Prediction of the Ultimate deformation capacity of RC Columns

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SUMMARY:
This paper presents an analytical approach for estimating the ultimate deformation capacity of flexurally critical RC columns. This analytical approach was used to evaluate the load-displacement diagrams for 14 flexurally-dominated RC column specimens. The accuracy of analytical results proved to be satisfactory as compared with the experiments, and was used to study the cause of strength loss at the ultimate state in details. It is observed that at the final stages of the column lateral capacity, the flexural strength provided by the core concrete is the main affective portion of the total moment strength. The analyses indicate that as the crushing spreads through the core, the neutral axis goes back to the original tensile region. The flexural strength deceases more severely once the neutral axis passes the theoretical plastic neutral axis. Beyond this point, the core becomes counterproductive, and produces negative resistance.

Keywords: Reinforced concrete , Column , Displacement capacity

1. INTRODUCTION:

True estimation of the lateral ductility of RC frames requires reasonable prediction of the ultimate displacement capacity of its columns. The ultimate lateral displacement capacity of a RC column is usually assumed as the displacement level at which a specific percent of the column maximum lateral strength is lost, Park et al. (1982), Sezen (2002), ASCE41(2007). However, an apparent point is observed in studying the experimental results of the lateral behavior of RC columns, after which severe decay of the lateral strength begins. This point was taken as the ultimate displacement capacity of studied columns and attempt was made to achieve a theory that can describe the reason of the loss of the lateral strength at the ultimate point.

Several empirical formulas have been presented for prediction of the ultimate deformation capacity based on an assumed form for the equation whose coefficients are so obtained by regression analysis to fit the experimental data. This approach has been applied in several studies such as those conducted by Park and Ang (1985), Panagiotakos and Fardis (2001), Rossetto (2002), Lam et al. (2003), Perus et al. (2006) , Haselton and Deierlein (2007) and Zhu et al. (2007). The conventional statistical approaches that are usually used for prediction of the ultimate deformation capacity of RC columns, use some parameters that are assumed to influence the displacement capacity. These parameters usually include axial stress ratio ($\frac{P}{f_c A_g}$), shear span to depth ratio, confinement index and slenderness ratio of rebars (s/d_b). Perus et al (2006) showed that there is a large scatter between the ultimate displacement of a large number of tested columns and each one of these significant parameters. In other words, none of these parameters can independently be used for prediction of the ultimate lateral displacement capacity of RC columns.

So as to find answers to the abovementioned questions, results of 16 experiments on RC columns with purely flexural failure modes were studied. An analytical procedure was developed and applied for

prediction of the experimentally obtained ultimate lateral displacement of these columns. After successful application of this analytical procedure, the mechanical events that happen during the lateral deformation of columns were comprehensively studied to obtain the event causing the start of severe decay of the lateral strength. Based on these studies, a theory is presented for analytical prediction of the ultimate lateral displacement capacity of flexurally dominated reinforced concrete columns. The theory was successfully applied in predicting the ultimate capacities of the studied column specimens.

2. DESCRIPTION OF THE PROPOSED THEORY

According to the rational classification made by ASCE41 (2007) there are four major failure modes for RC columns including flexural, shear, flexure-shear and lap-splice failures. General form of the lateral force-displacement curves of flexure-dominated RC columns is shown in figure 1, in which the first yielding occurs at point 1 and a sudden drop in the lateral strength is observed at point 2. This point was taken as the ultimate displacement capacity of columns in this study, instead of the conventional assumptions in the literature that take the point at which a specific fraction of the peak strength is lost as the ultimate point.

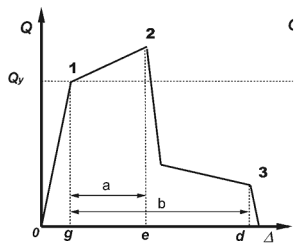

Figure 1. Idealized force-displacement curve for displacement-control columns (ASCE41)

The main objective of this study was finding the answers to the following questions:

1- What is the mechanical phenomena resulting in the start of severe decay of the lateral strength?
2- How can we predict the experimentally observed value of the displacement corresponding to this point analytically?

Attempts were made to find the answers to the above questions by studying the results of several tests on flexurally-dominated RC columns and comprehensive analyses on their models (which were validated by comparing with the experimental results). These attempts resulted in a theory that describes the reason of the loss of the lateral strength of RC columns. This theory can be independently used in an analysis to predict the displacement capacity of flexural columns, reasonably. This theory is described hereafter.

