

Dynamic Behavior of Flexible Rectangular Liquid Storage Tanks Subjected to Seismic Ground Motion

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SUMMARY:

An analytical method is proposed to determine the dynamic response of 3-D rectangular liquid storage tanks with four flexible walls, subjected to horizontal seismic ground motion. Fluid-structure interaction effects on the dynamic responses of partially filled fluid containers, incorporating wall flexibility, are accounted for in evaluating impulsive pressure. Solutions based on 3-D modelling of the rectangular containers are obtained by applying the Rayleigh-Ritz method using the vibration modes of flexible plates with suitable boundary conditions. Moreover, an analytic procedure is developed for deriving a simple formula that evaluates convective pressure and surface displacements in a similar rigid tank. The variation of dynamic response characteristics with respect to different tank parameters is investigated. A mechanical model, which takes into account the deformability of the tank wall, is developed. Accordingly, a simplified but an accurate design procedure is developed to improve code formulas for the seismic design of liquid storage tanks.

Keywords: rectangular tank, fluid-structure interaction, seismic design, impulsive response, convective response

1. INTRODUCTION

Liquid storage tanks are important components of lifeline and industrial facilities. They also play an important role in the rescue work after an earthquake. Based on observations from previous earthquakes, it is concluded that liquid storage tanks can be subjected to large hydrodynamic pressures during earthquakes. Consequently, high stresses can cause buckling failure in steel tanks. In concrete tanks, due to the large inertial mass of concrete, the stresses could be large and result in cracking, leakage or even collapse of the structure. The poor performance of some of these structures in past earthquakes has led engineers and researchers to study this problem, and to improve the behaviors of these structures.

There are some numerical and a few analytical methods that have been used for dynamic analysis of concrete rectangular liquid storage tanks. Hoskins and Jacobsen (1934) published the first report on analytical and experimental observations of rigid rectangular tanks under a simulated horizontal earthquake excitation. Housner (1963,1957) developed the most commonly used analytical model for estimating the dynamic response of a rigid rectangular tank. This model, with some modifications, has been adopted in most of the current codes and standards.

The 1964 Alaska earthquake (Haroun, 1983) caused the first large-scale damage to tanks of modern design of its time and initiated many investigations into the dynamic characteristics of flexible containers. Several studies were carried out to investigate the dynamic interaction between the deformable wall in the tank and the liquid, and showed that the seismic response of a flexible tank may be substantially greater than that of a similarly rigid tank. A three-degrees-of-freedom (3 DOF) model of the ground-supported cylindrical tank was developed by Haroun (1983), the application of which resulted in design charts used to estimate sloshing, impulsive and rigid masses.

For rectangular tanks, Haroun (1984) presented a very detailed method of analysis on the typical system of loadings. The hydrodynamic pressures were calculated by a classical potential flow approach. The formula of hydrodynamic pressures only considered the rigid wall condition. This may

be due to the fact that rectangular fluid containers are usually made of reinforced or prestressed concrete and may be considered quite rigid dynamically. Nevertheless, there are containers of this type for which flexibility must be taken into account in their dynamic response analysis, such as very large reinforced concrete structures used for the storage of nuclear spent fuel assemblies or prestressed concrete water tanks (Luft, 1984).

Some numerical methods that consider wall flexibility have been used for dynamic analysis of rectangular liquid storage tanks. Dogangun et al. (1996) and Dogangun and Livaoglu (2004) investigated the seismic response of liquid-filled rectangular storage tanks using the three-dimensional Lagrangian fluid finite element. Park et al. (1992) and Koh et al. (1998) studied the seismic response of rectangular tanks with four flexible walls by using a three-dimensional coupled boundary element-finite element method. Ghaemmaghami and Kianoush (2010) investigated the dynamic behavior of concrete rectangular tanks using the FEM in 2D space.

