# Design of FRP jackets for Confinement of Plastic Hinges in Reinforced Concrete Columns using Displacement Ductility

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**SUMMARY:**
A design method is presented for predicting the lateral deformation performance of concrete columns with a circular or rectangular cross-section confined with fiber reinforced polymer (FRP) composite jackets. A new stress-strain model based on damage is used in the development of a displacement-based design procedure for determining the FRP jacket required to achieve a target displacement ductility. The FRP jacket thickness is determined based on the target ultimate compressive strain and dilation of the confined concrete within the plastic hinge region. The design is based on the strain ductility increase provided by the FRP jacket and is thus a strain-based approach using performance-based design. A relationship is introduced for the minimum FRP jacket thickness required to preclude strain-softening behavior and uncontrolled dilation of the concrete. The design procedure compares favorably with experimental results from the literature for columns tested in single curvature that were upgraded with FRP jackets and demonstrated substantial displacement ductility.

*Keywords: confinement; ductility; FRP; seismic, design*

## 1. INTRODUCTION

Seismic rehabilitation of concrete structures using Fiber Reinforced Polymer (FRP) composites is undertaken for upgrading the performance of existing reinforced concrete (R/C) buildings and bridges to significantly improve their axial, shear, flexural, and ductile behavior during earthquakes. In particular, R/C buildings and bridges that were designed using inadequate seismic codes can benefit from seismic rehabilitation using FRP composites. The use of FRP composite jackets for improving the shear strength and ductility capacity of reinforced concrete members by improving confinement has become a popular structural rehabilitation option. The presence of FRP composite jackets within the plastic hinge region of a reinforced concrete beam-column element can induce the development of ductile flexural behavior, by increasing significantly the ultimate axial compressive strain of the confined concrete. FRP composite jackets have shown that they can inhibit premature lap splice, anchorage, or shear failure of a R/C column; this type of behavior is desirable for R/C sections subjected to cyclic loads such as those that occur in a large earthquake.

The analysis and design of existing structural systems that are rehabilitated with FRP composites requires an accurate estimate of the performance enhancement due to the confinement provided by FRP composite jackets (Seible 1997; Pantelides et al. 1999, 2004, 2007; Tastani and Pantazopoulou 2006; Binici and Mosalam 2007). In this paper, an analytical design method is presented for predicting the behavior of R/C columns confined with either concrete-filled FRP tubes (CFFT), referred to as unbonded FRP jacketed sections, or bonded FRP-confined concrete (BFCC) sections. The availability of procedures for the design of FRP jackets for plastic hinge confinement of R/C columns is somewhat limited. The most commonly used design procedures are the strain energy-based procedure by Seible et al. (1997), the multivariate regression analysis based on an upgrading index procedure by Monti et al. (2001), the target displacement and confining pressure dependent procedure by Tastani and Pantazopoulou (2006), and the lateral drift dependent procedure for circular columns by Ozbakkaloglu and Saatcioglu (2006).

## 2. DISPLACEMENT-BASED DESIGN OF FRP JACKET

In this paper, a new design procedure is introduced, in which the performance enhancement in compressive strength and strain ductility of FRP-confined concrete is expressed in terms of an internal damage-based stress-strain model (Moran 2011). No consideration is given to the increase in axial compressive strength of the FRP-confined concrete; this increase, even though beneficial, is secondary and is obtained from dilation of the FRP-confined concrete and the resultant transverse confining stresses provided by the elastic FRP jacket as transverse dilation progresses. The proposed design methodology is based on the strain ductility increase provided by the confining FRP jacket, and is thus a strain-based approach using performance-based design principles. The additional confinement and strain ductility provided by the available hoop reinforcement is ignored, because of the wide spacing and arrangement of the transverse steel, and possible corrosion of hoop reinforcement in existing R/C columns.