The mandatory equilibrium of the applied axial forces to the column section requires that the sum of the internal forces at the column section be equal with the externally applied axial load, as described by the following equation, in which C_c is the axial resisting force of concrete and Cs and Ts are the axial resisting force of compressive and tensile rebars, respectively and P is the applied axial load.

$$C_c + \sum C_{s,i} - \sum T_{s,i} = P \tag{2.1}$$

On the other hand the moment strength of column section can be calculated from the following equation.

$$M_{resistant} = C_c x_c + \sum C_s x_s + \sum T_s x'_s \tag{2.2}$$

In this equation, x_c, x_s and x'_s are the distances between the plastic center of the section (that coincides

with the centroid due to the symmetry of studied sections) to the points of resultant resisting forces of concrete, compressive and tensile rebars, respectively, as shown in the following figure.

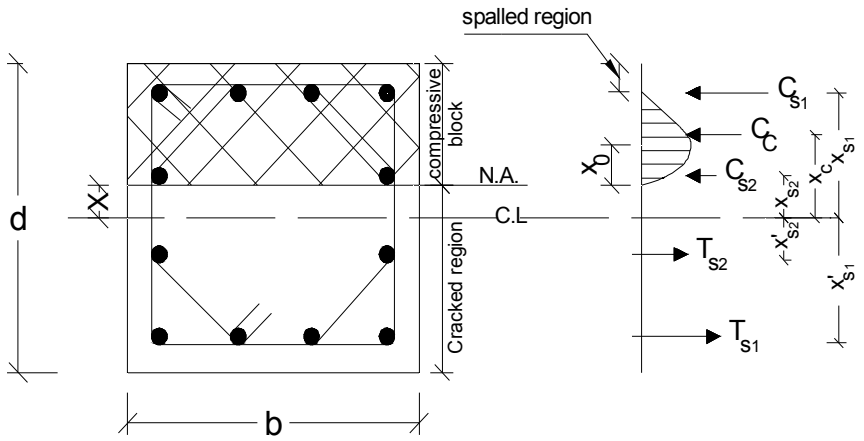

Figure 2. The schematic definition of the parameters used in the theory

A qualitative description of the stages of the lateral behavior of RC columns is expressed here.

1- At first, only the axial load is applied to the column. In this stage all parts of section are compressive and each one carries a specific fraction of the whole load. This axial load must be supported in all stages of the analysis, therefore the algebraic sum of the tensile and compressive forces of the section elements is constant and equals with the axial load.

2- By applying and increasing the lateral deformation, a crack is developed in the section. In this stage the slip of tensile longitudinal rebars produces additional lateral displacement in column. The moment strength in this stage is provided by tensile/compressive force of tensile/compressive rebars and compressive force of the compressive block of section (that consists of some parts of cover and core concrete).

3- By increasing the lateral deformation, the neutral axis moves forward to the compressive regions and since the stress of tensile rebars is increased, the compressive stress of the compressive block similarly increases to maintain the vertical load equilibrium. When the cover concrete stress is on the descending branch of its stress-strain curve, the core must compensate its contribution to the compressive resisting force.

4- After spalling of the cover (and parts of the core), the remained compressive block is not sufficient to provide the compressive force required to maintain the vertical load equilibrium. It should be noted that at this stage usually the contribution of the compressive rebars is decreased due to their buckling and on the other hand, the tensile rebars provide (almost) constant tensile force because they had been yielded formerly. The mandatory requirement of "maintaining the equilibrium with conservative (constant) applied axial load and constant yielding force of tensile rebars" enforces the neutral axis to set back to close some parts of the cracked region in order to utilize them in compression. This can even make some middle rebars of the section (that have been in tension before) compressive.

5- As the crushing spreads through the core, the neutral axis sets back to utilize the compressive capacity of previously tensile regions of concrete core. The set back of the compressive block of core concrete means that the point of the resultant compressive force of concrete sets back toward the section center. Until the neutral axis has not reached the center point of section (plastic center) during its withdrawal, the entire compressive concrete block produces resistant moment in the section. After the passage of the neutral axis over the section centroid, the passed parts of this block creates overturning moment (negative moment strength). By passing further parts of the compressive block of core concrete from centroid, this negative moment strength increases, which results in severe loss of the lateral strength. Therefore, the following theory can be explained as the reason of the strength loss of RC columns:

The ultimate lateral displacement capacity of flexurally dominated RC columns, after which the lateral strength starts its severe decay, is the displacement level at which the neutral axis reaches the section centroid during its withdrawal to the cracked regions.

