Nonlinear Rooftop Tuned Mass Damper Frame

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SUMMARY

Analytical studies of existing buildings demonstrate the effectiveness of Nonlinear Rooftop Tuned Mass Damper Frames (NRTMDF) for reducing seismic response. The technique utilizes a rooftop penthouse as a tuned mass damper with mass incorporated as the roof deck of the penthouse while targeted nonlinearity and energy dissipation are introduced through buckling restrained braces linking the penthouse roof to the structure below. This paper summarizes analytical studies of existing buildings, each modified with an NRTMDF. The studies demonstrate the effectiveness of this approach which stems from elastic and transient inelastic period shifts in the damper coupled with targeted energy dissipation in the penthouse buckling restrained braces. Analytical simulations show for many structures and sites that the approach decreases peak transient response. The technique also reduces demand on nonstructural elements and components reflected in developed floor spectra. Energy methods show that the NRTMDF approach enables reductions in energy demand on the structure.

Keywords: buckling restrained brace, dynamic, energy, nonlinear, tuned mass damper.

ABSTRACT

Innovative strategies driven toward improving the structural and nonstructural performance of existing buildings during large earthquakes are of paramount interest for researchers and practitioners of earthquake engineering. One such strategy is the use of a Nonlinear Rooftop Tuned Mass Damper Frame (NRTMDF). This technique utilizes a rooftop penthouse structure as a tuned mass damper with mass incorporated as the roof deck of the penthouse while targeted nonlinearity and energy dissipation are introduced into the system by virtue of buckling restrained braces which link the penthouse mass to the structure below. An analytical study of existing buildings, each modified with an NRTMDF, demonstrates the effectiveness of this approach for reducing seismic response. The effectiveness stems from two key actions enabled by the NRTMDF. First, the NRTMDF generates elastic and transient inelastic fundamental period shifts in the structure. For most sites the result is a decrease in the correlating seismic spectral acceleration response. Second, the NRTMDF introduces designated yielding elements within the system (buckling restrained braces). These elements serve as a channel through which seismic energy is dissipated in a safe, controlled and targeted manner. Analytical studies show that for many structures and site conditions the NRTMDF approach effectively decreases peak transient response parameters (base shear, story drift, displacement). In addition, the technique reduces the demand on nonstructural elements and components observed as diminished floor spectra derived from the analytical models. Energy methods demonstrate that the NRTMDF approach enables reductions in total energy demand on the structure in consideration of the complete response history.

1. INTRODUCTION

The research presented herein demonstrates the potential for the creative adaptation of existing technologies to reduce earthquake demand for either new construction or as a seismic rehabilitation strategy. The technique utilizes a Nonlinear Rooftop Tuned Mass Damper Frame (NRTMDF) which also functions as the penthouse enclosure, a common feature utilized for many structures to house mechanical equipment. The research focuses on two aspects of the rooftop tuned mass damper frame which affect the performance of the global structure. First, the NRTMDF enables the lengthening of the building's fundamental period with an initial elastic period shift and then with a transient inelastic period shifts enabled by yielding of the rooftop frame. The reduction of earthquake response due to a lengthened period is a well established fact for most seismically active regions of the world and is found within the prescriptive requirements of contemporary codes for new construction. Figure 1 demonstrates this concept. As shown, a lengthened period results in a decrease of the acceleration response. The NRTMDF also enhances the second mode participation with base structure inertial forces being offset by the opposing motion of the rooftop damper. Second, the NRTMDF offers the opportunity for introducing a designated earthquake input energy dissipating mechanism within the structure (buckling restrained braces). The energy dissipated in the NRTMDF is considered as a net seismic energy reduction (reduced seismic demand) on the global system.

Lengthening of the fundamental period enabled by the NRTMDF can be understood through consideration of primary variables, mass (*m*) and stiffness (*k*). Alteration of these variables either directly or indirectly affects natural vibration properties of a structure. An increase of mass and/or decrease of stiffness serve to lengthen a building's fundamental period, the results of which are often a decrease in the seismic acceleration response as demonstrated in Figure 1. In this figure, three period shifts are illustrated. The first is due to the elastic behavior of the NRTMDF while the second and third are transient and are enabled by nonlinear inelasticity as the buckling restrained braces yield creating an effective stiffness in the rooftop frame markedly less than the initial elastic stiffness. The inelastic period shifts are directly correlated to the effective stiffness of the NRTMDF at its peak nonlinear displacements for the earthquake acceleration records under consideration.