Moreover, there are a few analytical methods that have been used for dynamic analysis of rectangular liquid storage tanks. Kim et al. (1996) studied dynamic behavior of 3-D rectangular flexible fluid containers using the Rayleigh-Ritz method. In their study, only a pair of walls, orthogonal to the direction of the applied ground motion is assumed to be flexible, while the other pair remains rigid. Chen and Kianoush (2009) proposed a simplified method using the generalized SDOF system to study the dynamic response of liquid storage tanks. In the analytical methods that have so far been used for dynamic analysis of rectangular liquid storage tanks, neither the effect of sloshing nor the effect of wall flexibility have been appropriately considered. Moreover, the current design approach is inaccurate, as it does not fully consider all the major parameters affecting the response.

In this study, an analytical method is presented to investigate the dynamic response of flexible 3D rectangular liquid storage tanks with flexible walls on all four sides, subjected to horizontal seismic ground motion. Solutions based on three-dimensional modelling of the rectangular containers are obtained by applying the Rayleigh-Ritz method using the vibration modes of flexible plates with suitable boundary conditions. Trigonometrical functions that satisfy boundary conditions of the storage tank such that the flexibility of the wall is thoroughly considered are used to define the admissible vibration modes. The analysis is then performed in the time domain. The main objective of the final part of this study is to close the gap between analytical studies and practical design considerations. This is to provide the practicing engineers with a simple and sufficiently accurate tool for estimating seismic response of rectangular tanks. A mechanical model, which takes into account the deformability of the tank wall, is developed. The parameters of such a model can be obtained from developed charts and the maximum seismic loading can be predicted by means of a response spectrum characterizing the design earthquake. Accordingly, a simplified but an accurate design procedure is developed to improve code formulas for seismic design of liquid storage tanks.

2. FLUID MOTION IN RECTANGULAR TANKS

A rectangular tank with four flexible vertical walls of uniform thickness t_s and a horizontal rigid bottom is partially filled with incompressible and non-viscous liquid of depth H_L , as shown in Figure 1. The side lengths and height of the tank are $2L_x$, $2L_y$ and H_s , respectively. The walls of the tank are considered as thin plates made of linearly elastic, homogeneous and isotropic material and are assumed to perform transverse bending deflection but no in-plane deformation. The motion of the liquid is assumed to be frictionless and irrotational so that the velocity distribution of the liquid may be represented as a gradient of the velocity potential. According to the theory of fluid dynamics, the liquid velocity potential (Φ) should satisfy the Laplace equation and the hydrodynamic pressure at any point and time is given by

$$p(x, y, z, t) = -\rho_l \frac{\partial \Phi}{\partial t} \quad (1.1)$$

in which ρ_l is the mass density of the liquid. Considering the walls of the tank to be permeable and no cavitation on the liquid-wall interface, the liquid adjacent to the wall must move with it by the same velocity.