3. VERIFICATION OF THE PROPOSED THEORY

3.1. Description of the Applied Analytical Procedure

In order to examine the abovementioned theory, a database of the results of 16 RC columns with flexural mode of failure was collected as listed in table 1. To obtain the lateral behaviour of these columns, a uniform analytical procedure that was based on the sectional analysis was applied. Vast efforts were done to consider all the effective parameters to achieve a reasonable agreement between the analytical capacity curves with those obtained experimentally. For this aim, some modifications were made to the conventional models that will be expressed hereafter.

The lateral displacement of a RC column is assumed to consist of three components including flexural deformation, shear deformation and the displacement due to the slip of longitudinal bars at the column base. The analytical models used to obtain various components of the lateral displacements of studied columns are shown in figure 2.

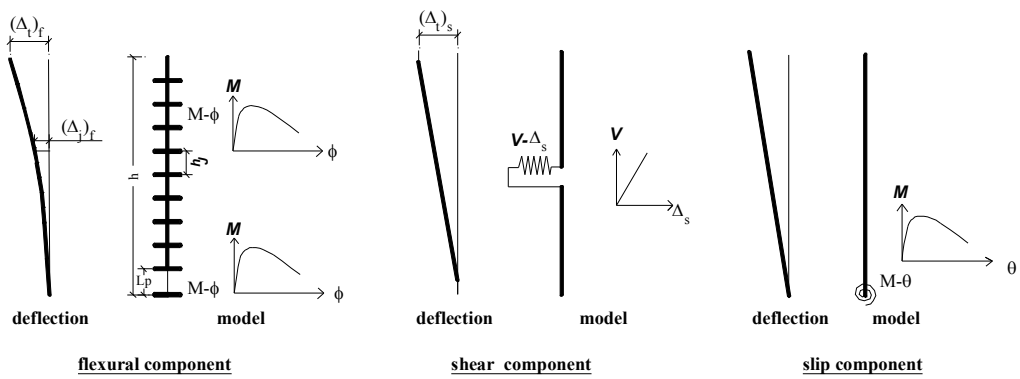

Figure 3. The applied models for determination of the three components of lateral displacement of RC columns

The methods of analysis, details of models as well as assumptions and modifications applied to obtain each one of the three components of lateral displacement of studied columns are described hereafter.

3.1.2. Flexural Displacement

The conventional method of sectional analysis was modified and applied. The following operations were conducted.

1- The column section was modelled by using the fiber element method and its moment-curvature relation was obtained. The stress-strain behaviours used for the three components of the section including cover, core and longitudinal rebars.

- Core concrete: The confinement model proposed by Mander et al (1988) was modified and applied to obtain the stress-strain curve of core concrete of the section. In this model, the lateral pressure provided by the confining steel is calculated based on its yield stress.

  In order to consider the non-uniform lateral pressure applied by the transverse reinforcements, the geometrical effectiveness coefficient proposed by EC8(2005) was applied, in which two distinctive effectiveness coefficients due to the arching actions between the rebars in section and along height of a RC column are considered. These dual inefficiency coefficients are multiplied and give K_e as:

$$K_e = \left(1 - \frac{s}{2b_o}\right)^2 \times \left(1 - \frac{\Sigma b_i^2}{6A_o}\right) \tag{3.1}$$

In which s is the clear spacing between transverse reinforcements, bi is the distance between consecutively restrained longitudinal reinforcements and b0 and A0 are the width and cross-sectional area of confined concrete, respectively.

So as to account for removal of the crushed concrete elements, the post-peak branch of this model was replaced by the proposed model of CEB model code 90 (1993). In this model, the post-peak branch is a line that interconnects the peak point to a point with stress and strain values of $0.85f_{co}$ and $\varepsilon_{co85}+0.2\frac{f_{ley}}{f_c}$, respectively. Such a post-peak branch was realized to be too conservative; therefore the stress value of $0.85f_{co}$ was replaced with $0.85f_{cc}$ and the post-peak line was extended to zero stress. The stress-strain curve of the modified model is compared with those of the original Mander et al.(1988) model and CEB model code 90(1993) in figure 3.

