can write the following relations between the displacements of beams and columns:

Absolute Horizontal and Vertical Displacements of \( i \)-th Beam Ends

Fig. 1. Model of a 3-story structure with rigid beams, mass-less columns, rotational springs and dampers, subjected to differential horizontal, vertical, and rocking components of ground motion.
Absolute Horizontal and Vertical Displacements of the Top of the Columns

1. 1st Story

\[ u_{x1} = u_{G1} + h_1 \sin \phi_1 \]
\[ v_{x1} = v_{G1} - h_1 (1 - \cos \phi_1) \]
\[ u_{x2} = u_{G1} + h_1 \sin \phi_2 \]
\[ v_{x2} = v_{G1} - h_1 (1 - \cos \phi_2) \]

2. \( i \)-th Story (\( i = 2,3 \))

\[ u_{x1} = u_{G1,1} + h_1 \sin(\theta_{G1,1} + \phi_{11}) \]
\[ v_{x1} = v_{G1,1} - h_1 \left[ 1 - \cos(\theta_{G1,1} + \phi_{11}) \right] \]
\[ u_{x2} = u_{G1,1} + h_1 \sin(\theta_{G1,1} + \phi_{12}) \]
\[ v_{x2} = v_{G1,1} - h_1 \left[ 1 - \cos(\theta_{G1,1} + \phi_{12}) \right] \]

By combining (1), and (2), we find the absolute motions of \( i \)-th rigid beam as follows:
\[ U_{G_i} = \frac{1}{2}(u_{r_1} + u_{r_2}) \]
\[ V_{G_i} = \frac{1}{2}(v_{r_1} + v_{r_2}) \]
\[ \sin \theta_{G_i} = \frac{v_{r_1} - v_{r_2}}{L} \quad i = 1, 2, 3 \]  \hspace{1cm} (3)

Next, we write the equilibrium equations. For the third story those are

\[ \sum F_X = 0 \Rightarrow -m_3 \ddot{U}_{G_3} + F_{33} + F_{34} = 0 ; \]  \hspace{1cm} (4a)
\[ \sum F_Y = 0 \Rightarrow -m_3 \ddot{V}_{G_3} + F_{31} + F_{32} - W_3 = 0 ; \]  \hspace{1cm} (4b)
\[ \sum M_{G_3} = 0 \Rightarrow -m_3 \ddot{\theta}_{G_3} + \frac{L}{2} \cos \theta_{G_3} (F_{31} - F_{32}) + \frac{L}{2} \sin \theta_{G_3} (F_{34} - F_{33}) = 0 ; \]  \hspace{1cm} (4c)
\[ \sum M = 0 \Rightarrow -m_2 \ddot{\theta}_{G_3} + F_{31} h_3 \sin \left( \theta_{G_3} + \phi_3 \right) - F_{32} h_3 \cos \left( \theta_{G_3} + \phi_3 \right) = 0 ; \]  \hspace{1cm} (4d)
\[ \sum M = 0 \Rightarrow -m_2 \ddot{\theta}_{G_3} + F_{32} h_3 \sin \left( \theta_{G_3} + \phi_3 \right) - F_{31} h_3 \cos \left( \theta_{G_3} + \phi_3 \right) = 0 ; \]  \hspace{1cm} (4e)

The equilibrium equations of \( i-th \) story (\( i=1,2 \)) are

\[ \sum F_X = 0 \Rightarrow -m_i \ddot{U}_{G_i} - F_{i(i+1)3} - F_{i(i+1)4} + F_{i3} + F_{i4} = 0 ; \]  \hspace{1cm} (5a)
\[ \sum F_Y = 0 \Rightarrow -m_i \ddot{V}_{G_i} - F_{i(i+1)3} - F_{i(i+1)2} + F_{i1} + F_{i2} - W_i = 0 ; \]  \hspace{1cm} (5b)
\[ \sum M_{G_i} = 0 \Rightarrow \]
\[ M_{Q_{i+1}G} + M_{Q_{i+2}G} + M_{Q_{i+1}G} + M_{Q_{i+2}G} \}
\[ -m_i \ddot{\theta}_{G_i} + \frac{L}{2} \cos \theta_{G_i} (F_{i1} - F_{i2}) + \frac{L}{2} \sin \theta_{G_i} (F_{i4} - F_{i3}) = 0 \]  \hspace{1cm} (5c)
\[ \sum M = 0 \Rightarrow -m_2 \ddot{\theta}_{G_i} + F_{i1} h_i \sin \left( \theta_{G_i} + \phi_i \right) - F_{i2} h_i \cos \left( \theta_{G_i} + \phi_i \right) = 0 ; \]  \hspace{1cm} (5d)
\[ \sum M = 0 \Rightarrow -m_2 \ddot{\theta}_{G_i} + F_{i2} h_i \sin \left( \theta_{G_i} + \phi_i \right) - F_{i1} h_i \cos \left( \theta_{G_i} + \phi_i \right) = 0 . \]  \hspace{1cm} (5e)