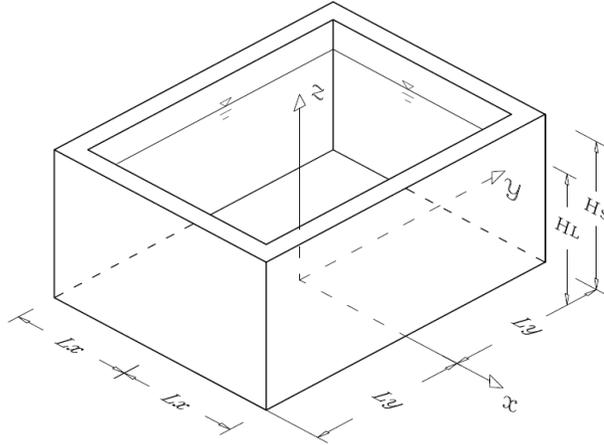


Figure 1. 3D model of rectangular container

3. SOLUTION OF HYDRODYNAMIC PRESSURE

The coupling between liquid sloshing modes and wall vibrational modes is weak; consequently, for the analysis it is sufficient to consider the two uncoupled systems separately. This includes the liquid-wall system and the free surface gravity waves in a similar rigid tank (Koh et al., 1998; Kim et al., 1996; Veletsos and Tang, 1987; Haroun, 1983, 1980). The solution for Φ will be expressed as the sum of an impulsive component, Φ_i , and a convective component, Φ_c . The impulsive component of the solution satisfies the actual boundary conditions along the tank wall and bottom and the condition of zero hydrodynamic pressure at $z = H_L$, whereas the convective component corrects for the difference between the actual boundary condition at $z = H_L$ and the one considered in the development of the impulsive solution. If the tank is subjected to an earthquake in the x direction (\ddot{u}_{gx}), using this fact that liquid velocity potential (Φ) should satisfy the Laplace equation and considering that the liquid adjacent to the wall must move with it by the same velocity, the condition of zero hydrodynamic pressure at $z = H_L$, the method of separation of variables and considering symmetry of both the liquid and the tank about the x - z plane and antisymmetry of them about the y - z plane, the impulsive velocity potential and the impulsive pressure can be analytically given in an infinite series form including the tank-wall dynamic deflection.

3.1. Impulsive responses

It is important to develop a set of suitable, admissible functions to describe the vibration of the tank wall in the Rayleigh-Ritz method. For this purpose the transverse motion of flexible wall is expressed as a linear combination of admissible functions:

$$w(x, y, z, t) = \sum_{m=1}^M \sum_{n=1}^N f_{mn}(t) \Psi_m^H \Psi_n^V \quad (3.1)$$

where coefficients $f_{mn}(t)$ = generalized coordinates to be determined; Ψ_n^V = nth vibration mode of a cantilever beam that is appropriate for wall displacement in a vertical direction; and Ψ_m^H = mth eigenfunction of vibrating beams, which boundary conditions are similar to those of the wall in horizontal direction. Because of the symmetry of both the liquid and the tank about the x - z plane and antisymmetry of them about the y - z plane only a quarter of the tank with appropriate boundary conditions is considered. As the tank wall can only provide transverse deflection but no in-plane deformation, the vibration of the tank wall can be equivalent to the vibration of a line supported

rectangular plate. The connecting line of the adjacent walls corresponds to the internal simply support which prevents the transverse motion of the plate but offers no resistance to the rotation. Using Eq. (3.1) the transverse motion of flexible wall can be rewritten in a matrix form as:

$$w = \Gamma \mathbf{f} \quad (3.2)$$

A matrix equation which governs the earthquake response of the undamped liquid-wall system can be obtained using the Green principle and by application of Hamilton's principle:

$$(\mathbf{M}_s + \mathbf{M}_{liquid})\ddot{\mathbf{f}} + \mathbf{K}\mathbf{f} = \mathbf{P}_{eff} \quad (3.3)$$

\mathbf{M}_s and \mathbf{K} are the mass matrix and stiffness matrix of the flexible wall that is modeled as a plate, respectively. \mathbf{M}_{liquid} and \mathbf{P}_{eff} are the added mass matrix due to the effect of the liquid and the effective earthquake load vector, respectively. This matrix equation of motion can be solved by the mode-superposition method. By equating the determinant of the left hand of Eq. (3.3) to zero, frequencies of impulsive modes of tank and eigenvectors pertinent to them are obtained.

$$\mathbf{f} = \sum_{k=1}^K \boldsymbol{\phi}_k q_k \quad (3.4)$$

$\boldsymbol{\phi}_k$ is the modal vector pertinent to the kth mode of vibration of the tank. $K = M \times N$ is the number of modes. Introducing damping into Eq. (3.3) and using Eq.(3.4), results in:

$$\ddot{q}_k + 2\zeta_f \omega_k \dot{q}_k + \omega_k^2 q_k = -\beta_k \ddot{u}_{gx} \quad (3.5)$$

where ζ_f , ω_k and β_k are damping ratio, frequency and modal participation factor of the kth impulsive mode respectively. The complete time history of $\mathbf{q}(t)$ and its time derivatives can be computed by a step-by-step application (Interpolation of Excitation). Once they are obtained, the displacement, acceleration and the impulsive pressure can be calculated.