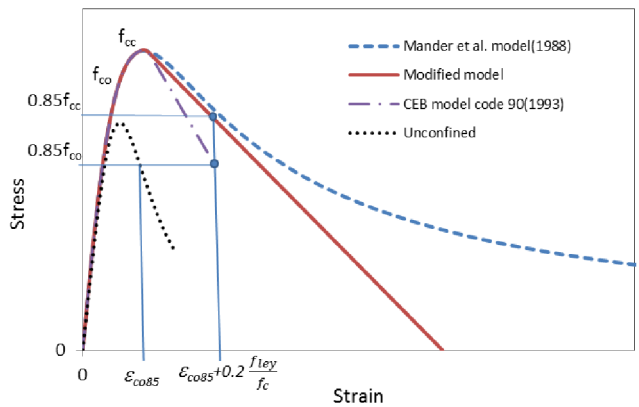

Figure 4. The proposed modification to the Mander et al. (1988) and CEB (1993) models for stress-strain curve of core concrete

- Cover concrete: The abovementioned modified Mander et al. (1988) model is used for the elements located at the cover concrete of section by considering zero lateral pressure. The post-peak branch was determined by using the original stress-strain formula of the Mander et al (1988) model up to a stress point of $0.85f_{co}$ but after this point it was linearly extended to zero stress, as shown in figure 3.

- Longitudinal reinforcements: The stress-strain behavior used for reinforcing rebars includes the strain hardening in tension and buckling in compression. The tensile stress-strain behavior of the applied longitudinal bars of each specimen was extracted from the results of coupon tests reported in the original documents. So as to predict the compressive stress-strain behavior of rebars, the model proposed by Sunjin et al. (1993) was found to perform well. In this model, the compressive stress-strain behavior of steel rebar is predicted based on its tensile behavior, the ratio of hoops spacing to the diameter of longitudinal bars and the diameter of bars.

2- To obtain the moment-displacement relationship of the column, the length of the plastic hinge at the column base is required. The formula proposed by Sunjin (2005) was applied to calculate this parameter, in which l_p is the length of plastic hinge, L/d is the shear span to depth ratio of column, A_s and A_g are steel and gross area of column section and P and P_0 are the applied axial load and nominal axial strength.

$$\frac{l_p}{h} = \left[0.3\left(\frac{p}{p_o}\right) + 3\left(\frac{A_s}{A_g}\right) - 0.1\right]\left(\frac{L}{d}\right) + 0.25 \geq 0.25 \quad (3.2)$$

3- So as to account for the nonlinear flexural deformation of the column height outside the plastic hinge, the remained height of the column outside the plastic hinge length was divided into 9 segments. This is in contrast to the conventional techniques that consider the concentrated plasticity only at the plastic hinge and could improve the accuracy of the results in comparison with the experimental results.

4- The effect of the secondary moment in each loading step was calculated based on the value of displacement obtained in the previous step.

3.1.2. Shear Displacement

The shear component of lateral displacement was calculated assuming a linear elastic shear behavior by applying the elastic shear modulus of concrete. The obtained values of the shear displacement were really small that is due to the flexural mode of the lateral behavior of selected columns used in this study.

3.1.3. Slip-Induced Displacement

One of the key sources of the lateral displacement of columns, especially flexural ones, is the lateral displacement created by the opening of the crack at the column-foundation interface and slip of the longitudinal reinforcement from foundation. This component of the lateral displacement was calculated based on the procedure applied by Alsiwat and Saatcioglu (1992). According to their procedure, the slip at each loading step is calculated based on the computed strain and stress of the longitudinal rebars.

3.1.4. Steps Of The Applied Analytical Procedure

Based on the above assumptions, the applied analytical procedure can be summarized as follows:

1- Assume a value for curvature at the column base $(\phi_b)_i$
2- Determine the corresponding moment from moment-curvature curve $(M_b)_i$
3- Determine the secondary moment by using the calculated top displacement of the previous step

$$(M_b)_s = P.(\Delta_t)_{i-1} \quad (3.3)$$

5- Calculate the corresponding net lateral force at the column top

$$V_i = ((M_b)_i - (M_b)_s)/h \quad (3.4)$$

6- Calculate the shear component of lateral displacement

$$(\Delta_t)_{s,i} = \frac{V_i . h}{G.A} \quad (3.5)$$

7- Divide the column height into 10 segments Determine the moment at each segment

$$(M_j)_i = V_i . h_j + P.(\Delta_j)_{i-1} \quad (3.6)$$

8- Determine the corresponding curvature of each segment $(\phi_j)_i$

9- Determine the moment-induced lateral relative displacement of each segment

$$(\Delta_j)_{j,i} = (\varphi_j)_i . h_j^2 \quad (3.7)$$

10- Calculate the flexural component of lateral displacement at the top

$$(\Delta_t)_{f,i} - (\varphi_b)_i . L_p^2 + \sum_1^n ((\varphi_j)_i . h_j^2) \tag{3.8}$$