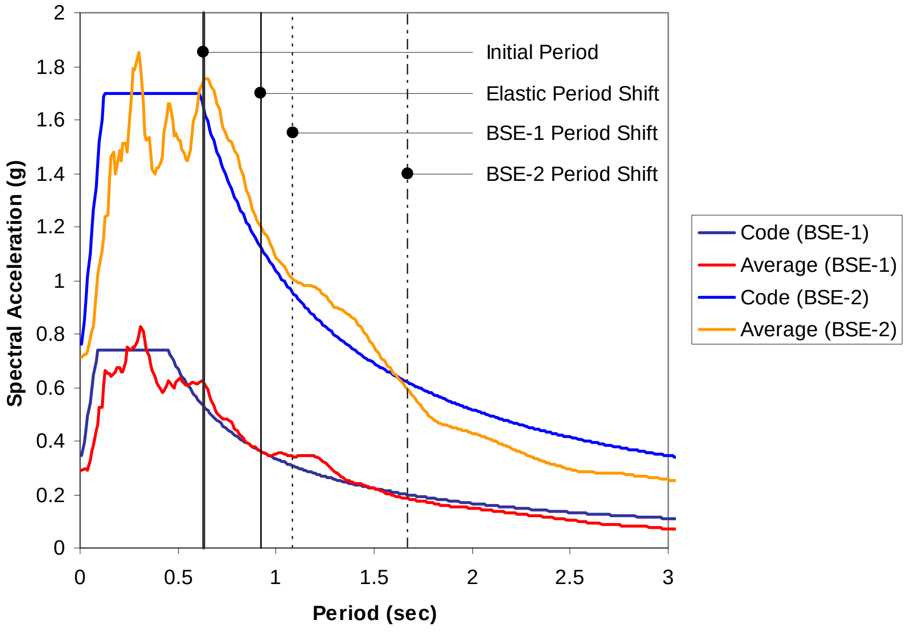

Figure 1. Code Design Spectra and Spectra from Acceleration Histories for Medium Sites with Building BF-4 Undamped and Damped Fundamental Periods and Period Shifts

The concept of requiring ductility in structural systems has been a feature of building codes for many years. When properly incorporated, ductility has the effect of creating increased hysteretic damping within the system. This can be qualitatively represented as a diminished response spectrum. When considered in tandem, the nonlinear period shifts and increased hysteretic damping provide an effective approach for seismic response reduction.

Code developers have realized that requiring ductile behavior is the most economically viable approach for building safe structures in regions of moderate to high seismicity. The concept of ductility in structural systems has long been understood but only recently has the behavior of ductile yielding mechanisms become the focal point behind high performance seismic systems, particularly within the context of performance based seismic design (PBSD). Whereas past prescriptive codes typically addressed ductility on a global scale for the structural system, contemporary codes are focused toward specific energy dissipating and yielding mechanisms. Current PBSD methods account for specific and deliberate elements to dissipate energy in a stable and controlled manner. Buckling restrained braces focus ductility into a yielding core and are deemed a reliable element for focused nonlinearity and energy dissipation. Upon being subjected to reversing cycles of strain, BRB’s dissipate seismic energy as the braces experience repeated cycles of elasto-plastic tension and compression. The effective energy dissipated within each cycle is quantified as the area within the hysteresis loop. The effectiveness of the BRB stems from axial loading action of the yielding core, enabling it to maintain the advantages of a stiff lateral system while still remaining capable of significant ductile behavior when subjected to high loads. As such, significant ductility is introduced into the frame while minimizing drifts and associated effects of geometric nonlinearity (P-Delta). Hence, this technology is deemed the most effective and stable approach for introducing designated yielding members in the NRTMDF frame.