### 2.1. Displacement Ductility

Consider an existing R/C column of height $L_C$. The displacement ductility of the column $(\mu_{\Delta_r})_{ex}$ can be found by performing a moment curvature analysis of the R/C cross section. Assuming a bilinear relationship, in which linear elastic behavior occurs up to the stage of first yield and plastic rotation is concentrated at the center of the plastic hinge (Priestley and Park 1987), as shown in Fig. 1, the displacement ductility $(\mu_\Delta)_m$ of the column is obtained as:

$$(\mu_\Delta)_m = \left(\frac{\Delta_u}{\Delta_y}\right)_m = \bar{M}_m + \frac{1}{C_\Phi}\left[(\mu_\Phi)_m - 1\right]\, 3\lambda_p\left(1 - \frac{\lambda_p}{2C_\Phi}\right) \tag{2.1}$$

$$(\mu_\Phi)_m = \left(\Phi_u/\Phi_y\right)_m = 1 + \left(\Phi_p/\Phi_y\right)_m \; ; \; \lambda_p = 0.12C_\Phi + 0.014\alpha_s\left(f_{ye}d_{bl}/L_c\right) \tag{2.2}$$

where subscript $m$ indicates two different conditions: $m = ex$ indicates an existing column and $m = up$ indicates the upgraded column; $C_\Phi = (L_v/L_c)$ is the column curvature coefficient: for single curvature bending $C_\Phi = 1.0$, and $C_\Phi = 0.50$ for double curvature; $(\Delta_u)_m$ and $(\Delta_y)_m$ are the analytical ultimate and yield displacement of the column, respectively; $\bar{M}_m$ is the moment capacity ratio of the column; $(\mu_\Phi)_m$ is the curvature ductility factor of the column; $(\Phi_u)_m$ and $(\Phi_y)_m$ are the ultimate and yield curvature of the column section, respectively; $\lambda_p = (L_p/L_c)$ is the normalized plastic hinge length (Panagiotakos and Fardis 2001); $L_p$ and $L_v$ are the analytical plastic hinge length and column shear span, respectively. In addition, $f_{ye}$, and $d_{bl}$ are the expected yield strength and bar diameter of the longitudinal steel, respectively; $\alpha_s$ is the reinforcing slippage coefficient: $\alpha_s = 1.0$ if slippage in the plastic hinge region is possible, and $\alpha_s = 0$ otherwise; the use of $\alpha_s = 1.0$ is recommended. The displacement ductility $(\mu_{\Delta f})_m$ of the existing or upgraded column in a structural system with elastic flexibility, as shown in Figure 1(c), can be found in terms of the displacement ductility $(\mu_\Delta)_m$ of the rigid system of Eqn. 2.1 as follows:

$$(\mu_{\Delta f})_m = 1 + C_s\left[(\mu_\Delta)_m - 1\right] \; ; \; C_s = \frac{1}{(1 + c_e)} \; ; \; c_e = \frac{\Delta_{es}}{\Delta_y} \tag{2.3}$$

