Determination of 2D shallow S wave velocity profile using waveform inversion of P-SV refraction data

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SUMMARY:
The evaluation of the shallow soil parameters happens to have a huge importance for earthquake engineering purposes. We proposed a new method to estimate the 2D inhomogeneous Shear wave velocity profile of shallow soils using a waveform inversion of P-SV refraction data. The numerical part of this method is based on a 2.5D finite difference staggered grid materializing the propagation wave field. The algorithm used for the inversion is the Hybrid Heuristic Search method proposed by Yamanaka (2007).Numerical experiments were conducted using synthetic observed waves. The inverted results show that we succeeded to reconstruct a 2D soil profile with irregular layer interface in a noisy environment using a single refraction shot. This approach is inexpensive and allows extracting more information contained in the seismic traces than the conventional seismic refraction method.

Keywords: seismic refraction, waveform inversion, deconvolution, 2.5D simulation.

1. INTRODUCTION

During earthquakes most of constructions interact directly with surface layers, structural irregularities in shallow soils may generate complex waves that will interfere to disturb the expected response of the buildings, detecting those irregularities and taking the necessary estimation for their effect to ground motion can have an important role to mitigate the earthquake damage upon manmade structures.

Many geophysical techniques are available to profile the shallow soils and each technique had specific advantages and disadvantages. The seismic refraction method remains one of the cheapest and simplest methods for shallow soils investigations, it had shown its efficiency to estimate shallow soil velocities, and is widely used for routine geotechnical engineering applications. Since this technique is based on the initial wave arrival, it gives very basic information about the soil and cannot provide lateral variations of the sedimentary layers. Also, this method suffers from some practical restriction in case of reversal velocity soils or the presence of a hidden thin layer. One of the major breakthroughs in seismic refraction is the development of the seismic refraction tomography. The recent advent on seismic refraction tomography had provided a significant new geophysical tool that performs well in many situations where traditional refraction techniques fail. However, one of the difficulties of the tomography imaging is to properly underline layers interfaces and can be relatively expensive than the conventional seismic refraction survey. The surface wave exploration became very popular in the last few years. When a vertical source is excited at the surface, the energy produced will scatter generating surface Rayleigh waves or P-SV waves that can be used for shallow soil exploration.

The objective of this research is to propose a simple and cheap method for shallow soil explorations able to reconstruct the 2D soil profiles using the surface P-SV waves obtained from the conventional seismic refraction survey. To exploit the maximum information from refraction seismograph, we apply wave form inversion analysis using the Hybrid Heuristic Search Method proposed by Yamanaka (2007). The algorithm will minimize the misfit between the observed waveforms and synthetic waveforms computed from numerically generated soils and recreate the optimal soil model that fits the observed waveforms. A primary study was initiated by Takekoshi and Yamanaka (2006), where they conducted a full wave form inversion of SH waves using numerical experiments and succeeded to reconstruct the 2D soil profile by inverting synthetic SH waves. Yamauchi and Yamanaka (2009) applied this method for a real case study using deconvoluted crude SH waveforms to get rid of the source effect.

2. WAVEFORM INVERSION

2.1. Forward Modelling

In this method, the soil model is computed by approximating the 2.5D P-SV equations of motion. The 2.5D simulation was initiated by Liner (1991) and allows to mimic a three dimensional geometrical spreading in a 2D plan with less computation time than 3D simulation. The simulation is performed by solving the P-SV equation of motion and stress in a cylindrical coordinates. The equations are approximated by the Finite difference staggered grid proposed by Virieux (1984) using the 4th order approximation for space and 2nd order approximation for time. The FD grid materializes numerically the propagation wave field and allows computing the outputted wave forms. A vertical point source simulates a plank hammering at the free surface and generates the waves to propagate among the numerical grid and intercepted by receivers that records the vertical component on the surface.

In order to check the validity of our 2.5D simulation we conducted a numerical experiment which consists of computing the waveforms recorded in a half space model, and calculate the attenuation curves issued from each 2D, 2.5D and 3D wave fields propagations. The model size is a 50m x 10m half space with a velocity Vs=400m/s, Vp=700m/s density ρ=1600 and a quality factor Qf=10. The computation was performed using a server equipped with 8 processors Intel® Xenon® 2.33Ghz.
A big difference on computation time is noticed between the 3D simulation which take almost 2 day by parallelizing 7 CPU, and the 2.5D simulation which takes only 4 minutes using a single CPU.
The normalized attenuation obtained from each simulation reflects that the 2.5D simulation allows obtaining a similar behavior to the 3D simulation rather than the 2D. Thus, using the 2.5D simulation is more realistic than the 2D simulation.

