Study on Vibration Damping Characteristics of a Spherical Shell and Damping Tendency of Shell and Spatial Structures

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SUMMARY:
Damping is one of the important parameter about vibration phenomena of structures. However it is impossible to evaluate damping ratios by theory or to know ones in apriority from structural systems because there are many environmental factors around and in the structures, for example, those are wind forces, seismic forces, and inner frictions in the structures and so on. Recently, accurate damping evaluation has been important role in design stage of structures. Damping ratios are tried to make clear through impact vibration experiments of a concrete spherical shell structure which is used for the planetarium of Funabashi city, Japan. In the experiments, velocity detectors are used and velocities in the vertical direction and two horizontal directions are measured at a measurement point. Further, damping characteristics of shell and spatial structures have been analyzed using about 50 data of damping of shell and spatial structures.

Keywords: shell and spatial structures, spherical concrete shell, damping ratio, impact vibration, velocity

1. INTRODUCTION

Damping is of great importance in dealing with vibration problems in buildings. However, due to the large number of factors involved, it is currently not possible to calculate a structure's damping ratio theoretically. Accurate estimates of the damping ratio at the design stage make understanding the dynamic behaviour of the structure possible, and with effective use of the building's inherent vibration properties, it is possible to reduce stress on structural members, reduce construction costs and make related energy savings. Also, with the greater emphasis on earthquake countermeasures since the Hanshin-Awaji earthquake, building response prediction has taken on great importance, and there is an urgent need to improve the accuracy of damping prediction (Tamura,1995).

In recent years, the expansion of databases of multi-storey buildings has, while limited to small-amplitude vibrations, begun to make rough estimates of damping ratio based on size, construction and materials possible (AIJ,2000). However, for shell and spatial structures the relative lack of absolute quantifiers, coupled with the difficulty of taking real measurements, mean there is very little real data available.

Shell and spatial structures hold large numbers of people and are widely used as emergency refuges in the event of disasters. Therefore there is a pressing social need for more research to improve accuracy of damping ratio and dynamic behaviour estimation.

With this in mind, experimental measurement and analysis of a number of shell and spatial structures has been carried out by Shingu Laboratory and others, and a database has been constructed with the goal of improving our understanding of their damping characteristics. The results of this research have been presented in conference papers (Shingu,2003-Yukawa,2009) and a book (AIJ,2008). However, the data accumulated and analysed to date is still not sufficient for a full picture of the damping characteristics of shell and spatial structures.
2. OBJECTIVES

Our research group has made experimental measurements and damping evaluations of a conical shell, two hyperbolic paraboloidal (HP) shells, a cylindrical shell, and a tensegric truss arch. However, this kind of research has yet to be conducted for a reinforced concrete spherical shell. This type of shell has also not so far been included in the database (Tatemichi, 2004), (Yukawa, 2009).

In this study we aim to clarify the damping characteristics of the prestressed concrete (PC) spherical shell of the planetarium at the Funabashi City General Education Centre (Enomoto Architectural Design Office, 1985), and to use this information to improve the damping database used for response analysis of shell and spatial structures. Additionally, this data is combined with the results of earlier work to conduct trend analysis of damping in shell and spatial structures.

3. OUTLINE OF SUBJECT STRUCTURE

The subject of measurement was the planetarium mentioned above (Photo 1), specifically its spherical shell. Vibration measurements were carried out, and the damping ratio estimated. The shell has a radius of 11.60m, a rise of 8.11m and a 12.60m radius of curvature. The shell thickness is 8cm, and the PC surfaces which fan out from the centre are held together with steel pretensioning strands. A PC cap forms the peak of the shell, and the gaps are filled with mortar. The shell is secured with bolts and concrete to the supporting lower structure. Figure 1 shows a cross-section of the structure.

4. EXPERIMENTAL METHOD

Vibration measurements were obtained using the impact from dropped sandbags. The purpose of impact vibration tests with sandbags is to ascertain damping characteristics at vibration amplitudes larger than normal microtremors.

4.1. Measurement system
In this study, servo velocity detectors (Photo 2) were utilized for their high accuracy and stability, and the data fed to a computer via portable vibration meters, where it was converted into velocity graphs. Velocity detector sensitivity specifications taken from the website of Tokyo Sokushin Co. Ltd. are given in Table 1 (Tokyo Sokusin URL).