\[ W_i = m_i g \]  \hspace{1cm} is the weight of \( i-th \) story. The moments in rotational springs are determined from

\[ M_{i} = k_\phi F(\phi_i - \theta_{G_i}) + c_\phi \dot{\phi}_i - \dot{\theta}_{G_i} \]
\[ M'_{i} = k_\phi F(\dot{\phi}_i) + c_\phi \dot{\phi}_i \]
\[ M_{1j} = k_\phi F(\phi_j - \theta_{G_i}) + c_\phi (\dot{\phi}_j - \dot{\theta}_{G_i}) \]
\[ M'_{1j} = k_\phi F(\dot{\phi}_j - \theta_{G_i}) + c_\phi (\dot{\phi}_j - \dot{\theta}_{G_i}) \quad (i = 1, 2, 3; j = 1, 2) \]  \hspace{1cm} (6)
where $F(\phi)$ is a nonlinear function of the type described in Fig. 2. By combining the above equations we obtain the independent equations of motion of the system as follows:

$$
-m_1 \ddot{v}_{o_1} - m_1 (\ddot{v}_{o_1} + g) G_1^* - B_1^* \ddot{\theta}_{o_1} - m_1 (\ddot{v}_{o_1} + g) H_1^* - G_1^* I_1^* = 0
$$

$$
-m_2 \ddot{v}_{o_2} - m_2 (\ddot{v}_{o_2} + g) G_2^* - B_2^* \ddot{\theta}_{o_2} - m_2 (\ddot{v}_{o_2} + g) H_2^* - G_2^* I_2^* = 0
$$

$$
-m_3 \ddot{v}_{o_3} + m_3 (\ddot{v}_{o_3} + g) G_3^* - B_3^* \ddot{\theta}_{o_3} + m_3 (\ddot{v}_{o_3} + g) H_3^* - G_3^* I_3^* = 0
$$

where $A_i^*, B_i^*, C_i^*, G_i^*$, and $I_i^*$ depend to $\phi_i$ and input ground motion. The system has six degrees of freedom—three independent and three dependent. Because of the assumed rigidity of three beams, their lengths are constant. Therefore, we can write the following relations for three beams:

$$
(L - u_{r_{12}} - u_{r_{11}})^2 + (v_{r_{12}} - v_{r_{11}})^2 = L^2
$$

$$
(L - u_{r_{22}} - u_{r_{21}})^2 + (v_{r_{22}} - v_{r_{21}})^2 = L^2
$$

$$
(L - u_{r_{32}} - u_{r_{31}})^2 + (v_{r_{32}} - v_{r_{31}})^2 = L^2
$$

Using (2), (3), (6), and (8) we can write (7) in terms of, $\phi_{11}$, $\phi_{21}$, and $\phi_{31}$ as follows:

$$
\begin{cases}
z_{12} \ddot{\phi}_{11} + z_{13} \ddot{\phi}_{12} + z_{13} = 0 \\
z_{21} \ddot{\phi}_{11} + z_{23} \ddot{\phi}_{21} + z_{23} = 0 \\
z_{31} \ddot{\phi}_{11} + z_{32} \ddot{\phi}_{31} + z_{33} = 0
\end{cases}
$$

where $z_{ij}$ is nonlinear function of $\phi_{ij}$ and input ground motion. The system of nonlinear equations of motion of the model in Fig. 2, which is described by (9), can be solved by numerical methods. We used the fourth-order Runge-Kutta method because of its self-starting feature and the long-range stability.