11- Calculate the slip-induced lateral displacement $(\Delta_t)_{sl,i}$
12- Calculate the top displacement

$$(\Delta_t)_i = (\Delta_t)_{f,i} + (\Delta_t)_{s,i} + (\Delta_t)_{sl,i} \tag{3.9}$$

13- Increase the value of the curvature at the column base and repeat the steps 1 to 12.

3.2. Comparison of the analytical and experimental results

The abovementioned analytical procedure was similarly applied to all of the 16 studied columns. Detailed information about the specimens was extracted from PEER structural performance database (2004) including geometrical and mechanical details of the columns. The moment-displacement curves obtained from the analytical procedure described above are compared with the experimental reported As can be seen in figure 5, the

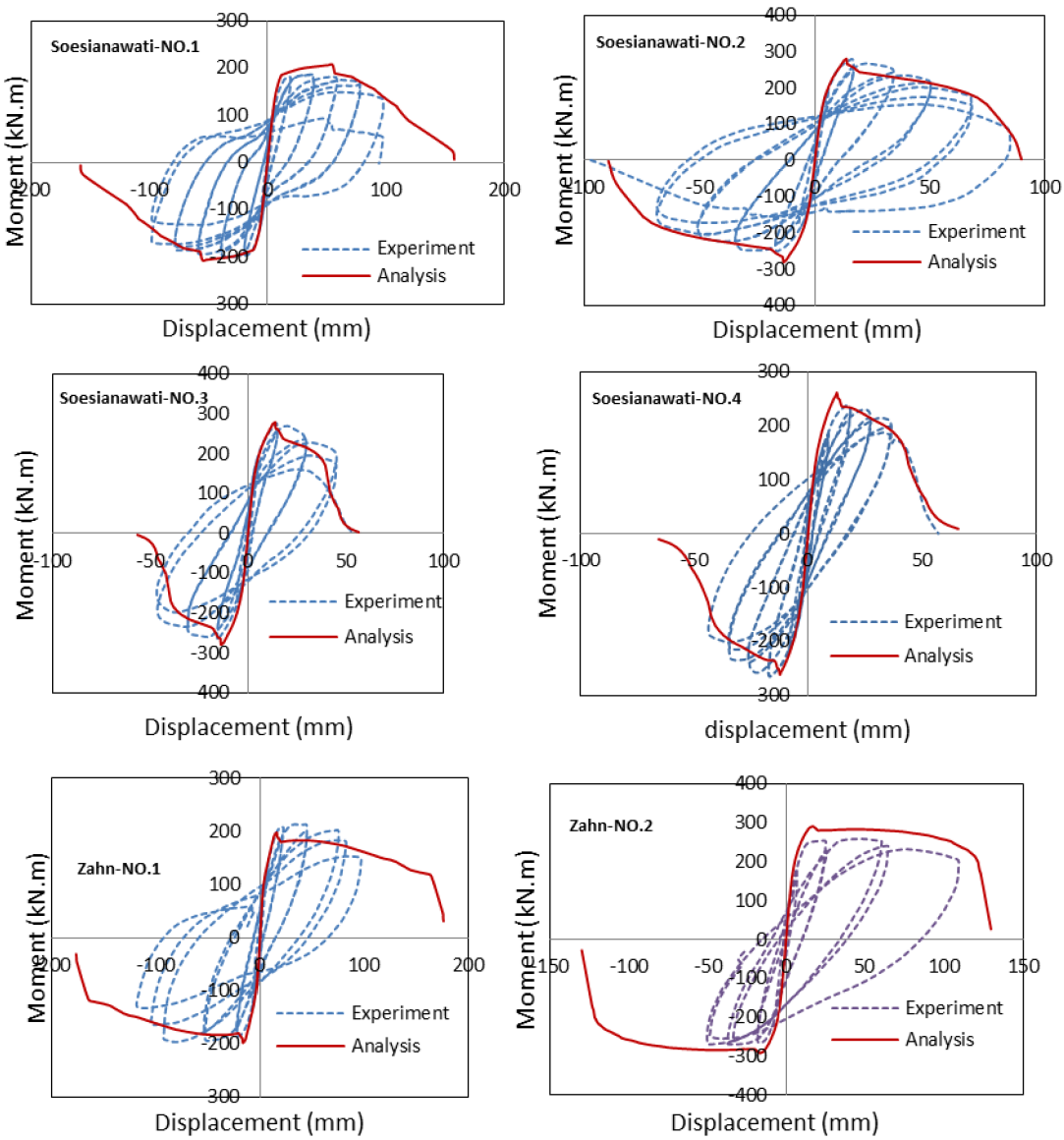

Figure 5. A comparison of the experimentally and analytically obtained moment-displacement behavior of the column specimens

3.3. Assessing the Mechanical Event Occurs at the Ultimate Displacement Level

Three major events have usually been assumed by various researchers as the significant event that occur concurrent with the loss of the lateral strength, including the first spalling of the cover concrete, the first spalling of core concrete and buckling of the longitudinal reinforcements. The analyses of the studied columns indicated that none of these events necessarily happens just at the point of ultimate lateral displacement. In the following figure, the ratios of the displacements at which either first spalling of the cover concrete or buckling of the longitudinal rebars occurs in a column to the column's ultimate displacement