2.2. Data processing

In data processing before the inversion, a waveform deconvolution was applied to the P-SV refraction data to get rid of the source influence; each sinusoid that composes the original signal can be changed in amplitude and/or phase as it passes through the undesired convolution. The deconvolution is the process of filtering a signal to compensate for an undesired convolution and recreate the signal as it existed before the convolution took place.
Let's consider a given system with the Green function $G_j(t)$ where we input waveforms with a source function $S(t)$, the waves travels among the system where they undergo a convolution caused by the soil proprieties and outputted as a receiver function $R_j(t)$. The outputted waveforms for a station *'j'* can be written in equation (2.1), where the asterisk stands for convolution:

$$R_j(t) = G_j(t) * S(t) \tag{2.1}$$

Deconvolution is almost impossible to understand in the time domain. In fact, if these signals were combined by addition or multiplication instead of convolution, the solution would be subtraction to remove or division to undo the convolution in our time series, however in time domain there is no simple inverse operation to allow the deconvolution process. Fortunately the deconvolution is quite straight forward in the frequency domain. After converting the signal to the frequency domain the convolution can be represented as a simple multiplication and it will be possible to manipulate the signal by dividing the power spectrum by the first station of our survey line $j=1$ that can be used as a reference station as follows:

$$T_j(\omega) = \frac{R_j(\omega)}{R_1(\omega)} = \frac{G_j(\omega) * S(\omega)}{G_1(\omega) * S(\omega)} \tag{2.2}$$

Where $T_j(\omega)$ is the normalized wave field and j is station number, $R_j(\omega)$ is the receiver function in the frequency domain for each j station. In the Equation (2.2), the source spectrum cancels its self, so that the normalized wave field will include only the Green function of the soil.
All we need to do is to transform back to time domain and we can obtain the data free from the source effect.

In order to prove the deconvolution theorem, we conducted numerical experiments where we compute the waveforms of a simple two layered model using a vertical point source of 50Hz, and another one of 100Hz. For both cases we used the first station as a reference station for our deconvolution then compared the deconvoluted waveforms for each case.

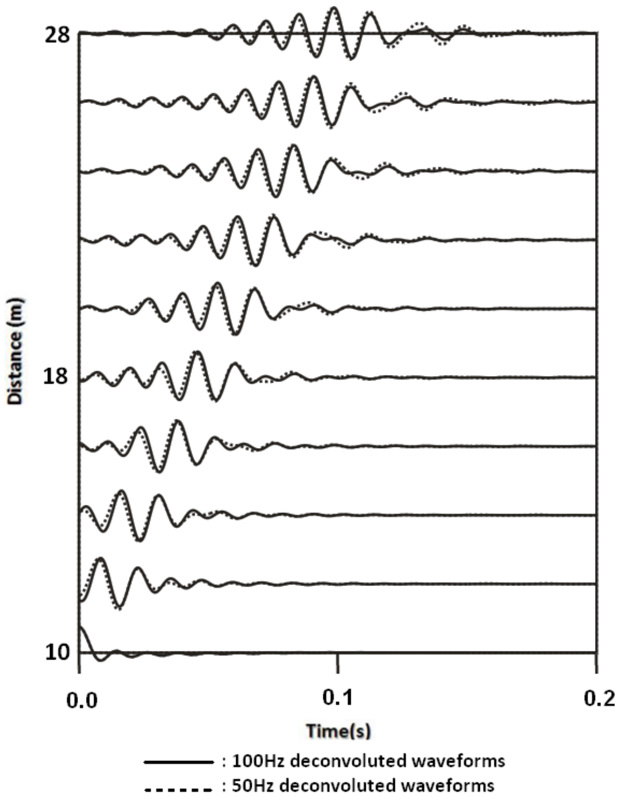

Figure 1. Comparing deconvoluted wave forms from different source frequencies.

Fig. 1. show the comparison of the deconvoluted waveforms for both 50Hz an 100Hz pulse. The comparison underlines a perfect fitting of the deconvoluted waveforms despite the difference of the inputted source frequencies. The spectrum at the first station (reference station) cancels itself and cannot be used to obtain any information. These results allow concluding that we succeeded to get rid of the source influence from our signal using the waveform deconvolution.