![Photo 2](Servo velocity detectors)

### Table 1  Velocity detector sensitivity

<table>
<thead>
<tr>
<th>Component</th>
<th>1 component (horiz./vert. switchable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring frequency</td>
<td>0.2~100Hz</td>
</tr>
<tr>
<td>Range</td>
<td>Velocity: ±0.1m/s (±10cm/s)</td>
</tr>
<tr>
<td></td>
<td>Acceleration: ±20m/s²</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>Velocity(H): 0.1V/m/s</td>
</tr>
<tr>
<td></td>
<td>Velocity(L): 100V/m/s</td>
</tr>
<tr>
<td></td>
<td>Acceleration: 500mV/m/s²</td>
</tr>
</tbody>
</table>

#### 4.2. Measuring method (sandbag drop test)

A 15kg sandbag was dropped from head height at point "B" (Figure 2) and allowed to fall freely onto the surface of the shell to create a damping vibration waveform. Measurements were taken using servo velocity detectors at a sampling frequency of 200Hz over a period of 30 seconds, for a total of 6,000 data points. This was repeated 40 times.

#### 4.3. Measurement points

Velocity detectors were placed at the top of the shell (point "A"), a point 2m horizontally removed from the centre (point "B") and a point on the shell’s circumference (point "C"), as shown in Figure 2. At point "B", the 2m-radius "cap" portion of the shell connects to the PC panels of the roof, which are held together by the prestressed concrete steel, and they are connected with rubber and mortar. Measurements at point “B”, then, are of the shell sides only, and not the cap.

Velocity detectors were placed at the top of the shell using a 40m, 1990kg basket lift (Photo 3).

![Figure 2](Measuring positions)
5. WAVEFORM ANALYSIS METHOD

The 40 damping free vibration waveforms measured in the sandbag drop test were used to calculate
damping ratios using the envelope curve, and averaged separately for each channel to evaluate
damping ratio. And the envelope curve parameters shown in Equation 1 are approximated manually.

\[ v_n = \alpha \exp\left(-h n \Delta t\right) \]  

where,  
- \( v_n \): velocity at sampling number n,  
- \( \alpha \): initial amplitude  
- \( h \): damping ratio,  
- \( n \): sampling number \((n=1,2,3\ldots)\),  
- \( \Delta t \): time interval (0.005s)

6. RESULTS

The results of experimental measurements and subsequent analysis are as follows.
An example of one of the velocity-time waveforms obtained from the sandbag drop test is shown in
Figure 3.

The damping free vibration waveforms obtained from the 40 sandbag drop tests were used to calculate
damping ratios using the envelope curve. The average figures obtained from the 40 repeats were used,
and these are shown in Table 2.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Damping ratio</th>
<th>Channel</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1</td>
<td>6</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>2.3</td>
<td>7</td>
<td>3.1</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>8</td>
<td>4.0</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>9</td>
<td>3.4</td>
</tr>
<tr>
<td>5</td>
<td>2.4</td>
<td>Ave.</td>
<td>2.8</td>
</tr>
</tbody>
</table>
Table 3 shows the damping ratios obtained from the time response curves from the sandbag drop test, averaged for each of the three measuring points.

<table>
<thead>
<tr>
<th>Measuring point</th>
<th>Sandbag drop test</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.5</td>
</tr>
<tr>
<td>B</td>
<td>2.9</td>
</tr>
<tr>
<td>C</td>
<td>2.1</td>
</tr>
<tr>
<td>Ave.</td>
<td>2.8</td>
</tr>
</tbody>
</table>

7. DAMPING TREND ANALYSIS

For the purposes of this trend analysis, the results from the PC spherical shell were added to a database prepared in an earlier study, along with results from two papers presented by other researchers (Nakazawa, 2009), (Miyashita, 2011) to make a total population of 51 shell structures.

7.1. Investigation of size dependence

The dimensions of the 51 shell structures in the database are shown as a scatter plot in Figure 4.

It has been suggested that height be used as a parameter to estimate damping in multi-story buildings. So, for shell and spatial structures, we will examine the size parameters of span and rise and look for a correlation.

