Calibration of a source spectrum model and construction of spectral strong motion attenuation relationships from accelerogram records

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SUMMARY:
A general methodology for the calibration of source spectrum attenuation relationships, based on accelerogram records of any region, is presented. The source spectrum model allows calculation of the expected Fourier amplitude spectrum at any site, induced by the occurrence of an earthquake of a given magnitude and hypocentral distance, for different strong motion measures (acceleration, velocity, displacement or energy). Using random vibration theory the expected intensity value, in the time domain, is computed. The model is calibrated against an accelerogram records database by means of a genetic algorithm, in order to minimize the estimation mean error and deviation. This methodology allows the construction of theoretically-based attenuation models, calibrated with the instrumental information available in any accelerograph network.

Keywords: Attenuation, Source spectrum, Seismic hazard.

1. INTRODUCTION

Seismic hazard assessment requires the definition of attenuation relationships that adequately represent the transformation processes suffered by seismic waves travelling through the earth's cortex. These processes are mainly related to the expansion of the wave front and anelasticity of the travelling medium. Source spectrum models allow the definition of the Fourier Amplitude Spectrum (FAS) of ground strong motion given a magnitude and hypocentral distance, while taking into consideration the mentioned processes. The sources are modelled as discontinuities in a homogeneous and isotropic medium and earthquakes are assumed to be pure shear dislocations. Using these basic assumptions, the shape of the bedrock FAS can be computed at any location. Once FAS is computed, the mathematical expectation of peak strong motion parameters, in the time domain, can be computed by means of random vibration theory.

The abovementioned approach allows calculation of strong motion parameters, such as peak ground acceleration, velocity or displacement at any location. However, FAS depends on several seismological variables that define the rupture process and the quality of the medium. Source spectrum basically depends on stress drop, radiation pattern, quality factor, wave velocity of the medium and corner frequency. Therefore, the use of source spectrum to compute the FAS is not straightforward. In this paper a numerical calibration process is presented in order to determine the set of seismological variables that best fit the observed strong motion values, recorded by an accelerogram network. This process can be applied to seismic records of any region of the world in order to derive calibrated strong motion attenuation relationships.

2. SOURCE SPECTRUM MODEL

Aki (1967) deduced the shape of displacement FAS given the pass of shear waves at a far-field location,

$$U(f) = \frac{R_{\theta\phi}}{4\pi\rho\beta^3 R} \frac{M_0}{1+\left(\frac{f}{f_c}\right)^2} \tag{1}$$

where ρ is the density of the wave travelling medium, β y the shear wave velocity, R is the hypocentral distance, M_0 is the seismic moment $R_{\theta\phi}$ is the radiation pattern and f_c is the corner frequency.

The FAS in Eqn. 1 has a plateau proportional to M_0 in the low frequency region, and decays in the high frequency region as a function of the frequency to the square. The frequency where decay starts is the corner frequency. This frequency can be computed as (Brune, 1970):

$$f_c = 4.9 \cdot 10^6 \beta^3 \sqrt{\frac{\Delta\sigma}{M_0}} \tag{2}$$

where $\Delta\sigma$ is the stress drop in the seismic source after the dislocation has occurred. Brune (1970) transformed the displacement spectrum $U(f)$ in Eqn. 1 to acceleration, $A(f)$.

$$A(f) = \frac{R_{\theta\phi}}{4\pi\rho\beta^3 R} \frac{M_0 \cdot f^2}{1+\left(\frac{f}{f_c}\right)^2} \tag{3}$$

Strong motion intensity diminishes with distance not only due to geometric attenuation, but also to physical conditions in the path travelled by seismic waves. Only some of the total energy transmitted by seismic waves is transformed into elastic deformation of the medium. The rest is dissipated due to anelasticity of the rocks. Therefore, this effect has to be included in the model in order to correctly quantify strong motion. Anelasticity is controlled by the Q parameter (Knopoff, 1964). An adequate way to include anelasticity in the source spectrum is by means of an exponential decreasing filter, so FAS is now defined as in Eqn. 4.

$$A(f) = \frac{R_{\theta\phi}}{4\pi\rho\beta^3 R} \frac{M_0 \cdot f^2}{1+\left(\frac{f}{f_c}\right)^2} \exp\left(\frac{-\pi f R}{\beta Q}\right) \tag{4}$$