2.3. Inversion algorithms

In computing, what is called 'heuristic programming' solves a problem by a method of trial and error, in which the success of each attempt at solution is assessed and used to improve the subsequent attempts, until a solution acceptable within defined limits is reached. For our purpose, the most important feature about heuristic approaches is that it allows minimization of the misfit between the observed and the synthetic data, this will lead to a progressive reduction of the difference until it reaches the minimum fitting at the end of the inversion.

The inversion algorithm used in our research is the hybrid heuristic search method proposed by Yamanaka (2007), based on the genetic algorithms and simulated annealing. The misfit function to be minimized can be written as follows:

$$E = \frac{1}{M}\sum_{j=2}^{M}\left\{\frac{\sum_{i=1}^{N}\left[\overline{D_j^o(t_i)} - \overline{D_j^c(t_i)}\right]^2}{\sum_{i=1}^{N}\left[\overline{D_j^o(t_i)}\right]^2}\right\} \qquad (2.3)$$

Where, $D^o_j(t_i)$and $D^c_j(t_i)$ are respectively the observed and the calculated vertical waveforms after deconvolution, at each M station along N number of data.

In tomography inversion, subsurface structure is often modelized using many cells, but it will be very difficult to invert a tomographic image with so many unknown parameters. In order to reduce the number of parameters to be inverted, and obtain a lateral variation of the model we used the soil parameterization model proposed by Aoi et all (1995).This method allows dividing each layer into few blocks with velocities and depths to be inverted independently. Each block is materialized using a linear combination of a basis function $C_x(x)$ and a coefficient P_k the interface depth $d(x)$ can be written as follows:

$$d(x) = \sum_{k}^{L} P_k C_x(x) \qquad (2.4)$$

Where L is the number of basis function. Each basis function is defined in equation (2.5), with Δ and X_k as parameters specified in advance.

$$C_x(x) = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{\Delta(x - x_k)}\right) x_{k-1} \le x \le x_{k+1} \\ 0 \qquad otherwise. \end{cases} \qquad (2.5)$$

2.4. Validation of the inversion algorithm

To confirm the reliability of our inversion algorithm, we conduct an inversion trial using a simplified two layered model with a flat interface as shown in Fig. 2b. This experiment is meant to verify if our algorithm is able to invert a simple soil structure. The model used for this purpose is a 30m x 9m soil with a grid of 0.1m. The physical proprieties for each layer as well as the inversion parameters are displayed in Table. 2.1.

Table2.1. Model parameters used for soil with a flat interface

Layers	Vp(m/s)	Vs(m/s)	Density(kg/m^3)	Interface depth(m)	Search limits	
					Vs(m/s)	Depth
1	425	250	1600	2	130-350	0.5-4.0
2	1300	650	2200	-	650	-

A vertical point source is excited on the surface at 4m and the signal is recorded by 10 stations deployed on the free surface. The surface layer is divided into ten blocks with unknown depth and velocities. A smoothing factor is implemented between the blocks in order to reduce the trade off relation between velocity and depth during the inversion.
The comparison of the final computed waveforms and the observed ones are displayed in Fig. 2a. The comparison shows a perfect similarity between the calculated waveforms and the observed ones. Fig. 2c. illustrates the most optimal model inverted from the calculated waveforms, comparing with the target model on Fig. 3b, the inverted soil model succeeded to reconstruct the layer interface at 2m depth, the velocity of the blocks is averaged to 243.9 m/s which is close to the target one set to 250m/s.

The standard deviation (SD) of the average velocity for the blocks of the surface layer is SD=4.78m/s.

The results of this experiment shows that our inversion algorithm is able to invert two dimensional soil models with a flat interface within a satisfactory level of accuracy.