7.2. Relation between Damping Ratio and Span

Figure 5 shows the correlation between damping ratio and span.
There is a significant spread of damping ratios, and the correlation is not strong, but we can see that damping ratios tend to increase with span. The least-squares approximation of this relation is shown as Equation 2.

\[ h = 0.02L + 1.32 \]

where, \( h \) : damping ratio (%) \( L \) : span (m)

### 7.3. Relation between damping ratio and rise

Figure 6 shows the relation between damping ratio and rise. Damping ratios are mostly concentrated in the 0.5%-2.5% range, but a clear correlation cannot be seen.
8. NATURAL VIBRATION ANALYSIS

To understand the vibration characteristics of a spherical PC shell, a finite element model was constructed, and natural vibration analysis carried out to find natural frequencies and natural vibration modes. This analysis was carried out after the experimental evaluation of the spherical shell’s vibration properties, so the results of the former were not considered when conducting the latter. Model construction and analysis was carried out using the common ANSYS finite element analysis software. The lower part of the building to which the dome is attached was assumed to be rigid enough to be disregarded, and only the shell itself was modelled as a single, continuous body. The details of the finite element model of the subject structure are as follows.

8.1. Finite element model

A “SOLID45” finite element model was constructed to model the spherical shell roof of the Funabashi General Education Centre’s planetarium. The beams that for the joint area of the shell are modelled using the “BEAM4” element type, and analysis carried out with pin support boundary conditions at the edge of the model. The equivalent young modulus value of 3.3N/m² used in this analysis was derived as the average of 2.9N/m² (reinforced concrete) and 3.7N/m² (prestressed concrete), values used in previous studies (Kato,1959), (Kato,1960). The model elements and parameters used are shown below.

Shell:
Element type: SOLID45
Material properties
- Equivalent Young’s modulus: $E = (2.9, 3.3, 3.7) \times 10^{10}$N/m²
- Mass density: $\rho = 3.8 \times 10^3$kg/m³, Poisson’s ratio: $\nu = 0.2$

Beams:
Element type: BEAM4
Material properties
- Equivalent Young’s modulus: $E = (2.9, 3.3, 3.7) \times 10^{10}$N/m²
- Mass density: $\rho = 3.8 \times 10^3$kg/m³, Poisson’s ratio: $\nu = 0.2$
- Geometrical moment of inertia: $I = 5.94 \times 10^{-3}$m⁴

8.2. Analytical results

The results of natural vibration analysis of the above model are shown below. Modal analysis was carried out using the block Lanczos method. An overview of natural frequencies is given in Table 4, and examples of natural vibration modes are shown in Figures 7-10.

<table>
<thead>
<tr>
<th>Order</th>
<th>Equivalent Young’s modulus $\times 10^{10}$N/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.9</td>
</tr>
<tr>
<td>1</td>
<td>21.94</td>
</tr>
<tr>
<td>2</td>
<td>27.00</td>
</tr>
<tr>
<td>3</td>
<td>27.01</td>
</tr>
<tr>
<td>4</td>
<td>36.50</td>
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<tr>
<td>5</td>
<td>36.55</td>
</tr>
<tr>
<td>6</td>
<td>42.45</td>
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<td>7</td>
<td>42.73</td>
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<td>8</td>
<td>48.93</td>
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<td>9</td>
<td>52.98</td>
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<td>10</td>
<td>54.05</td>
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<tr>
<td>11</td>
<td>56.98</td>
</tr>
<tr>
<td>12</td>
<td>57.42</td>
</tr>
<tr>
<td>13</td>
<td>58.37</td>
</tr>
</tbody>
</table>
8.3. Comparison of experimental data and analytical results

The following is a comparison of data obtained experimentally with the results of the natural vibration analysis. Waveform processing of the damping free vibration data from the sandbag drop test has not been carried out. However, for reference, an example Fourier spectrum from the edge beam part and main body of a PC spherical shell is shown in Figures 11 and 12.
A peak is visible at around 25Hz. This is a much larger response than in microtremor observation, so this is thought to indicate the effect of the shell itself at the edge beam.

In the Fourier spectrum from near the top of the dome, we can see that, as with microtremor observations, there are peaks between 25Hz and 40Hz. Therefore in future research it is necessary to carry out waveform processing for predominant frequencies over 25kHz.

9. CONCLUSIONS

1) The damping ratio of this structure is 2.8%.
2) The following formulas were obtained relating size and damping ratio, where $h$ (%) is the damping ratio, $L$ (m) is the span of the shell

$$ h = 0.02 L + 1.32 $$

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