In general, a benefit of ductile systems is found in their ability to lengthen the fundamental period of the structure. For relatively stiff soil sites located near rupturing faults, longer period structures generally respond with a reduced acceleration response when compared to short period structures. This is due to matching of frequency contents which yields a higher structural response when sites and structures are closer to matching and lower response when frequency contents between the two are far apart. Hence, a structure whose frame yields due to seismic inertial forces will undergo a period shift that will typically lessen the seismic acceleration response and reduce the event’s overall impact for stiff and moderately stiff sites. Engineers have long understood this to be true and have developed systems such as seismic base isolation to deliberately lengthen a building’s fundamental period. In the author’s past research the potential for lengthening the fundamental period and thereby lowering seismic response by utilizing a mass tuned damper in the form of a limber frame atop relatively stiff buildings was investigated (Johnson et al. 2003). The research indicated that a reduction in seismic response was possible for a set of well suited specific parameters including the input ground motion and the dynamic properties of the original structure.

The current research investigates the application of designated yielding frame members (buckling restrained braces) incorporated in rooftop tuned mass damper frames and assesses the feasibility and effectiveness of this approach for reducing seismic response. The methods for measuring performance demonstrated within this research are aimed toward the quantification of structural demand and its reduction utilizing the NRTMDF method. This technique introduces a new and inexpensive alternative for the mitigation of damage caused by earthquakes. This translates to safer structures with lower repair costs following a large event or less expensive but more resilient new structures. It also translates to reduced ancillary costs associated with bracing of nonstructural elements and components since the approach not only reduces demands on the structure itself but on all of the nonstructural systems within the building. This method represents a new tool and a new avenue of study for researchers and engineers to consider in an attempt to mitigate the costly effects of significant earthquakes. Figure 2 depicts the geometry of the NRTMDF added to a conventional braced frame structure.

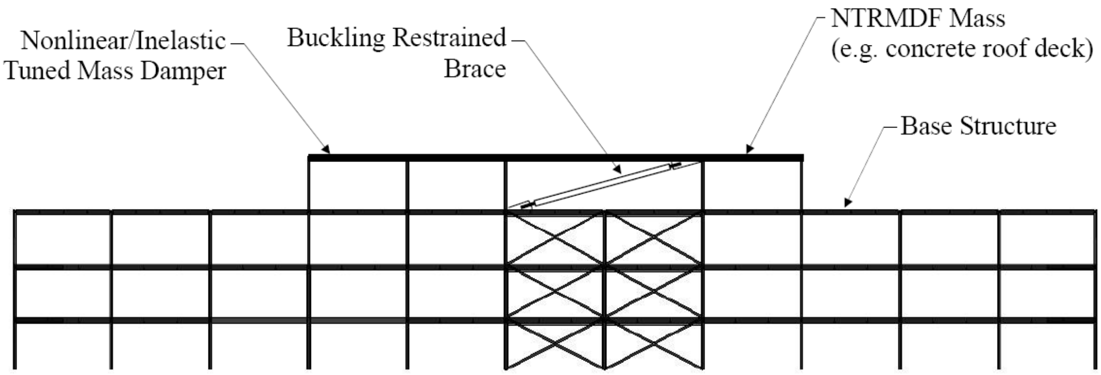

Figure 2. Structured Modified with Nonlinear/Inelastic Rooftop Tuned Mass Damper Frame

2. LITERATURE REVIEW

A challenge in the design of isolation systems is that, to withstand very severe or near-fault motions, bearings often become so large, stiff and strong that they provide little isolation during moderate seismic events (Morgan and Mahin 2010). An alternative to base isolation is the use of tuned mass dampers. While the concept of tuned mass dampers in structural applications is not new, its use has been confined primarily to high-rise structures for the purpose of suppressing wind-induced vibrations. The inventory of research dealing with dampers for suppression of seismic-induced vibration is growing with some adaptations reflecting the concepts addressed in this research.

Johnson et al. (2003) investigated the concept of a tuned mass damper comprised of a limber rooftop moment frame which formed a penthouse enclosure. For this, the analytical modeling indicated reductions in peak rooftop displacement as high as 46%. This research also demonstrated that significant nonlinear behavior of the rooftop frame would be required in order to develop advantageous dynamic behavior.