3.2. Convective responses and surface displacements

The convective pressure p_c can be evaluated with reasonable accuracy by considering the tank wall to be rigid. It needs to satisfy Laplace's equation. Solution of Laplace's equation such that it satisfies the boundary conditions may be expressed as:

$$p_c(x, y, z, t) = \rho_l L_x \sum_{j=0}^{\infty} D_j A_j(t) \sin(\alpha_j x) \cosh(\alpha_j z) \quad (3.6)$$

in which

$$\lambda_j = (2j+1)\pi/2 \quad (3.7)$$

$$\alpha_j = \lambda_j / L_x \quad (3.8)$$

$$D_j = -2(-1)^j / [\lambda_j^2 \cosh(\lambda_j \mu)] \quad (3.9)$$

$$\mu = H_l / L_x \quad (3.10)$$

$$A_j(t) = \omega_j^e \int_0^t \ddot{u}_{gx}(\tau) \sin[\omega_j^e(t-\tau)] d\tau \quad (3.11)$$

where $A_j(t)$ represents the instantaneous pseudo acceleration of an undamped SDOF system which has

a circular natural frequency ω_j^c equal to that of the j th sloshing mode of vibration of the liquid and is excited by a base acceleration $\ddot{u}_{gx}(t)$.

$$\omega_j^c{}^2 = g\alpha_j \tanh(\alpha_j H_l) \quad (3.12)$$

Damping of convective responses (ζ_c) can be introduced into Eq. (3.11) easily. The vertical displacement, $\eta(x, y, t)$, of an arbitrary point at the liquid surface can be determined:

$$\eta(x, y, t) = -L_x \sum_{j=0}^{\infty} \frac{2(-1)^j A_j(t)}{\lambda_j^2 g} \sin(\alpha_j x) \quad (3.13)$$

On making use of identities (Standard Math Tables (Beyer 1976)):

$$\sum_{j=0}^{\infty} \frac{2}{\lambda_j^2} = \lim_{x \rightarrow 0} \frac{\tanh(x)}{x} = 1 \quad (3.14)$$

Using Eqs. (3.13) and (3.14) the maximum surface displacement near the wall, η_{\max} , is determined:

$$\eta_{\max} \leq (S_{ac}/g) L_x \quad (3.15)$$

in which S_{ac} is the maximum of $A_0(t)$, or the spectral acceleration corresponding to the natural frequencies $\omega_c = \omega_j|_{j=0}$ and damping ζ_c . Eq.(3.15) is the same as ACI 350.3-06 formula used to determine the vertical surface displacement.

4. SEISMIC DESIGN FORCES

The most common standards and codes currently used for the design of tanks are ACI 350.3 (2006), Eurocode-8 (2006) and New Zealand standard (2004). Seismic loads in these standards are based on the mechanical model derived by Housner (1963,1957) for rigid tanks with some modifications. The design procedure considers two response modes of the contained liquid: (1) The impulsive response in which the portion of the liquid accelerates with the tank walls, and (2) the convective response caused by the portion of the liquid sloshing in the tank. It should be noted that in these codes, the importance of the effects of wall flexibility has been recognized and the corresponding increase in the acceleration coefficients has been adopted. However, the effect of wall flexibility has not been thoroughly accounted for by a reasonable and accurate method. The main objective of this part of the study is to devise a practical approach which would allow, from an engineering point of view, a simple, fast and sufficiently accurate estimate of the seismic response of rectangular storage tanks.