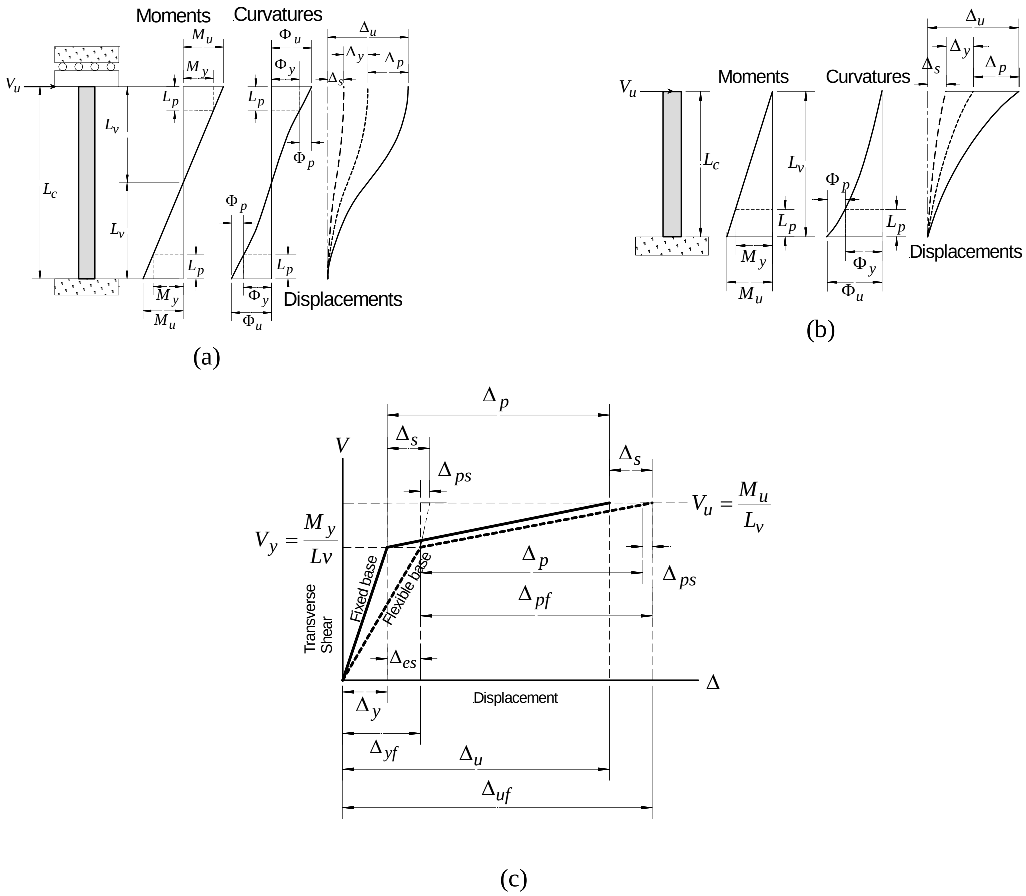



**Figure 1.** Moment, curvature and displacement of a column: (a) double curvature, (b) single curvature, and (c) effect of elastic flexibility

where $\Delta_y$ is the analytical column yield displacement in a rigid system and $\Delta_{es}$ is the elastic displacement due to system flexibility; $C_s$ is a system flexibility coefficient that accounts for the elastic flexibility of the structure (i.e. soil-structure interaction, beam-column connection, beam flexibility, footing-column connection, etc.), where typically $0.67 \le C_s \le 1.0$, $C_s = 1.0$ indicates a rigid support (i.e. $c_e = \Delta_{es} = \Delta_{ps} = 0$); $C_s = 0.67$ corresponds to $c_e = 0.50$.

### *2.1.1. Upgrading Indexes*

Upgrading of an existing column with inadequate ductile capacity $(\mu_{\Delta f})_{ex}$ may be required to achieve a desired level of performance during a seismic event. By selecting a given target ultimate displacement ductility $(\mu_\Delta)_{up}$, the designer can establish a target displacement upgrading index $I_{\Delta f}$ of the system defined as:

$$I_{\Delta f} = (\mu_{\Delta f})_{up} / (\mu_{\Delta f})_{ex} \tag{2.4}$$

The curvature ductility of the existing column $(\mu_\Phi)_{ex}$ can be found using $(\mu_{\Delta f})_{ex}$ of Eqn. (2.3) determined from a nonlinear section analysis of the existing column; the curvature ductility of the upgraded column $(\mu_\Phi)_{up}$ can be found by substituting the following displacement ductility $(\mu_\Delta)_{up}$ into Eqn. 2.1:

$$(\mu_\Delta)_{up} = 1 + I_{\Delta f}\left[(\mu_\Delta)_{ex} - 1\right] + \frac{(I_{\Delta f} - 1)}{C_s} \tag{2.5}$$

Thus, the target curvature ductility $(\mu_\Phi)_{up}$ can be obtained in terms of the selected target displacement upgrading index $I_{\Delta f}$ of Eqn. (2.4) and the displacement ductility of the as-built column $(\mu_\Delta)_{ex}$ of Eqn. (2.1). Upon selecting a target displacement upgrading index $I_{\Delta f}$, using Eqs. (2.1)-(2.5), assuming that plane sections remain plane, and considering that at yield $(\Phi_y)_{up} \cong (\Phi_y)_{ex}$, results in the following curvature upgrading index $I_\Phi$ :

$$I_\Phi = \frac{(\mu_\Phi)_{up}}{(\mu_\Phi)_{ex}} \cong \frac{(\Phi_u)_{up}}{(\Phi_u)_{ex}} \tag{2.6}$$