For attenuation purposes, Q should also depend on the frequency, given the fact that frequency is relevant in energy dissipation. Q can be defined, in terms of frequency, as

$$Q = Q_0 \cdot f^{\varepsilon} \tag{5}$$

where Q_0 and ε depend on the characteristics of the medium. Brune's spectrum (Eqn. 3) predicts constant amplitude for frequencies higher than f_c. This is not possible since it implies infinite energy availability. Also, real earthquakes show that high frequencies are attenuated faster than low frequencies, causing strong motion intensity to be attenuated faster than predicted by geometric attenuation. Singh et. al. (1982) proposed a low-pass exponential filter to include this characteristic in source spectrum. Including this filter, FAS is now defined as in Eqn. 6

$$A(f) = \frac{R_{\theta\phi}}{4\pi\rho\beta^3 R} \frac{M_0 \cdot f^2}{1+\left(\frac{f}{f_c}\right)^2} \exp\left(\frac{-\pi f R}{\beta Q}\right) \exp(-\pi f k) \tag{6}$$

where k controls how fast amplitude decays as a function of frequency. Since high frequency decay depends on distance, k can be expressed as,

$$k = k_1 + \frac{R}{Q_1} \tag{7}$$

where k_1 and Q_1 depend on the characteristics of the medium. Finally, the source spectrum is multiplied by $(1/2)^{0.5}$ to decompose it into two orthogonal components. Also, is multiplied by 2 to consider free surface amplification. The resulting source spectrum (Eqn. 8) allows a good description of strong motion FAS for the far-field (Gallego, 1999).

$$A(f) = \frac{2}{\sqrt{2}} \frac{R_{\theta\phi}}{4\pi\rho\beta^3 R} \frac{M_0 \cdot f^2}{1+\left(\frac{f}{f_c}\right)^2} \cdot \exp\left(\frac{-\pi f R}{\beta Q_0 f^{\varepsilon}}\right) \cdot \exp\left(-\pi f \left(k_1 + \frac{R}{Q_1}\right)\right) \tag{8}$$

In order to compute near-field strong motion FAS, the approach proposed by Singh et. al. (1989) is applied to the source spectrum in Eqn. 8. In this approach, rupture area is considered as an infinite set of area differentials, each of which can be modelled using Eqn. 8. The resulting near-field source spectrum is (Gallego 1999),

$$A(f)^2 = \frac{R_{\theta\phi}}{4(\pi\rho\beta^3)^2} (M_0 f^2)^2 \frac{\exp\left(-2\pi f \left(k_1 + \frac{R}{Q_1}\right)\right)}{r_0^{\,2}} \left[E1(\alpha R) - E1\left(\alpha\sqrt{r_0^{\,2} + R^2}\right)\right] \tag{9}$$

where r_0 is the circular rupture area radius, $E1$ is the exponential integral (Gautschi & Cahill, 1965) and $\alpha = 2\pi/\beta Q_0$. In summary, FAS is modeled using Eqn. 8 for the far-field and Eqn. 9 for the near-field.

3. RANDOM VIBRATION THEORY

Given the random nature of strong motion records, random vibration theory can be used to determine the expected value of strong motion intensity in the time domain ($E\{a(t)\}$), as a function of its FAS. From Cartwright & Longuett-Higgins (1956) and Davenport (1964) it can be established that,

$$E\{a(t)\} = \sqrt{2\ln\left(\frac{1}{2\pi}\sqrt{\frac{m_4}{m_2}}T_d\right)}\sqrt{m_0} + \frac{\gamma}{\sqrt{2\ln\left(\frac{T_d}{2\pi}\sqrt{\frac{m_4}{m_2}}\right)}}\sqrt{m_0} \tag{10}$$

where T_d is the duration of the intense phase, γ is Euler's constant ($\gamma = 0.577...$) and m_n are the n-order moments of FAS,

$$m_n = \frac{2^{n+1}\pi^n}{T_d}\int_{-\infty}^{\infty} f^n A(f)df \tag{11}$$

If the moments in Eqn. 11 are computed using the source spectrum model of Eqns. 8 and 9, then the expected value of strong motion intensity in Eqn. 10 corresponds to the peak ground acceleration (PGA). The procedure is equivalent for any other intensity measure (velocity, displacement of energy).

4. CALIBRATION PROCEDURE

Calibration is performed using a strong motion records database. The database has to be previously debugged in order to discard wrong data or records of too low PGA. As a general recommendation, PGA should be at least one order of magnitude higher than the sensitivity of the accelerometer. The database should be divided in as many as focal mechanisms are identified in the study area, in order to compute a different attenuation relationship for each. The accelerograms in this database need to fulfil some basic requirements: i) they need to be recorded at bedrock, ii) magnitude and hypocentral distance (i.e. epicentral distance and depth) have to be known, iii) and magnitude has to be given in M_w scale.