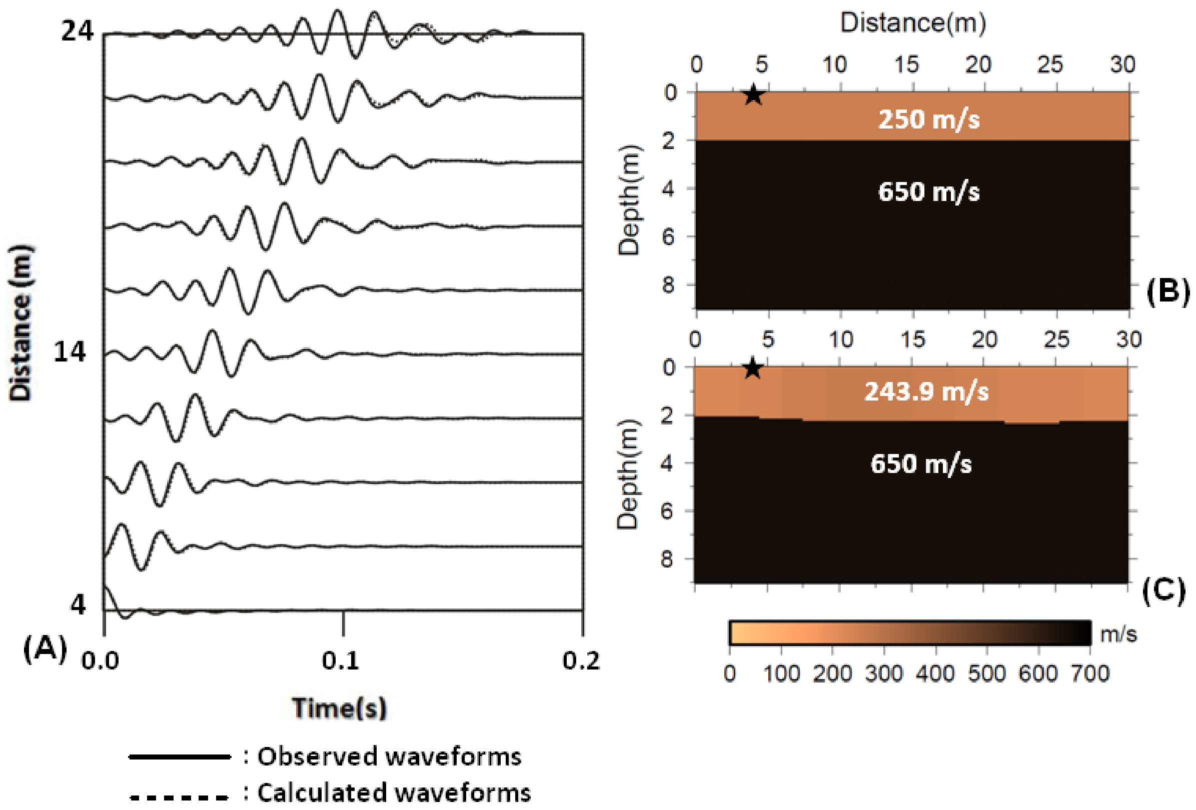

Figure 2. Inversion Results of the two layer flat interface soil model:
(A) Comparison of the observed and calculated waveforms.
(B) Target soil profile. (C) Inverted soil profile.

3. NUMERCIAL EXPERIMENTS

3.1. Two layers model with an irregular interface

The P-SV inversion program works successfully in soils with horizontal series. However, complex near surface geology may not fit into the assumption of a series of horizontal layers. Our next experiment aims to verify the capacity of the algorithm to reconstruct a two layered model with an irregular interface.
For this purpose, we utilize the same model previously described with the same physical proprieties and include an irregularity as a dome shaped interface that rise the interface depth from 4m to 2m as illustrated in Fig. 4b. The irregularity is at a distant location from the source in order to allow a better refraction along the interface.

The vertical source is located at 4m and for a more accurate acquisition the whole surface is covered by receivers from 4m to 24m with an interval of 1m. Our zone of interest is marked with a red dashed box in Fig. 3a. and Fig. 3b. Areas outside the red dashed box are out of discussion since no recordings are available.

The number of generations in the inversion is 100 and the search limit for the surface layer Vs are between 130m/s and 350m/s and 1m to 6m for depth. The surface layer is divided into ten blocks, and for more accurate results we fix the basement velocity to 650m/s.