where $(\Phi_u)_m = (\varepsilon_{cu}/c_u)_m$ ; $(\varepsilon_{cu})_{ex}$ and $(c_u)_{ex}$ are the ultimate compressive strain and neutral axis depth of the existing column, respectively, determined from analysis of the existing column section; $(\varepsilon_{cu})_{up}$ and $(c_u)_{up}$ are the unknown target ultimate compressive strain and neutral axis depth of the FRP-upgraded column, respectively. The unknown target ultimate compressive strain $(\varepsilon_{cu})_{up}$ of the FRP-upgraded column can be conservatively found using:

$$(\varepsilon_{cu})_{up} = I_c I_\Phi (\varepsilon_{cu})_{ex} \;\; ; \;\; I_c = \frac{(c_u)_{up}}{(c_u)_{ex}} \approx 1.08 - 0.20(I_\Phi - 1)^{0.38} \leq 1.0 \tag{2.7}$$

where $I_c$ is the neutral axis upgrading index, a geometric parameter of the upgraded FRP-confined R/C column; determined from a parametric study using moment-curvature analysis of over 400 R/C columns of various cross sections. The index $I_c$ of Eqn. (2.7), shown as a dashed line in Fig. 2, represents a mean plus three standard deviations prediction of $I_c$ ; the solid line shown in this figure, is the best fit curve determined from regression analysis.

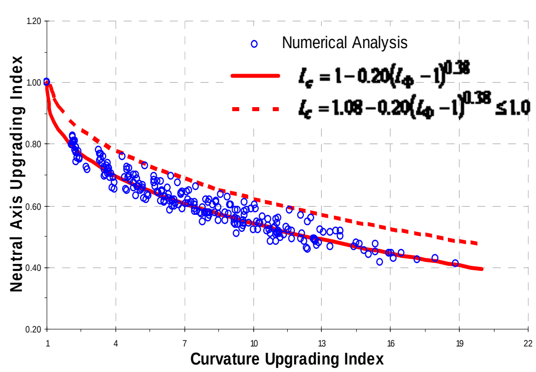



**Figure 2.** Neutral axis upgrading index versus curvature upgrading index

Upon establishing the target compressive strain $(\varepsilon_{cu})_{up}$ of Eqn. (2.7), an FRP jacket having a transverse modulus $E_j$ and an ultimate tensile coupon failure strain $\varepsilon_{fu}$ is selected by the designer. The actual rupture strain of the confining FRP jacket $(\varepsilon_{ju})_{up}$ typically occurs at much lower strains than $\varepsilon_{fu}$ due to stress-concentrations at the jacket-to-concrete interface from axial strain-induced damage (internal cracking, aggregate sliding or crushing, void compaction or nucleation) of the confined concrete, from triaxial stresses in the FRP jacket, and from stress concentrations at the rounded corners of rectangular FRP jacketed sections (Lam and Teng 2003; Eid et al. 2009). As a result, a design jacket strain of $(\varepsilon_{ju})_{up} = \kappa_{sh}\varepsilon_{fu}$ is recommended, where $\kappa_{sh}$ is a shape dependent strain reduction coefficient which determines the influence of the cross-section's shape on the premature failure of the FRP jacket for circular and rectangular jackets with rounded corners, as shown in Fig. 3:

$$\kappa_{sh} = (\varepsilon_{ju})_{up} / \varepsilon_{fu} = \left[\frac{1-2\alpha_R(1-\sin\theta_a)}{3(1-\alpha_R\alpha_{sh})\cos\theta_a}\right] \tag{2.8}$$

$$\alpha_{sh} = \frac{H_c}{B_c} = \tan\theta_d \;\; ; \;\; \alpha_R = \frac{R_j}{H_c} \;\; ; \;\; \theta_a = \theta_d - \sin^{-1}(\sin\theta_d - \cos\theta_d) \tag{2.9}$$

where $\alpha_{sh}$ and $\alpha_R$ are the section and jacket corner aspect ratio, respectively. For circular sections $\alpha_{sh} = 1.0$, $\theta_d = 45.0^\circ$ and $\alpha_R = 0.50$; $\theta_a$ is the diagonal jacket strain angle shown in Fig. 3; for circular sections $\kappa_{sh} = 2/3$. At high compressive strains, the FRP-confined concrete core exhibits significant transverse strains that result from dilation of the confined concrete. The rate of dilation of the confined concrete core is defined herein as the plastic dilation rate $\mu_{jp}$ of the FRP-confined concrete section (Moran 2011); this dilation rate is plotted in Fig. 4 versus the FRP-jacket stiffness $K_{je}$.