The methodology presented allows the calculation of the expected value of any strong motion intensity measure as a function of its FAS. Therefore, strong motion intensity can be expressed as a function of all parameters involved in its formulation.

$$E\{a(t)\} = f\left(M_0, R, \Delta\sigma, Q_0, \varepsilon, k_1, Q_1, \rho, \beta, R_{\theta\phi}\right) \tag{12}$$

In this study, calibration is focused on all the parameters that define strong motion intensity. However, not every parameter in Eqn. 12 is susceptible of calibration. Since distance and magnitude are known quantities, M_0 and R have to be removed from the group of parameters. Particularly, M_0 is a function of magnitude (M_w) (Hanks & Kanamori, 1979).

$$Log_{10}(M_0) = 1.5M_w + 16.1 \tag{13}$$

Given that computation of the expected value of strong motion is a highly non-linear problem, no classical statistical analysis can be performed to obtain the set of seismological parameters that best fit the recorded accelerations. Therefore, in this study a genetic algorithm was implemented in order to perform the calibration. The steps followed by this algorithm are:

1. Selection of parameters for calibration. In this study ρ and β have been set to their commonly accepted values (2.5 Ton/m^3 y 3.5 Km/s respectively), so the rest are the final set of parameters for calibration ($\Delta\sigma$, Q_0, ε, k_1, Q_1 and $R_{\theta\phi}$).

2. Definition of variation ranges for each parameter. This allows constraining the result to only physically logical values.

3. Construction of a population of N different sets of parameters, selected randomly from within the previously defined ranges. This means that N different seismological models are defined.

4. For each seismological model, expectation of strong motion intensity is computed and residuals are calculated for each record in the database. Residuals (Re) are computed as follows:

$$\mathrm{Re} = \ln\left(\frac{a_{rec}}{a_{com}}\right) \tag{14}$$

where a_{rec} is the nominal recorded intensity of each record and a_{com} is the computed intensity using the source spectrum model. Nominal intensity is defined as:

$$a_{rec} = \sqrt{\frac{{a_x}^2 + {a_y}^2}{2}} \tag{15}$$

where a_x is the x orthogonal component of the strong motion record (typically the E-W component) and a_y is the y orthogonal component (typically the N-S component). Vertical component is not taken into account.

5. For each seismological model, the first and second central moments are computed (i.e. bias and variance, respectively).

6. Weights are assigned to each seismological model as a function of its capacity to minimize the bias. Those seismological models with bias close to zero will have a larger weight in the population.

7. Reproduction. Here, a loop starts where all defined models interact. Models start to reproduce between each other, having those models of higher weight a higher chance for reproduction. Reproduction follows a random mix of the parameters of a couple of seismological models, so the resulting child model has information from both parent models. This process is controlled in a way that the population size *N* remains constant.

8. The process in step 7 is repeated for as many generations as wanted.

The result of the application of this algorithm is the seismological model with the lowest bias for the calibration database. Given the high non-linearity of this problem, it is not possible to ensure that the resulting model corresponds to the one with a global minimum bias. Mutation in seismological models can be performed during the analysis in order to avoid stagnation in a local minimum bias.

This algorithm allows the calibration of attenuation models, based on theoretical seismology models, with recorded strong motion from real earthquakes.

5. CASE STUDY: COLOMBIA

The methodology presented was used to calibrate attenuation functions for Colombia. Strong motion records were obtained from the National Geological Survey of Colombia (SGN). The database contains 285 acceleration records with three components each (E-W, N-S and vertical). 118 were identified as intraplate earthquakes and 166 as subduction earthquakes. The initial ranges for the seismological variables were established as shown in Table 1.

Table 1. Value ranges for seismological parameters

Parameter	Units	Minimum	Maximum
$\Delta\sigma$	Bar	75	250
Q_o	-	700	800
ε	-	1	2
k_1	-	0.005	0.015
Q_1	Km	3500	4000
$R_{\theta\phi}$	-	0.55	0.63

The seismological parameters resulting of applying the calibration methodology are shown in Table 2.

Table 2. Resulting seismological models for different focal mechanism

Focal mechanism	Bias	Standard deviation	$\Delta\sigma$ [bar]	Q_o	ε	k_1	Q_1 [Km]	$R_{\theta\phi}$
Intraplate	0.0003	1.03	108.4	754.9	1.77	0.0075	3716.2	0.623
Subduction	0.0006	1.05	183.4	765.8	1.88	0.014	3573.5	0.608

Fig. 1 presents the attenuation curves for PGA, defined for hypocentral distances between 5 and 500 Km and moment magnitudes between 4 and 8.5.