Wong and Johnson (2009) investigated the effects of multiple tuned mass dampers located at various stories of a midrise structure. Their study demonstrated reductions in plastic energy and hinging regions of moment frame beams by as much as 70%.

Nawrotski (2006) demonstrated reductions in peak rooftop displacements as high as 40%. For his research a rooftop masses in the form of solid slabs were analytically added to mid-rise shear wall and braced frame models. The masses connected to the roofs of the structures by virtue of helical spring devices with integrated dampers. In addition to the reduction in peak transient response parameters, Nawrotski reports an increase in effective damping from 5% to 15%.

Villaverde (1998) and Villaverde et al. (2005) investigated the concept of a base isolated penthouse structure. The research included the use of viscous dampers for enhanced energy dissipation between the penthouse and building interface. The 1998 study utilized elastomeric isolators atop a 5-story, one-bay stiff moment frame structure. Tuning a modified version of the Secretaria de Comunicaciones y Transportes (SCT) accelerogram (Mexico City, 1985 Earthquake) to resonate with the undamped structure and then running the same tuned record on the damped structure yielded reductions in peak response parameters as high as 84 percent. For the 2005 study, Villaverde et al. (2005) used steel ovaling isolators in lieu of elastomeric isolators, fundamentally repeating the earlier study but with the different bearing for the penthouse. This study yielded a reduction in peak response parameters of 55%.

Buckling restrained brace frames (BRBF) are currently being considered in the construction of tall buildings for performance-based seismic design (Moehle et al. 2011). Some of the criteria imposed for the design are as follows: (1) for frequent events, with a return period of 25 years, the building should remain at the service level at which building components remain elastic, with only minor yielding in BRBF components and a drift limit of 0.5%. In extreme events (at the MCE level) the building should withstand shaking without collapse, with a maximum drift ratio limit of 3%; (2) For the serviceability earthquake, an event with the 43-year return period, the requirement is that the demand-to-capacity ratio does not exceed 1.5.

Though tuned mass dampers have been used primarily for suppression of steady-state vibrations of mechanical equipment and wind-induced building vibrations, past research suggests the potential for significant reduction of seismic response. Further research is required to corroborate this approach as a valid technique for reducing structural response induced by seismic ground motion.

3. METHOD OF ANALYSIS

Analytical modeling served as the primary method for assessing the effectiveness of the NRTMDF approach. Ten existing structures with varying undamped dynamic properties served as a broad-based reflection of structures deemed sensitive to seismic motion. For these, comparison of peak response parameters between undamped and damped structures demonstrates the effectiveness of the NRTMDF approach. Likewise energy methods demonstrate the validity of the approach by considering the complete acceleration time-history. Beyond assessing specific performance parameters for each structure, development of floor spectra enables an assessment of potential seismic forces on nonstructural elements and components for the undamped and damped structures.

For the NRTMDF assessment, ten actual structures (Table 1) were analytically modeled using analysis applications developed by CSI Berkeley (ETABS, SAP, Perform 3D, see www.csiberkeley.com). The structures varied from 2 to 9 stories and were comprised of braced frames, eccentric braced frames, shear walls and moment frames. Undamped structural periods varied from 0.25 to 2.0 seconds. As such, the majority of structures fall at or near resonance with sites and ground motions when compared to spectra representing considered ground motions. In the interest of brevity, one test model, designated BF-4 (4 story braced frame structure) is deemed the most appropriate candidate to characterize the complete study. The results for this test model are presented in this paper. For the remaining buildings only summary results will be presented.