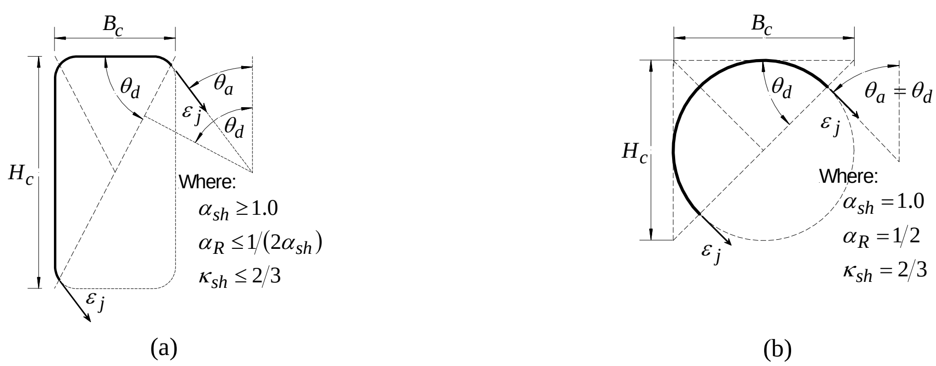



**Figure 3.** Typical geometry and definition of terms: (a) rectangular or square and (b) circular FRP jacketed sections

The plastic dilation rate and FRP jacket stiffness can be obtained as (Moran 2011):

$$\mu_{jp} = \left|\frac{\partial\varepsilon_j}{\partial\varepsilon_c}\right|_p \cong \left|\frac{\varepsilon_{ju}}{\varepsilon_{cu}}\right|_p = \nu_{ci} + \frac{\sqrt{2}-\nu_{ci}}{\left(1+\frac{K_{je}}{35}\right)^2} \;\; ; \;\; K_{je} = t_j\left(\frac{k_e C_{sh}}{H_c}\right)\left(\frac{E_j}{f'_{co}}\right) \tag{2.10}$$

$$C_{sh} = \frac{[(1+\alpha_{sh}) - (4-\pi)\alpha_R \alpha_{sh}]}{\left[1 - \alpha_{sh}(4-\pi)(\alpha_R)^2\right]} \; ; \; k_e = 1 - \frac{2\left[(1-2\alpha_{sh}\alpha_R)^2\right]}{3\left[1-\alpha_{sh}(4-\pi)(\alpha_R)^2\right]} \tag{2.11}$$