4.1. Hydrodynamic base shear

The instantaneous hydrodynamic base shear $Q(t)$ is given by:

$$Q(t) = 2 \int_{-L_y}^{L_y} \int_0^{H_l} p|_{x=L_x} dz dy \quad (3.16)$$

The hydrodynamic base shear due to wall deformation relative to the ground $Q_f(t)$ using Eqs. (1.1), (3.4) and (3.5) can be expressed as:

$$Q_f(t) = \sum_{k=1}^K Q_k^f \ddot{u}_k(t) \quad (3.17)$$

where $u_k(t)$ is the solution of the differential equation:

$$\ddot{u}_k(t) + 2\zeta_f \omega_k \dot{u}_k(t) + \omega_k^2 u_k(t) = -\ddot{u}_{gx}(t) \quad (3.18)$$

For the applicable tank, all frequencies of impulsive modes are large. Acceleration spectral spectrums accepted in codes show that the maximum accelerations which correspond to significant modes are close to each other. Therefore the mode that has the maximum Q_k^f is the predominant mode. The dimensionless fundamental natural frequency, $\Omega_f = (\rho_s \omega_f^2 t_s H_s^4 / D)^{1/2}$ where D is the flexural rigidity of the tank wall, that corresponds to the predominant mode of vibration is displayed for tanks completely filled with water in Figure 2 assuming normal density concrete is used. The fundamental natural frequency (ω_f) that is pertinent to Ω_f can be determined for different values of the aspect ratio. Similar charts for different thicknesses can be found in Hashemi et al. (2012). Figure 2 shows that the first mode is predominant when $L_x \leq 1.2L_y$, otherwise another mode could become predominant. Therefore, in general, for considering the seismic response, it is better that the effect of all modes is considered. The value of $\ddot{u}_f(t)$ corresponding to ω_f can be obtained by the solution of Eq.(3.18) for the predominant mode. It is assumed that the maximum accelerations corresponding to significant modes are equal to the maximum of $\ddot{u}_f(t)$. Therefore Eq.(3.17) can be rewritten approximately:

$$Q_f(t) = m_f \ddot{u}_f(t) \quad (3.19)$$

where m_f is obtained by combining Q_k^f for all modes ($k=1,2,\dots,K$) by the Complete Quadratic Combination (CQC) method (Kiureghian (1980)). The value of m_f can be approximated using Eq.(3.20) obtained by curve fitting the numerical results. The numerical results show that m_f is independent of wall thickness.

$$\frac{m_f}{m_l} = \tanh \left[0.866 \left(\frac{2L_x}{H_L} \right) \right] / \left[1.732 \left(\frac{2L_x}{H_L} \right) \right] \quad (3.20)$$

where m_l is the total mass of liquid. Investigations show that when $L_y/H_L < 0.65$, a correction factor for m_f, c_f , should be considered:

$$c_f = 1.25 - 0.71(L_y/H_L - 0.3) \quad (3.21)$$

To determine the base shear one can consider only the first mode of convective response while the next modes are negligible:

$$Q(t) = m_c \ddot{u}_c(t) + m_f \ddot{u}_f(t) + m_r \ddot{u}_{gx}(t) \quad (3.22)$$

where m_r, m_f and m_c are equivalent masses corresponding to forces associated with ground motion,

wall deformation relative to the ground, and liquid sloshing, respectively. $\ddot{u}_c(t) = A_0(t)$ is the absolute acceleration of a SDOF system which has a circular natural frequency ω_c . Since the base shear and moment due to wall deformability are proportional to the relative acceleration of the wall, one can rearrange Eq. (3.22) in order to estimate the maximum seismic loads by means of a response spectrum. And subsequently, the maximum base shear can be estimated by:

$$|Q_{\max}| = \sqrt{(m_c s_{ac})^2 + (m_f s_{af})^2 + [(m_r - m_f)(\ddot{u}_{gx})_{\max}]^2} \quad (3.23)$$

in which S_{af} is the spectral accelerations corresponding to the natural frequencies ω_f .