Table 1. Summary of Structures

Designation	No. Stories	Lateral System	Use	Period (sec)
BF-1	2	Braced Frame	Office Building	0.25
BF-2	2	Braced Frame	Educational/Research	0.3
BF-3	3	Braced Frame	Office Building	0.4
BF-4	4	Braced Frame	Office Building	0.63
BF-5	9	Braced Frame	Office Building	1.05
EBF-1	4	Ecc. Braced Frame	Computation Facility	0.54
SW-1	6	Shear Wall	Office Building	0.35
SW-2	6	Shear Wall	Research Facility	0.41
MF-1	5	Moment Frame	Office Building	1.4
MF-2	8	Moment Frame	Office Building	2

The ten test models reflect a broad-base of structures typically deemed sensitive to seismic ground motion. In the interest of assessing performance on the basis of soil types, suites of ground motions reflecting hard, medium and soft soils were selected (soil types C, D, E). For each of these, suites of 7 ground motions were selected reflecting the prescribed approach from *Seismic Rehabilitation of Existing Buildings* (ASCE 41-2006). The methods presented in ASCE 41 provide the basis for the selection, scaling and development of ground motions utilized for the study. As such, a suite of 7 records were selected reflecting BSE-1 ground motions (10% in 50 year probability of exceedence) along with 7 records for BSE-2 (2% in 50 year probability of exceedence). Inasmuch as medium (Type D) soils reflect the majority of conditions in a realistic sense, the motions reflecting this are presented in detail for Building BF-4 while the balance of structures with the medium site condition will only be presented in summary form.

4. ANALYSIS RESULTS

Test model BF-4 is a four-story concentric braced frame office building with an undamped fundamental period of 0.63 seconds. As such, this structure responds with relatively high acceleration response for medium soil conditions in its undamped condition. Figure 1 depicts the medium site average spectra from the ground motions for BSE-1 and BSE-2 superimposed with the targeted code spectra for each. On this, the ordinate for the undamped period of BF-4 demonstrates a relatively high expected acceleration response for each suite of ground motions. Noteworthy with respect to these spectra is that a theoretical increase in fundamental period for BF-4 will result in a decrease of the spectral acceleration response.

Upon developing an effective NRTMDF for BF-4 and the medium site condition, analyses indicate a fundamental elastic period shift to 0.92 seconds. Other dynamic alterations, effective inelastic period shifts and changes in peak response parameters for this case are shown in Table 2. Effective period lengthening due to inelastic actions is considered in the design in modern tall buildings including BRBFs for performance based seismic design (Moehle et al. 2011).

Table 2 shows that the average reductions in peak rooftop displacement are over 40% while base shear reductions are markedly less. This reflects the nonlinear analysis methods and the inability of this structure to develop horizontal reactions which are higher than the inelastic lateral capacity of the bottom story of the structure.

Figure 3 depicts a rooftop displacement history for this case for the LA24 ground motion taken from the SAC ground motion suite (http://nisee.berkeley.edu/data/strong_motion/sacsteel/ground_motions.html) which was included within the BSE-2 medium site ground motions for the study. As depicted, peak rooftop displacements as well as rooftop displacements in general are reduced for the damped

Table 2. Average Change in Peak Output Parameters for BF-4, Medium Sites

NRTMDF Properties

Mass = 719186 kg

Initial Stiffness = 71.8 kN/mm

Damper Yield Strength = 2440.5 kN

Initial Damped Period = 0.92 sec

BSE-1 Average Effective Stiffness = 34.56 kN/mm

BSE-1 Average Peak Damped Period = 1.09 sec

BSE-2 Average Effective Stiffness = 11.42 kN/mm

BSE-2 Average Peak Damped Period = 1.67 sec

Ground Motion Suite	Base Shear (% Change)	Rooftop Displacement (% Change)	NRTMDF Displacement (mm)
BSE-1			
Average	-19.9%	-43.6%	108
BSE-2			
Average	-2.5%	-40.9%	367