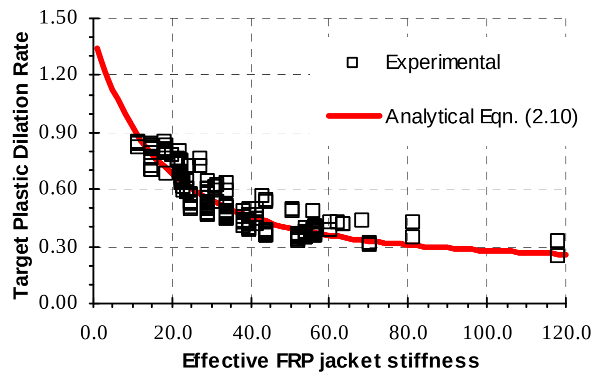



**Figure 4.** Plastic dilation rate versus FRP jacket stiffness of FRP-confined concrete sections

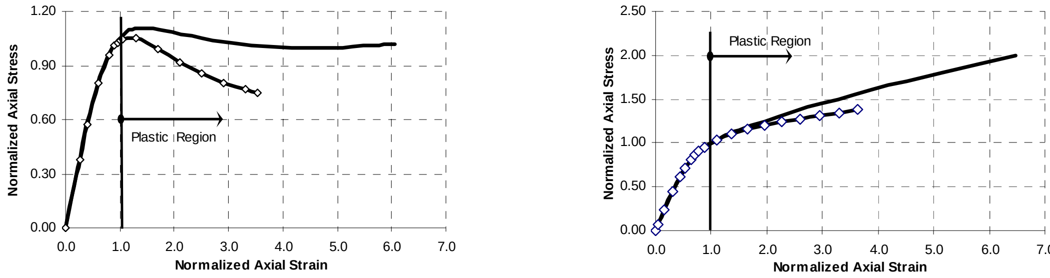
where $\nu_{ci}$ is the initial Poisson's ratio of concrete, where $0.15 \le \nu_{ci} < 0.25$, $\nu_{ci} = 0.20$ is recommended; $k_e$ is the shape-dependent confining efficiency of the FRP jacket; $C_{sh}$ is the FRP jacket reinforcement ratio coefficient, for circular sections $C_{sh} = 2.0$ and $k_e = 1.0$; $f_{co}^{'}$ is the unconfined concrete peak compressive strength. The plastic dilation rate $\mu_{jp}$ of Eqn. (2.10) indicates that the dilation behavior of FRP-confined concrete depends on the lateral kinematic restraint provided by the FRP jacket as measured by the FRP jacket stiffness $K_{je}$ of Eqn. (2.10), and the geometry of the FRP-confined concrete section $(\alpha_{sh}, \alpha_R)$.

*2.1.2. Control of Dilation and Strain Softening*

Using the plastic dilation rate of the upgraded column one can obtain the required FRP jacket stiffness from Eqn. (2.10). Setting $\mu_{jp} = (\mu_{jp})_{up} = (\varepsilon_{ju}/\varepsilon_{cu})_{up}$ and $K_{je} = (K_{je})_{up-dil}$ in Eqn. (2.10), and solving for $(K_{je})_{up-dil}$ gives the minimum FRP jacket stiffness required to control the transverse dilation behavior of the FRP-confined concrete core:

$$(K_{je})_{up-dil} = 35\left(\sqrt{\frac{\sqrt{2} - \nu_{ci}}{(\mu_{jp})_{up} - \nu_{ci}}} - 1\right) \tag{2.12}$$

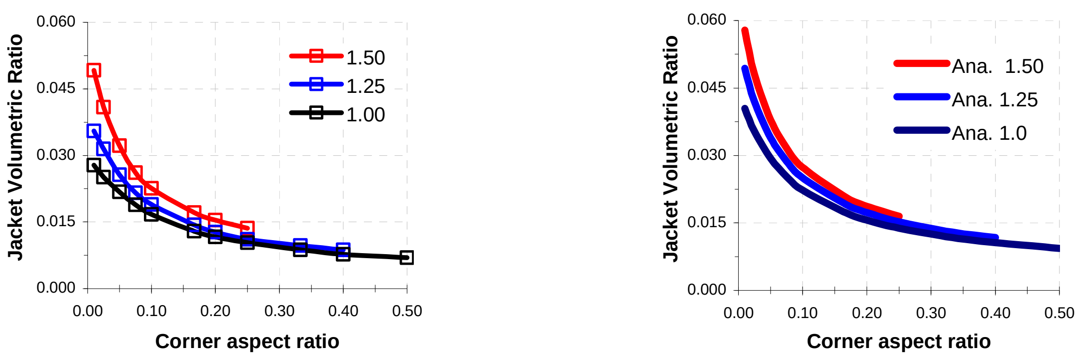
Analysis of the dilation behavior of FRP-confined concrete sections suggests that at very low jacket stiffness, the FRP jacket is not effective in providing adequate lateral restraint against unstable crack growth. The effectiveness of the FRP jacket in curtailing this unstable crack growth increases as the jacket stiffness increases, due to an increase in the lateral restraint provided by the jacket. As a result, concrete sections confined by low stiffness FRP jackets can exhibit an undesirable strain-softening behavior [Fig. 5(a)], since the FRP jacket can experience premature failure due to rupture; the confined concrete core may also exhibit significant bulging due to uncontrolled dilation which can result in premature buckling of the vertical column reinforcement. In additions, columns confined by low stiffness FRP jackets tend to exhibit a small increase in strain energy (area under the stress-strain curve), and a small increase in curvature and displacement ductility of the R/C column, when compared to well confined columns which exhibit strain-hardening compressive behavior [Fig 5(b)], whose strain energy, axial strain, curvature ductility and displacement ductility increase proportionally with an increase in FRP jacket stiffness.