4.2. Overturning moment

The overturning moment $M(t)$ induced on a section of the tank immediately above its base is given by:

$$M(t) = 2 \int_{-L_y}^{L_y} \int_0^{H_L} p|_{x=L_x} z dz dy \quad (3.24)$$

One may consider only the first mode of convective response to determine the overturning moment:

$$M(t) = m_c h_c \ddot{u}_c(t) + m_f h_f \ddot{u}_f(t) + m_r h_r \ddot{u}_{gx}(t) \quad (3.25)$$

where h_c , h_f and h_r are, respectively, the heights at which the convective component of the liquid mass, m_c , equivalent mass corresponding to force associated with ground motion, m_r , and equivalent mass corresponding to force associated with wall deformation relative to the ground, m_f , are considered to be concentrated. A proper function is obtained by curve fitting the numerical results:

$$h_f/H_L = 0.58 - 0.12 \tanh[2.5(L_x/H_L - 0.25)] \quad (3.26)$$

The numerical results show that the effect of variation of thicknesses or L_y/H_L on h_f/H_L is negligible. Using Eqs.(3.22) and (3.25) a mechanical model that is equivalent with the rectangular liquid storage tank has been developed and shown in Figure 3. The maximum overturning moment applied to the bottom of the wall is given by:

$$|M_{\max}| = \sqrt{(m_c h_c s_{ac})^2 + (m_f h_f s_{af})^2 + [(m_r h_r - m_f h_f)(\ddot{u}_{gx})_{\max}]^2} \quad (3.27)$$

5. VALIDITY VERIFICATION

In this section, a numerical example is presented to investigate the convergence and validity of the present method. A computer program was first written to check the validity of the theoretical formulation in computing seismic responses of the flexible storage tank.

Since rectangular tanks are used most often for the wet-type storage of nuclear spent fuel assemblies, a typical dimension for those tanks is selected (Koh et al., 1998): the height of the wall, $H_s=10$ m; the wall thickness, $t_s=1$ m; the water depth, $H_L=9$ m; the length of the short side wall, $2L_x=20$ m; and the length of the long side wall, $2L_y=50$ m, and typical material properties for the concrete tanks: the density, $\rho_s=2400$ kg/m³; the Young's modulus, $E=2.1 \times 10^{10}$ N/m²; and the Poisson's ratio, $\nu=0.17$. The N-S component of the 1940 El Centro Earthquake records is used as an input motion in x direction. The tank is assumed to be fixed to the ground, and to have 3 percent structural damping.