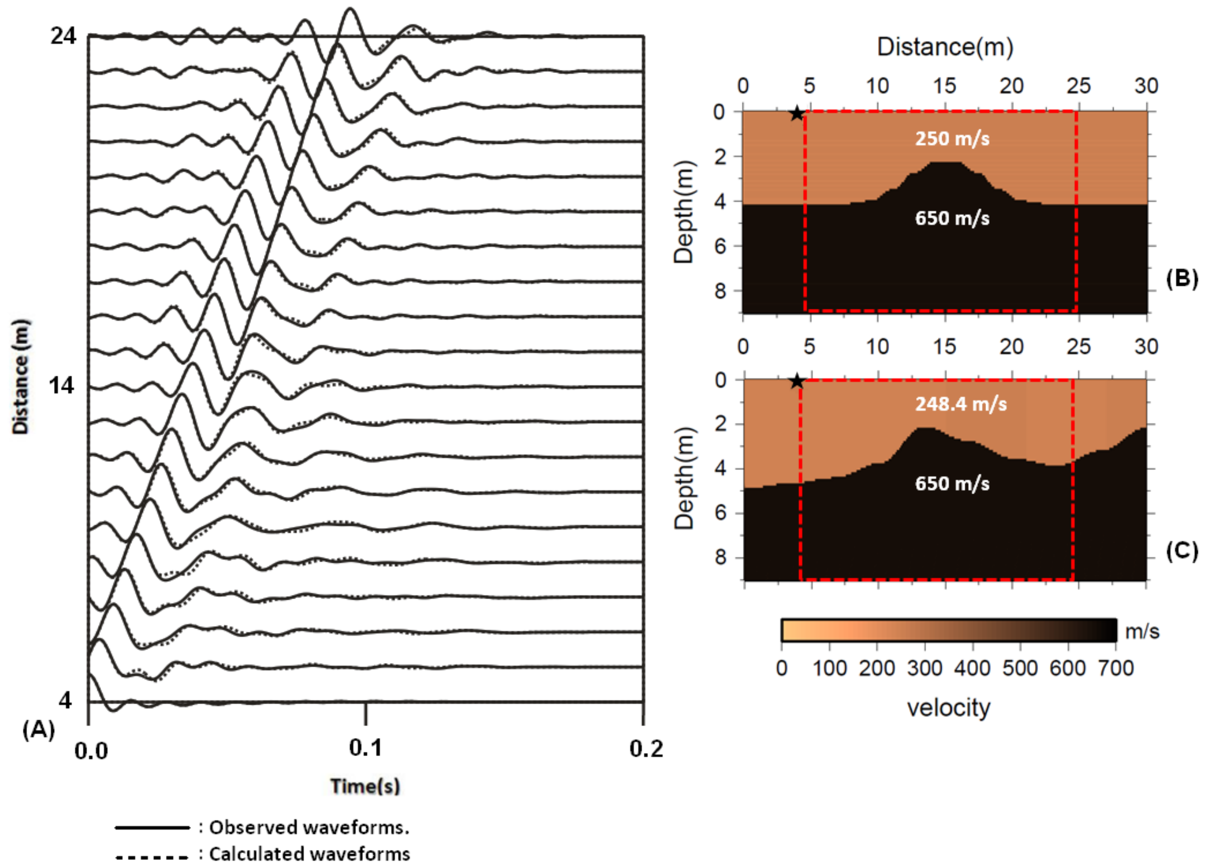

Figure 3. Inversion results of the two layers model with irregular interface
(A) Comparison of the observed and calculated waveforms.
(B) Target soil profile. (C) Inverted soil profile.

Fig. 3a. synthesizes the comparison between the observed results in solid line and the computed waves in dashed lines, the calculated wave forms shows an acceptable fitting with the observed ones proving that the algorithm succeeded to approximate the velocity of the surface, the final optimized model have an averaged velocity of 248.4m/s with a standard deviation SD=3.48m/s.
The inverted soil model in Fig. 3c. shows that the algorithm succeeded to speculate the existence of the dome and reconstruct its shapes between 10m to 20m as well as its depth between 4m and 2m.

The left side of the dome was faithfully reproduced because it is located on the front side of the source, the refracted waves sweep the left side of the dome and refract along the slop interface providing detailed information of its structure, however some inaccuracies occurred to perfectly reproduce the right slope of the dome since not enough information are contained in our signal, for a better profiling of the right slop another right shot will be required.
As stated above, the areas outside the survey line are not to be considered because no data are available; the attribution of the depth on these areas can be random and have no effect in our computed waveforms.

The inversion conducted previously deals with a pure numerical signal free from noise. For a more realistic approach and to check the performance of the inversion in a noisy environment, we corrupted our signal with a white noise calculated from 30% of the maximum amplitudes at each station, and tried to invert the same model previously discussed.