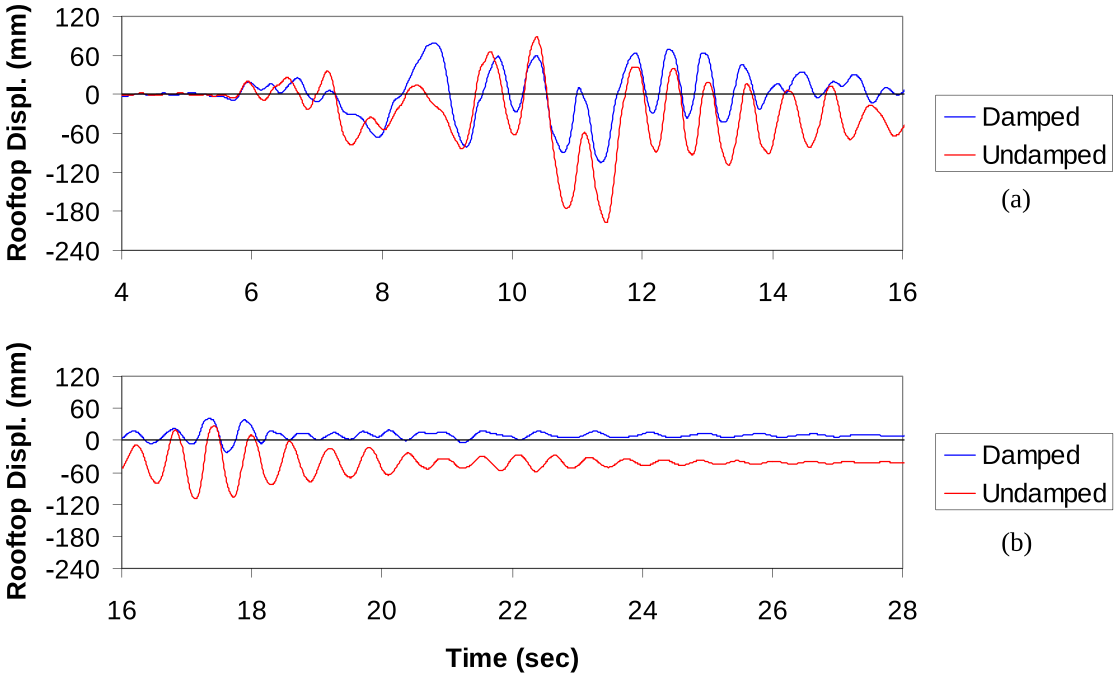

Figure 3. BF-4 Rooftop Displacement History for Medium Site, BSE-2, LA24 Motion, 1-16 seconds(a) and 16-28 seconds(b)

structure. In addition, the damped structure has almost no permanent inelastic deformation, whereas the undamped has approximately 50mm of permanent inelastic deformation. Figure 4 demonstrates the hysteresis loop of the NRTMDF for this example. Quantitative assessment of the area within each loop over the entire duration of the earthquake enables the determination of total energy dissipated through nonlinear action of the NRTMDF. This energy is dissipated through the NRTMDF through a targeted mechanism. As such, it is not available as energy in either kinetic or strain form which may cause damage elsewhere in the structure.

Another noteworthy result for this example is that the analytical modeling suggests that for BSE-1, the NRTMDF predicts a prevention of brace buckling and other nonlinear phenomena associated with structural nonlinearity in the primary structure. Results of analyses predict that the NRTMDF prevents buckling of braces in the primary structure for the BSE-1 ground motion suite while for the BSE-2 ground motion suite the nonlinear performance of the braces is similar to the undamped structure subject to the BSE-1 ground motion suite. As such, the results suggest that the NRTMDF offers the potential for an increase by one full performance measure as predicated by codes for rehabilitation (ASCE 41-2006) for which such an increase in performance may be a shift from Collapse Prevention to Life Safety for the same ground motion.

Consideration of energy methods depicts a fundamental validity to the NRTMDF approach and the potential not only for reduction of peak transient response parameters but for a reduction in response for the duration of the earthquake acceleration history. Park and Ang (1985), and Fardis (1995) each developed measures of energy dissipation reflected in a calculated damage index. For this, a damage index of 1.0 or more predicts failure while a smaller damage index reflects the potential for satisfactory