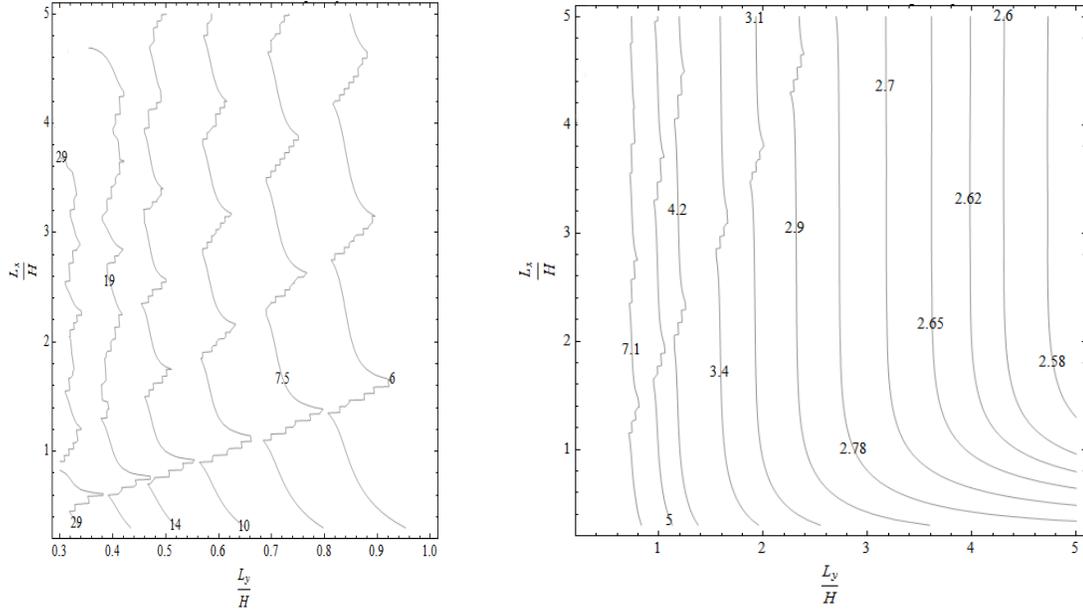


Figure 2. Dimensionless fundamental natural frequency (Ω_f) for tanks completely filled with water ($t_s/H_L=0.1$)

The time history of the resultant force acting on the long side wall of the tank model is compared with those presented by Koh et al. The comparison shows that they are in good agreement and the maximum of the resultant force predicted by the present method (9460 kN) is very close to that obtained by Koh et al. (9244 kN). The accuracy of the present method can be further verified by pressure distribution over the long side wall when the resultant forces reach their peak values. The predictions by the proposed method are compared with the results from the coupled 3 dimensions boundary element-finite element method (Koh et al., 1998), a Lagrangian fluid finite element (Dogangun and Livaoglu, 2004) and the results obtained by using the Eurocode-8 (Dogangun and Livaoglu, 2004) as shown in Figure 4. Time history of the sloshing motion at the middle cross-section of the long side wall are presented and compared with those using the indirect boundary-element-finite-element method (Koh et al., 1998) and finite element method (Ghaemmaghami, 2010) in Figure 5. The predictions by the proposed method are virtually identical to those of mentioned references.

Consider the tank discussed in the preceding example is full of water and its impulsive and convective responses are damped 5% and 0.5%, respectively. The maximum ground acceleration of the N-S component of the 1940 El Centro earthquake records is $(\ddot{u}_{gx})_{\max} = 0.313g$. The fundamental natural frequency of sloshing, $\omega_c = 1.19 \text{ rad/sec}$, is obtained from Eq.(3.12); and consequently, the spectral acceleration corresponding to it for a damping ratio of $\zeta_c = 0.5\%$ can be found from spectral response ($S_{ac} = 0.049g$). The fundamental impulsive frequency of liquid-wall vibration for values of $L_x/H_L = 1$, $L_y/H_L = 2.5$ and $t_s/H_L = 0.1$ is determined from Figure 2; $\Omega_f = 2.87$ and $\omega_f = 24.9 \text{ rad/sec}$. The spectral acceleration for a damping ratio of $\zeta_f = 5\%$ is $S_{af} = 0.799g$. The remaining parameters can be obtained from Eqs.(3.20) and (3.26): $m_f/m_l = 0.271$, $h_f = 0.464H_L = 4.64m$, and using ACI 350.3: $m_r/m_l = 0.542$, $h_r = 0.4H_L = 4m$, $m_c/m_l = 0.473$ and $h_c = 0.582H_L = 5.82m$.

The maximum surface displacement near the wall, the maximum base shear and the maximum overturning moment induced on a section of the tank immediately above its base, respectively, are determined using Eqs. (3.15), (3.23) and (3.27) as $(\eta_{\max})_{app} = 0.49m$, $(Q_{\max})_{app} = 22.99 \times 10^6 N$ and $(M_{\max})_{app} = 103.4 \times 10^6 N \cdot m$ that are in good agreement with the exact results that are $(\eta_{\max})_{exact} = 0.4556m$, $(Q_{\max})_{exact} = 19.32 \times 10^6 N$ and $(M_{\max})_{exact} = 86.13 \times 10^6 N \cdot m$.