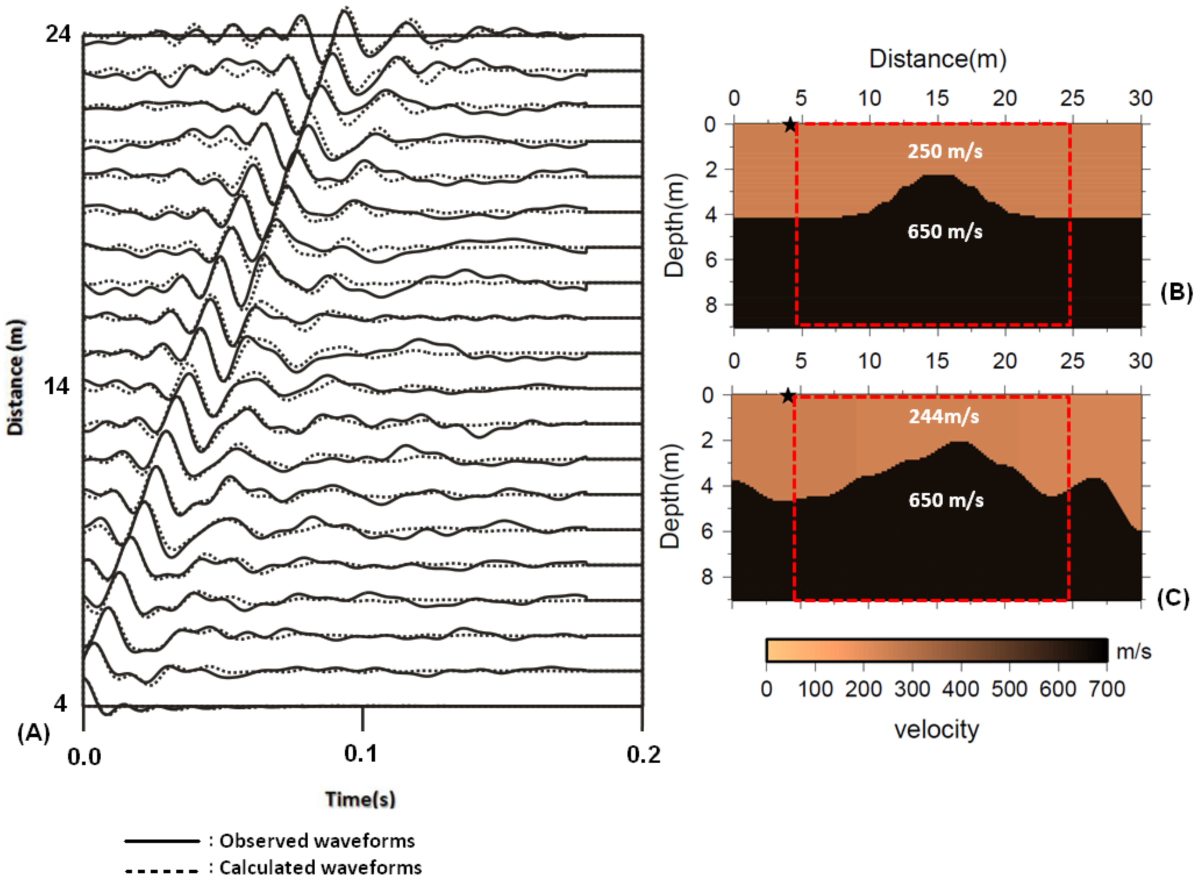

Figure 4. Inversion results of the two layers model with irregular interface using a noisy signal
(A) Comparison of the observed and calculated waveforms.
(B) Target soil profile. (C) Inverted soil profile.

The comparison of the final inverted waveforms and the observed ones in Fig. 4a. shows that the algorithm had no difficulty to invert the velocity of the layer despite the noisy signal, averaged to 244m/s with a standard deviation SD=6.97m/s.

The algorithm also detected the existence of the dome interface; however some inaccuracies occurred on precisely tracking down its position as well as reconstructing its slopes, those inaccuracies are mainly due to the noise ripples in our signal that disturbs the refracted waves, the information contained in the signal is altered and can lead to some inaccuracies during the inversion.

Through this experience we can conclude that our method is also effective to invert velocity soil profiles with irregular interfaces and able to speculate the existence of those irregularities with a certain level of accuracy in a very noisy environment.

CONCLUSION

Through this research we succeeded to reconstruct 2D soil profile using waveform inversions of P-SV refraction data.

Synthetic waveforms are computed from a numerically materialized soil model using a Finite Difference Staggered grid by approximating the 2.5D P-SV wave equations of motion and stress which gives same geometrical spreading as the 3D simulation. In order to get rid of the source

signature in our signal, deconvolution is applied to the waveforms.
Numerical experiments were conducted for some difficult cases to obtain using the conventional seismic refraction method. We succeeded to determine the 2D soil profile with irregular interfaces in a noisy environment using a single shot.

This method is still at its early stage of development, a further development consists of applying two sided shots to increase the resolution. Attempting to invert Vp and Vs and mapping the two dimensional Poisson ratio of soils, and finally see the performance of this method for real refracted data.

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