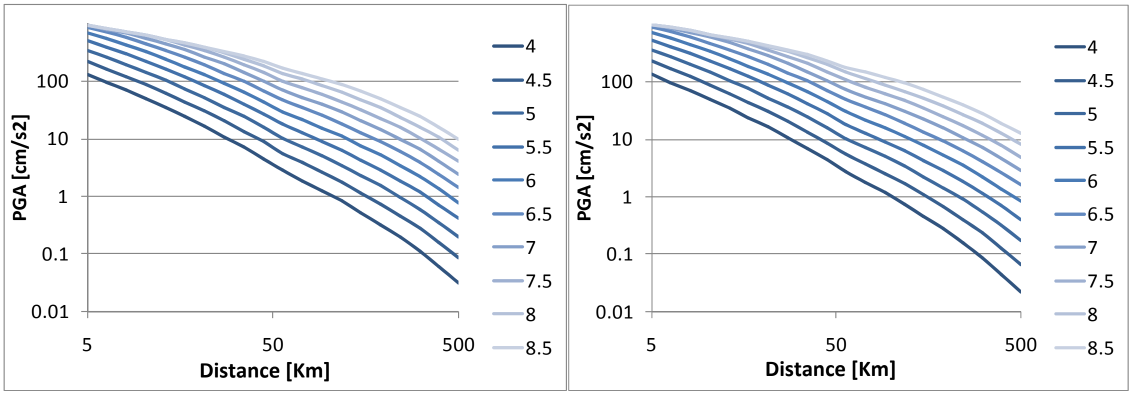

Figure 1. Attenuation relationships obtained for Colombia, for intraplate and subduction earthquakes

One of the advantages of using calibrated source spectrum models to derive attenuation relationships is the possibility to extend the analysis to spectral ordinates. Multiplying the FAS by the transfer function of a single degree of freedom oscillator, characterized by a structural vibration period and a damping ratio, gives the FAS of the oscillator response. Applying random vibration theory to this spectrum allows calculation of the mathematical expectancy of spectral acceleration. Therefore, spectral attenuation relationships can be derived from a calibrated source spectrum model. Fig. 2 shows two response spectra computed following this procedure.

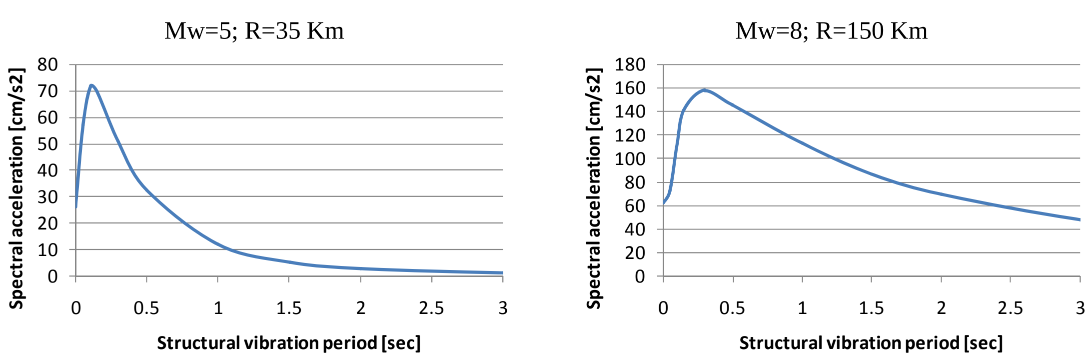

Figure 2. Response spectra computed from calibrated source spectrum for Colombia

6. CONCLUSIONS

The presented source spectrum model is useful for representing the FAS of strong motion given an earthquake magnitude and hypocentral distance. Two kinds of source spectra are used: far-field and near-field. Then, by using random vibration theory, the mathematical expectation of strong motion intensity, in the time domain, can be computed from its FAS. Peak ground intensity measures as well as spectral intensities can be computed following this methodology.

Source spectrum parameters can be calibrated using the proposed genetic algorithm, which seeks to minimize the bias between observations (acceleration records database) and predictions (source spectrum model). The result is a theoretically based attenuation model, calibrated with the available strong motion information from real earthquakes.

Source spectrum parameters are calibrated for Colombia, deriving in a new attenuation relationship for the country, for intraplate and subduction earthquakes. Applying the source spectrum model and random vibration theory methodology presented in this paper, using the resulting calibrated parameters, allows the computation of attenuation relationships for peak ground acceleration, velocity or displacement, as well as for spectral intensities given a single degree of freedom oscillator characterized by a structural period and a damping ratio.

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