are drawn. It is obvious that none of these two significant events have a meaningful correlation with the ultimate displacement capacity of columns. In most cases both crushing and buckling occurs prior to reaching the ultimate deformation capacity. Therefore they cannot be considered as the reason of the severe loss of the lateral strength. On the other hand, the displacement at which the neutral axis reaches the section centroid during its set back to the cracked parts of the section has been really very close to the values of the ultimate displacement of all of the studied columns.

Table 1. List of the column specimens applied for examination of the proposed theory

Column specimen	$\frac{\Delta_{spalling}}{\Delta_{ultimate}}$	$\frac{\Delta_{buckling}}{\Delta_{ultimate}}$	$\frac{\Delta_{x=0}}{\Delta_{ultimate}}$
Soesianawati et al. 1986, No.1	0.37	0.74	0.98
Soesianawati et al. 1986, No.2	0.49	0.98	1.03
Soesianawati et al. 1986, No.3	0.78	1.15	0.83
Soesianawati et al. 1986, No.4	0.40	1.07	0.97
Zahn et al. 1986, No. 7	0.14	0.47	1.06
Zahn et al. 1986, No. 8	0.16	0.60	1.03
Tanaka and Park 1990, No. 5	0.14	0.46	0.91
Tanaka and Park 1990, No. 6	0.12	0.42	0.97
Tanaka and Park 1990, No. 7	0.20	0.88	1.15
Tanaka and Park 1990, No. 8	0.14	0.84	0.96
Xiao and Martirossyan 1998, 8L19-T10-0.1P	0.25	0.69	0.99
Xiao and Martirossyan 1998, 8L19-T10-0.2P	0.28	0.74	0.87
Xiao and Martirossyan 1998, 8L16-T10-0.1P	0.56	1.16	0.93
Xiao and Martirossyan 1998, 8L16-T10-0.2P	0.19	0.56	1.11

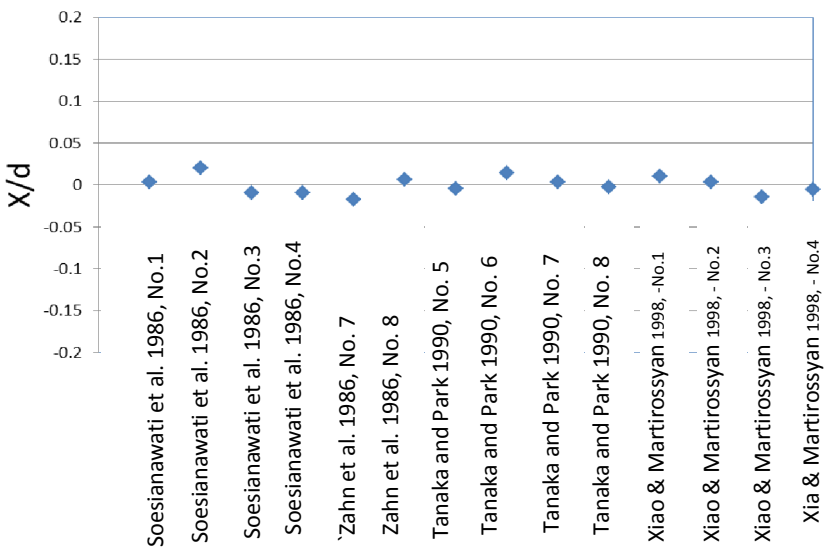

Figure 6. The ratio of the distance between neutral axis and centroid to depth of section for studied column specimens