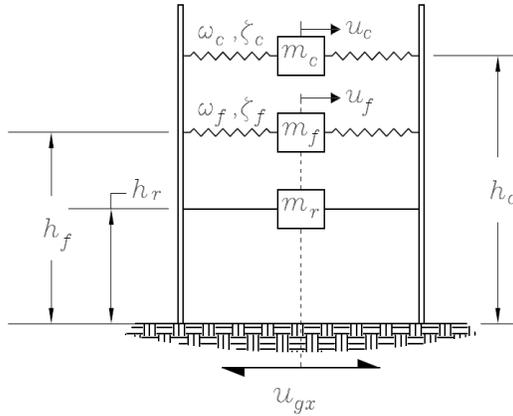


Figure 3. Mechanical model of flexible tank

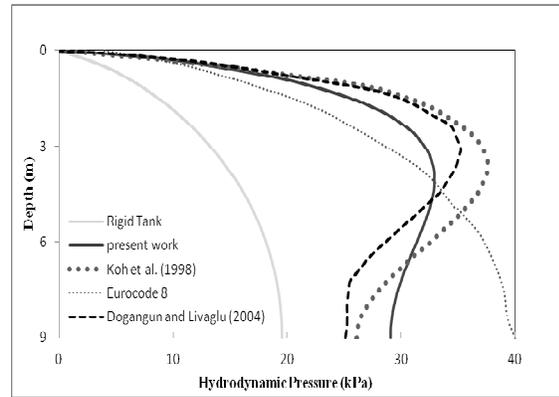


Figure 4. Comparison of pressure distributions along the height of the middle cross-section of the long side wall

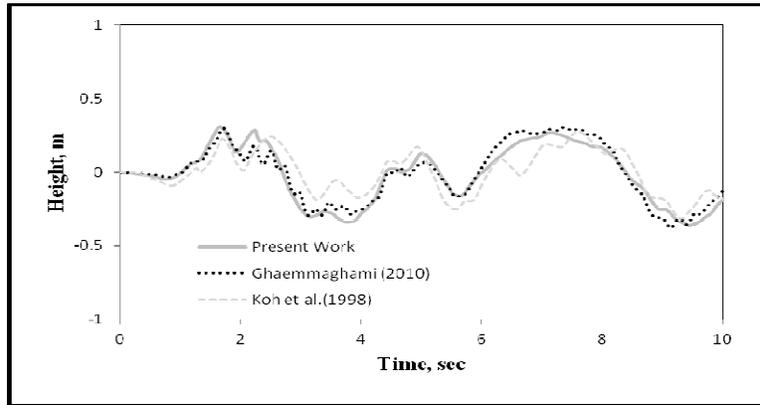


Figure 5. Time histories of liquid surface elevation at the middle cross-section of the long side wall

6. Conclusions

Analytical solution methods are developed that can be used for the analysis of the dynamic behavior of partially filled rectangular fluid containers under horizontal ground excitations.

Using the combination of the superposition method and the method of separation of variables, the exact analytical solution of the impulsive velocity potential is derived. Solutions based on three-dimensional modeling of the rectangular containers are obtained by applying the Rayleigh-Ritz method using the vibration modes of flexible plates with suitable boundary conditions. An analytical procedure is developed for deriving a simple formula that estimates convective pressure and surface displacements in a similar rigid tank.

The results of this study show that hydrodynamic pressure distributions for assuming rigid and flexible walls differ from each other in magnitude and in shape. The hydrodynamic pressures in the middle of the wall for flexible storage tanks are generally larger than for rigid storage tanks. Moreover, hydrodynamic pressure varies not only in the vertical direction but also in the horizontal direction over the wall surface. The validity and convergence of the proposed methods are confirmed through numerical examples. The predictions by the proposed method are in good agreement with the results

from a Lagrangian fluid finite element, FEM, and the results obtained by using the coupled BEM-FEM.

A mechanical model, which takes into account the deformability of the tank wall, is developed. The maximum seismic response of a deformable rectangular tank can therefore be estimated by means of a response spectrum. It is recommended that the effect of wall flexibility on hydrodynamic pressures should be considered in design codes and standards.

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