(a) (b)

**Figure 5.** Normalized axial stress versus normalized axial strain curves of sections: (a) strain-softening and (b) strain-hardening compressive behavior

Suppression of strain-softening compressive behavior in FRP-confined concrete within the plastic hinge region is warranted. A parametric study was performed to determine the optimal FRP jacket reinforcement ratio $(\rho_j)_{SH}$ required to preclude strain-softening compressive behavior in rectangular, square, and circular FRP-confined concrete sections. The variables used were: $f_{co}^{'}$, $E_j$, $t_j$, $B_c$, $\alpha_R$, and $\alpha_{sh}$. The results are given in Fig. 6(a), in which the optimal FRP jacket reinforcement ratio is plotted versus the jacket corner aspect ratio $\alpha_R$ of FRP jacketed sections with $\alpha_{sh} = 1.0,\ 1.25, \text{and}\ 1.50$.




(a) (b)

**Figure 6.** Strain-hardening jacket volumetric ratio versus jacket corner aspect ratio of rectangular FRP-confined concrete sections with section aspect ratios of 1.50, 1.25 and 1.00: (a) parametric and (b) analytical

As can be observed in Fig. 6(a), $(\rho_j)_{SH}$ depends on the geometry of the confined concrete core $(\alpha_{sh}, \alpha_R)$, the FRP jacket transverse modulus $E_j$ and the unconfined concrete compressive strength $f_{co}^{'}$. A conservative estimate of $(\rho_j)_{SH}$ can be found using the following expression:

$$\left(\rho_j\right)_{SH} = \left(\frac{2 f_{co}^{'}}{k_e E_j}\right) K_{je} = \lambda_{SH} \sqrt[3]{f_{co}^{'}} \left(\frac{f_{co}^{'}}{E_j}\right) \; ; \; \lambda_{SH} = 2\alpha_{sh}\left[\frac{1+12(1+\alpha_{sh}\alpha_R)}{1+12\alpha_{sh}\alpha_R}\right] \tag{2.13}$$

where $\lambda_{SH}$ is an FRP shape-dependent strain-hardening coefficient. The value of $\left(\rho_j\right)_{SH}$ from Eqn. (2.13) is plotted in Fig. 6(b). Solving for $K_{je}$ in Eqn. (2.13) and setting $\left(K_{je}\right)_{up-SH} = 2\,K_{je}$ [to account for strain gradient effects (Binici and Mosalam 2007), for detrimental effects of excess pore-water (Imran 1994; Moran 2011), and for stress-concentrations that can occur at vertical and transverse column reinforcement locations (Tastani et al. 2006; Karabinis et al. 2007)] yields:

$$\left(K_{je}\right)_{up-SH} = k_e \lambda_{SH} \sqrt[3]{f_{co}^{'}} \tag{2.14}$$

The FRP-jacket stiffness of the upgrading FRP jacket $\left(K_{je}\right)_{up}$ required to prevent both uncontrolled dilation and strain-softening behavior in the FRP jacketed plastic hinge region of an R/C column is the maximum value found using Eqs. 2.12 and 2.14. Using Eqn. (2.10) and solving for the minimum thickness of the upgrading FRP jacket $\left(t_j\right)_{up}$ yields:

$$\left(t_j\right)_{up} = \left(K_{je}\right)_{up}\left(\frac{H_c}{k_e C_{sh}}\right)\left(\frac{f_{co}^{'}}{E_j}\right) \tag{2.15}$$

## 3. DISPLACEMENT-BASED DESIGN PROCEDURE

A flow chart of the proposed displacement-based design procedure for the plastic hinge confinement of RC columns is provided in Figure 8.

## 4. DESIGN EXAMPLES

Using the design procedure outlined in Fig. 8, two physical examples from the literature, a circular and rectangular column tested by Seible et al. (1997) are used to compare the proposed method with experimental results.