So as to examine the proposed theory, the values of the distance between the neutral axis and centroid of the section (the critical distance noted as X in figure 2) at the ultimate lateral displacement point (start of severe decay of the lateral strength) were extracted from the analytical models of all of the studied specimens. Figure 6 shows the ratio of this critical distance to the section depth at the load step corresponding to the ultimate displacement capacities of these columns. As can be seen, for all of these specimens, the neutral axis has really reached the section centroid at the ultimate lateral displacement point.

3.4. Detailed Tracing of the Mechanical Behaviour of One of the Studied Specimens

In this section, the analytically obtained mechanical behaviour of the column specimen tested by Soesianawati et al. (1986) (i.e. Soesianawati et al. 1986, No.4) is discussed. Figure 7 shows the moment-displacement capacity curve of this column. As can be seen in this figure, there is a good agreement between the analytical and experimental results, confirming the predictions of the model.

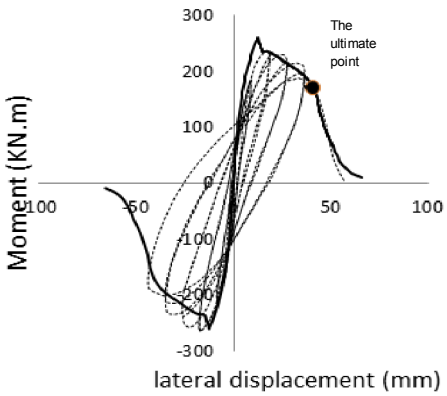

Figure 7. A comparison between the analytical and experimental capacity curves of the column specimen tested by Soesianawati et al. (1986)

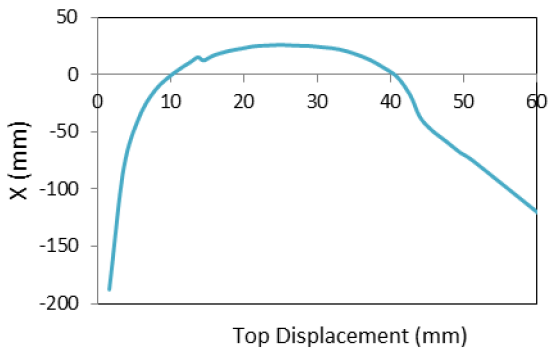

Figure 8. The variations of the neutral axis location by the lateral displacement from analytical model of the column specimen tested by Soesianawati et al. (1986)

In order to inspect the proposed theory for this column specimen, the location of the neutral axis (i.e. very close to the crack tip) was traced during the lateral deformation of this column. As shown in figure 8, the neutral axis moves significantly during the lateral deformation. In this figure the variable "X" is as shown and defined in figure 2 is the distance between the neutral axis to the centroid of the section. This distance is the key parameter defining the proposed theory. At the first step of loading, that the curvature is zero, obviously this distance tends to infinity.

By increasing the curvature, the neutral axis passes the section centroid and moves forward to the compressive half of the section. After spreading the crushing over the cover and core concrete, the mandatory equilibrium with axial compressive force requires that the neutral axis set back to utilize more area of concrete. This is obvious in figure 8, in which at a displacement level of 40.8 mm, the value of x equals with zero, that means that neutral axis has passed over the section centroid. This displacement level is exactly the point of start of severe decay of the lateral strength of the column from analytical model as marked in figure 7 and very close to the experimentally observed ultimate point.