Floor masses and story stiffness vary linearly from top to bottom as follows. Their relative values are so proportioned that the fundamental period of vibration of the building is 0.1N, $N$ being the number of stories in the building (Gupta and Trifunac 1989).

$$
m_j = (1.0 - 0.4 \frac{j-1}{N-1}) m_i
$$

$$
k_{\theta j} = (1.0 - 0.4 \frac{j-1}{N-1}) k_{\theta i}
$$

$$
\begin{array}{c}
\begin{align*}
&= 1, 2, \ldots, N \\
&= 1, 2, \ldots, N
\end{align*}
\end{array}
$$

We suppose that the floor damping coefficients vary linearly from top to bottom as follows:

$$
c_{\theta j} = (1.0 - 0.4 \frac{j-1}{N-1}) c_{\theta i}
$$

$$
\begin{array}{c}
\begin{align*}
&= 1, 2, \ldots, N
\end{align*}
\end{array}
$$

Fig. 4. Fault-parallel, $d_{\parallel}(t)$, and fault-normal, $d_{\perp}(t)$, displacements with magnitude $M=7$. 
3. NEAR-FAULT GROUND MOTION

We describe the ground motion by $d_F$ (fault-normal pulse) and $d_N$ (fault-parallel permanent displacement) and select their amplitudes and duration consistent with the variables which describe near fault motions. Fig. 4, shows a fault schematically with these two characteristic motions, $d_N$, and $d_F$, which describe monotonic growth of the displacement toward the permanent static offset, and a pulse, here assumed to be perpendicular to the fault and associated with failure of a nearby asperity or passage of dislocation under or past the observation point (Haskell 1969). Further discussion and motivation for selecting these simple strong motion displacement functions are described in our previous work (Jalali and Trifunac 2007, 2008, 2009).

An important physical characteristic of $d_N$ and $d_F$ is the large initial velocity associated with the onset of these motions. It is proportional to the stress drop on the fault and even in the presence of nonlinear site response that can be in the range of hundreds of cm/s (Trifunac 2008, 2009).

For the fault-normal pulse, we chose (Fig. 4 -center)

$$d_F(t) = A_F e^{-\alpha_F t},$$

where the values of $A_F$, and $\alpha_F$, versus earthquake magnitudes, are given in Trifunac (1993, 2009).

For the fault-parallel permanent displacement, we consider (Fig. 4 -bottom)

$$d_N(t) = \frac{A_N}{2} (1 - e^{-\tau_N t}),$$

where the values of $A_N$, and $\tau_N$, versus earthquake magnitudes, are given in Trifunac (2009).

The amplitudes of $d_F$ and $d_N$ have been studied in numerous regression analyses of recorded peak displacements at various distances from the fault and in terms of the observed surface expressions of fault slip. The latter are traditionally presented as average dislocation amplitudes, $\bar{u}$, and are related to $d_N$, as $\bar{u} = 2d_N$ (see Fig. 4 -top).

4. STRUCTURAL RESPONSE

The relative motion of individual column foundations and of the entire foundation system will depend on the type of foundation and stiffness of the connecting beams and slabs, the characteristics of the soil surrounding the foundation, the type of incident waves, and the direction of wave arrival (Trifunac 1997; Trifunac et al. 1999). At the base of each column, the motion has six degrees of freedom, which will depend on the foundation-soil interaction and on the degree to which the nonlinear deformations occur in the structure and in the soil. In this paper, we consider only simultaneous action of horizontal, vertical, and rocking components of near-fault ground motion at the base of the columns $(u_{g1}, v_{g1}, \theta_{g1})$, for earthquake with $M = 5$ to 7, but we disregard the effects of foundation-soil interaction. We assume that the building is near a fault and that the longitudinal axis of the building (X-axis) coincides with the radial direction (r-axis) of the propagation of waves from the earthquake source so that the absolute displacements of the bases of columns are different only because of the wave passage. We assume that the ground motion can be described approximately by linear wave
theory. By considering the wave propagation from left to right in Fig. 1, we assume that the excitations at the bases of two columns have the same amplitude, but different phase. The phase difference (or time delay $\tau$) between the two ground motions depends on the distance between columns and the horizontal phase velocity of the incident waves (Trifunac and Todorovska 1997). As is seen from Fig. 1, the system is excited by $u_{g_i}, v_{g_i}, \theta_{g_i}, i = 1, 2$, at the two bases, so that