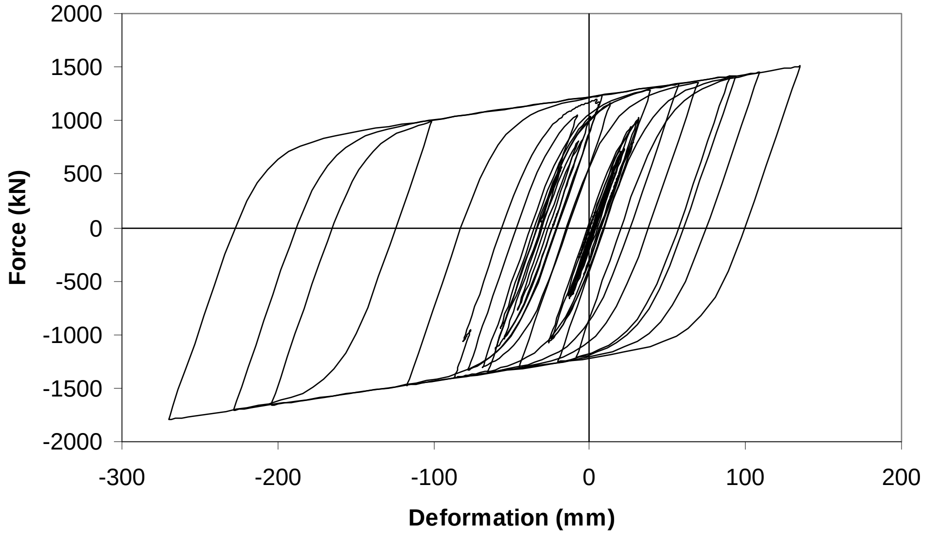

Figure 4. BF-4 NRTMDF Buckling Restrained Brace Hysteresis Loop for Medium Site BSE-2, LA24 Motion

performance. For the BF-4 model, the Park and Ang method yields a reduction in damage index of 31.4% and 34.0% for BSE-1 and BSE-2 respectively. Likewise the damage indices for the Fardis method are reduced by 52.0% for BSE-1 and 65.3% for BSE-2.

The reduction of peak transient response parameters reflects improved performance for the structure and a greater potential for seismic performance and damage reduction. When considering the effectiveness of the NRTMDF on nonstructural elements and components, floor spectra developed from story levels of the structure for the undamped and damped structures provide a demonstration of the reduced accelerations. Figure 5 demonstrates an example of reduced spectral accelerations enabled by the NRTMDF. This reflects the potential to reduce damage to nonstructural systems, which often comprise the majority of a building's value.

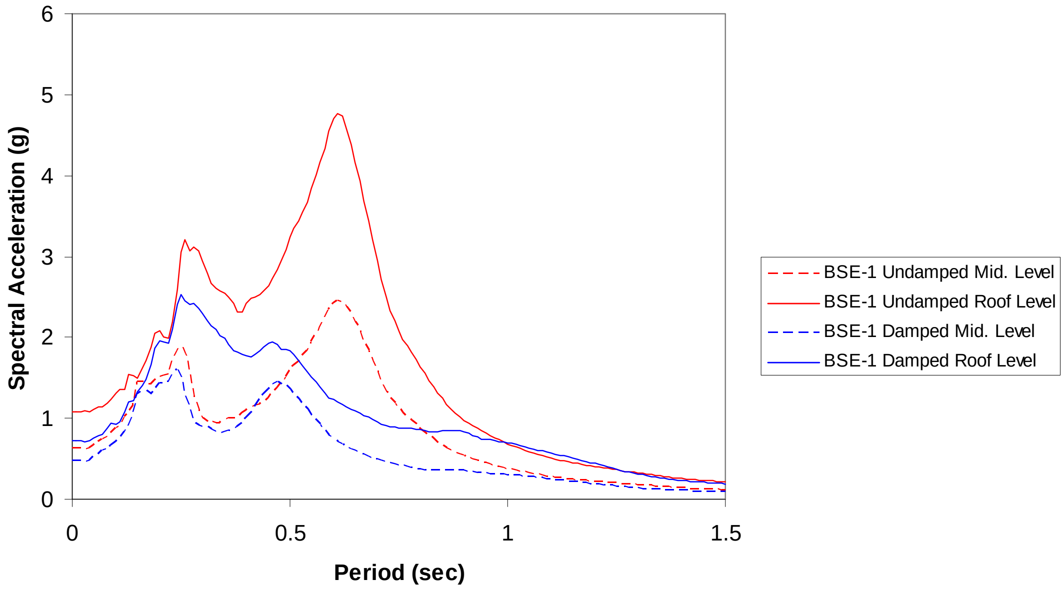

Figure 5. Average Floor Spectra for BF-4, BSE-1 Ground Motions, Medium Site

While the developed spectra of Figure 5 qualitatively demonstrate the reduced accelerations at the respectively floors, the quantitative results cannot be pragmatically assessed within the context of this research since they rely specifically upon the dynamic properties of the assembly under consideration. Calculating the average spectral acceleration values from 0 to 1.0 second periods is deemed an appropriate measure for comparative purposes. For the medium site condition, analyses of test model BF-4 show a reduction of average spectral accelerations of 41.6% and 29.1% for BSE-1 and BSE-2 respectively.