4. CONCLUSION

In this paper a theory was proposed that can explain the reason of the strength loss of the flexural RC columns and be used to predict the ultimate displacement of these columns. According to this theory, the loss of the lateral strength of these columns happens when the neutral axis passes the section centroid during its set back toward the tensile parts of the section. This is mainly because some

fraction of the concrete compressive force becomes counterproductive and producing negative moment strength. Results of 16 flexure-dominated RC column specimens reported in the literature were applied to examine the proposed theory. A unified analytical procedure was applied by modifying the conventional sectional analysis that could predict the experimentally reported force-displacement diagrams of studied columns. To examine the proposed theory, the verified analytical models were studied. It was observed that at the low levels of section curvature, the neutral axis moves toward compressive parts of the section and by increasing the curvature and consequent crushing of parts of cover and core concrete, the remained compressive concrete area of the section is not enough to satisfy the equilibrium and therefore the neutral axis sets back toward the tensile part of the section. Whenever it passes the centroid, some fraction of the moment strength provided by concrete becomes counterproductive and therefore the lateral strength significantly decays. The distance between the neutral axis and the centroid was less than 0.3 times the section depth for all of the studied specimsns that proves the theory. The proposed theory could predict the ultimate displacement of the studied columns with less than 10 percent for most of the studied columns.

REFERENCES

Alsiwat J. M and Saatcioglu M. (1992). "Reinforcement anchorage slip under monotonic loading." *ASCE J. Struct. Eng.,* 118(9), 2421-2438.

ASCE (2007). "ASCE/SEI 41: Seismic Rehabilitation Of Existing Buildings." ASCE.

Berry M. , Parrish M. and Eberhard M. (2004). "PEER structural performance database: user's manual (version 1.0)." *Pacific Earthquake Engineering Research Center, University of California, Berkeley.*

CEB-FIP (1993). "Model Code 90." *Thomas Telford.*

CEN. (2005). "Eurocode 8: design of structures for earthquake resistance. Part 3: Assessment and retrofitting of buildings."

FEMA 273 (1997). "NEHRP guidelines for the Seismic rehabilitation of buildings." FEMA.

FEMA 356 (2000). "prestandard and commentary for the Seismic rehabilitation of buildings." FEMA.

Haselton C. B. and Deierlein G. G. (2007). "Assessing seismic collapse safety of modern reinforced concrete moment frame buildings." *PEER ReportNo. 2007/08. Pacific Earthquake Engineering Research Center,University of California, Berkeley.*

Lam S. S. E. , Wu B. , Wong Y. L. , Wang Z. Y. , Liu Z..Q. and Li C. S. (2003). "Drift capacity of rectangular reinforced concrete columns with low lateral confinement and high-axial load." *ASCE J. Struct. Eng.*, 129(6), 733–742.

Mander, B. J., Priestley J. N. M. And Park R. (1988). "Theoretical Stress-Strain Model For Confined Concrete.", *ASCE J. Struct. Eng.*, 144(8), 1804-1826.

Moghaddam H. , Samadi M. and Pilakoutas, K. (2010). "Compressive behavior of concrete actively confined by metal strips, part B: analysis." *J. Mat. Struct. (Springer)*, 43(10),1383–1396. DOI 10.1617/s11527-010-9589-5.

Panagiotakos T. B. and Fardis M. N. (2001). "Deformations of reinforced concrete members at yielding and ultimate." *ACI Struct. J.,* 98(2):135–148.

Park R., Priestley M. J. N. and Gill W. D. (1982). "Ductility Of Square Confined Concrete columns." *J. Struct. Div., ASCE.,* 108(4), 929–951.

Park Y. J. and Ang A. H. S. (1985). "Mechanistic seismic damage model for reinforced concrete." *ASCE J. of Struct. Eng.,* 111(4), 722–739.

Perus I., Poljansek K. and Fajfar P. (2006). "Flexural deformation capacity of rectangular RC columns determined by the CAE method." *Earthq. Eng. Struct. Dyn.,* 35:1453–1470.

Rossetto T. (2002). "Prediction of deformation capacity of non-seismically designed reinforced concrete members." *Proc. 7th U.S. nat. conf. on earthq. Eng., Boston.*

Soesianawati M. T., Park R. and Priestley M. J. N. (1986). "Limited Ductility Design of Reinforced Concrete Columns." *Report 86-10, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand*, 208 pages.

Sungjin B. (2005). "Seismic performance of full-scale reinforced concrete columns." *Ph.D. thesis, University of Texas at Austin.*

Tang C. , Chen H. and Yen T. (2003). "Modeling confinement efficiency of reinforced concrete columns with rectilinear transverse steel using artificial neural networks." *J. Struct Eng.,* 129(6),775–783.

Zhu L., Elwood K. J., Haukaas T. (2007). "Classification and seismic safety evaluation of existing reinforced concrete columns." *ASCE J. Struct. Eng.*; 133(9), 1316–1330.