$$u_{g_2}(t) = u_{g_1}(t - \tau), \quad v_{g_2}(t) = v_{g_1}(t - \tau), \quad \theta_{g_2}(t) = \theta_{g_1}(t - \tau), \quad \tau = L/C_x,$$

where $C_x$ is the horizontal phase velocity of incident waves. In this study, for simplicity, we assumed that $v_{g_2}(t) = \pm u_{g_2}(t)$, considering the upward and downward ground motions, and the functional form of $u_{g_2}(t)$ is defined by (12) and (13) for the fault-normal pulse and fault-parallel displacements, respectively. In Figs. 5 through 8 we illustrate the results for $v_{g_2}(t) = +u_{g_2}(t)$, and in Figs. 9 and 10 for $v_{g_2}(t) = -u_{g_2}(t)$. The rocking component of the ground motion is approximated by (Trifunac 1982; Lee and Trifunac 1987) $\theta_{g_2}(t) = (-1/C_x)\dot{v}_{g_2}(t)$, where $\dot{v}_{g_2}(t)$ is the vertical velocity of ground motion. For body waves, $C_x$ will depend on the shear wave velocity in the half space ($\beta$) and the incident angle ($\gamma$). For surface waves, $C_x$ will depend on the dispersion characteristics of the medium ($C_x(\omega)$ will be different for each of the surface wave modes). For plane waves, the value of $C_x$ varies between $\beta$ and infinity ($\beta < C_x < \infty$). In this paper, the horizontal phase velocity will be assumed to vary between 100 m/s and infinity ($100 < C_x < \infty$), and the typical value of $L$ is in the range from 10 to 100 m. For illustrations in this work, it is assumed that $L = 10$ m, and for different phase velocities different time delays are selected ($\tau = 0.0, 0.01, 0.03, 0.05, 0.1$). The height of each story is $h_1 = h_2 = h_3 = 3.5$ m, and the first period of the system is assumed to be $T_1 = 0.3$ s. The damping ratio of the first mode is taken to be $\zeta_1 = 0.02$. In nonlinear analyses, the material is assumed to be elastoplastic, and the yielding limit of rotational springs at all stories is supposed to be $\phi_y = 0.01$.

![Fig. 5. Maximum linear drift along the height of the building, excited by fault-normal pulse, for horizontal only (left) and horizontal, vertical and rocking strong motion (right), and time lags from 0 to 0.10 s.](image)
5. RESULTS AND CONCLUSIONS

Figures 5 through 10 illustrate the variation of maximum linear and nonlinear drifts along the height of the building subjected to fault-normal pulse and fault-parallel displacement, respectively. It is observed from Figs. 5, 7, 9 and 10 that for linear system the multi-component differential-ground-motion effect is mainly important for the first story and depending on the time delay and earthquake magnitude this effect can amplify the first story drifts by more than 2 or 3 times relative to the drift computed for synchronous horizontal ground motion ($\tau = 0$). This is in excellent agreement with theoretical prediction of these effects by Trifunac and Todorovska (1997). The drift of the top stories is not amplified and is relatively insensitive to differential motion effects. However, because of the high velocity of the ground motion near earthquake faults the story drifts quickly exceed the typical design levels and fault-normal pulses produce more intense drift demand than the fault-parallel displacement.

From Figs. 6, 8, 9 and 10 it is seen that for nonlinear system the effect of vertical and rocking differential ground motions becomes more prominent for the top-story drifts as well. In this condition
the simultaneous action of horizontal, vertical, and rocking differential ground motions can amplify the drift of top stories by more than 2 or 3 times relative to the common analysis for uniform horizontal ground motion and for the cases we analyzed this occurs for fault-normal pulse.

Fig. 8. Maximum nonlinear drift along the height of the building, excited by fault-parallel displacement, for horizontal only (left) and horizontal, vertical and rocking strong motion (right), and time lags from 0 to 0.10 s.

Fig. 9. Linear (left) and nonlinear (right) peak drifts in the response to fault-normal pulse, with negative vertical and rocking components of motion, and for time lags from 0 to 0.10 s.

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Fig. 10. Same as Fig. 9 but for fault-parallel displacement.