While the aforementioned results reflect the summary of test model BF-4 and the effect of the NRTMDF on this model, Table 3 reflects the summary of peak transient response parameters for all ten test structures and the medium site condition. Similar, but quantitatively different values in damage index and floor spectra can be observed in all nine remaining structures.

Table 3. Summary of Average Changes in Peak Output Parameters for BSO, Medium Sites

Structure	Base Shear (% Change)	Rooftop Displacement (% Change)	NRTMDF Displacement (mm)
		BSE-1	
BF-1	-8.0%	-16.8%	47
BF-2	-10.6%	-25.4%	37
BF-3	-2.9%	-12.7%	96
BF-4	-19.9%	-43.6%	108
BF-5	-30.7%	-36.0%	130
EBF-1	-5.1%	-29.3%	89
SW-1	-10.8%	-13.3%	51
SW-2	-1.0%	-13.3%	95
MF-1	-25.5%	-27.3%	189
MF-2	-37.2%	-30.2%	184
		BSE-2	
BF-1	0.4%	-7.8%	143
BF-2	-2.1%	-15.6%	91
BF-3	-0.7%	-16.2%	243
BF-4	-2.5%	-40.9%	367
BF-5	-6.6%	-26.8%	481
EBF-1	-8.1%	-30.4%	314
SW-1	-8.8%	-10.5%	176
SW-2	-2.6%	-13.3%	259
MF-1	-20.0%	-31.3%	705
MF-2	-8.4%	-26.1%	803

BSO = Basic Safety Objective (Analyses considering BSE-1 and BSE-2)

5. CONCLUSION

Simulations demonstrate that the NRTMDF method represents a valid technique for achieving two dynamic alterations which promote a diminished response to structural vibrations. First, the technique drives shifts in fundamental period of the structure. An initial period shift is enabled by virtue of elastic behavior of the NRTMDF and further shifts are enabled as the NRTMDF develops active nonlinear behavior driven by the yielding of its buckling restrained braces. The magnitude of this inelastic period shift is dependent on the degree of nonlinear strain of the NRTMDF with higher nonlinear strains correlating to longer period shifts. Second, the NRTMDF becomes an effective mechanism for targeted energy dissipation. Analyses show that the energy dissipation through the NRTMDF of the damped structures is significant and comprises a significant portion of the total energy balance.

Traditional methods of seismic analysis address peak transient response parameters and do not provide measures of performance for complete acceleration time-histories. Energy methods provide performance measures addressing peak transient response parameters in conjunction with demand through the entire acceleration history. Similar to traditional approaches which consider only peak transient response parameters, the energy methods demonstrate that the NRTMDF provides the potential for a significant reduction of seismic response.

Studies of floor spectra of damped and undamped structures demonstrate the potential for the NRTMDF to reduce seismic forces imposed on a building's nonstructural elements and components. Spectra developed from acceleration response histories extracted from key stories of analytical models show average reductions in spectral accelerations as high as 40% for the period range of 0 to 1 seconds.

The analyses demonstrate the potential for seismic response reductions for many classes of structures and soil types. Structures lying on the dynamic threshold of a significant reduction in spectral acceleration response by virtue of a positive shift in fundamental period are the best candidates for the approach. This can be qualitatively demonstrated by observation of the applicable spectra under consideration and the apparent potential for reduction in spectral acceleration response. This study identifies medium period structures (with a fundamental period of 0.5 to 0.8 seconds) located on medium site conditions (Type D soils) as the classification of structures and sites for which the greatest performance enhancements are enabled by virtue of the NRTMDF.

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