### 4.1. Circular FRP-Upgraded Column

For the as-built and FRP upgraded circular cantilevered concrete column tested by Seible et al. (1997) the following is found. For the as-built column $\Phi_y = 5.69x10^{-6}$, $\bar{M}_{ex} \approx 1.05$, $C_s = 0.913$, $C_e = 0.10$, $C_\phi = 1.0$, $\lambda_p = 0.123$, $\left(\Phi_u\right)_{ex} = 2.65x10^{-5}$, $\left(C_u\right)_{ex} = 198.0$ mm, $\left(\varepsilon_{cu}\right)_{ex} = 0.00524$, $\left(\mu_{\Delta f}\right)_{ex} \approx 2.2$, $\left(\mu_\Delta\right)_{ex} \approx 2.31$; and $\left(\mu_\Phi\right)_{ex} = 4.65$. For the upgraded column, $\bar{M}_{up} = 1.25$, setting $\left(\mu_{\Delta f}\right)_{up} = 8.0$, $I_{\Delta f} \approx 3.64$, $\left(\mu_\Delta\right)_{up} = 8.67$, $\left(\mu_\Phi\right)_{up} = 22.39$, $I_\Phi = 4.82$, $I_c = 0.747 \le 1.0$, $\left(\varepsilon_{cu}\right)_{up} = 0.0189$, $\left(\varepsilon_{ju}\right)_{up} = 0.00667$, $\left(\mu_{jp}\right)_{up} = 0.353$, $\left(K_{je}\right)_{up} = \max(63.45, 16.29) = 63.45$, $\left(t_j\right)_{up} = 5.37$ mm. This thickness is approximately 5.3 % larger than the 5.10 mm jacket used in the cantilever column test by Seible et al. (1997), which performed to a displacement ductility of approximately $\mu_{\Delta f} \approx 10.0$ that is greater than the target value of $\left(\mu_{\Delta f}\right)_{up} = 8.0$.

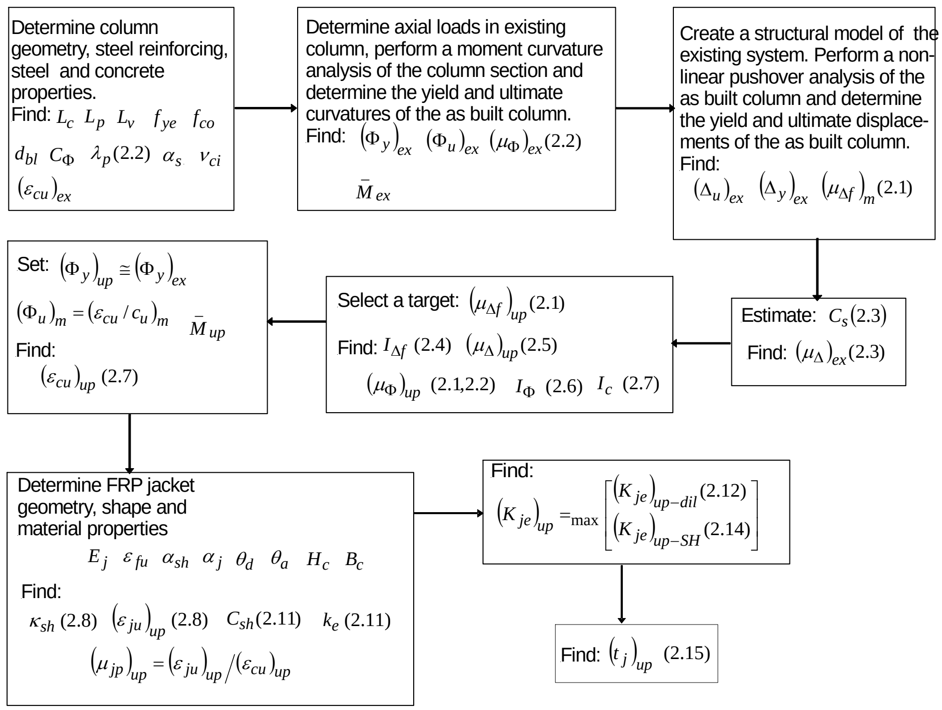



**Figure 8.** Flow chart of proposed displacement-based design procedure

### 4.2. Rectangular FRP-Upgraded Column

For the as-built and FRP upgraded rectangular concrete column tested by Seible et al. (1997) the following is found. For the as-built column $\Phi_y = 3.99x10^{-6}$, $\bar{M}_{ex} \approx 1.05$, $C_s = 0.91$, $C_e = 0.10$, $C_\phi = 1.0$, $\lambda_p = 0.136$ , $(\Phi_u)_{ex} = 2.66x10^{-5}$, $(C_u)_{ex} = 182.9$ mm, $(\varepsilon_{cu})_{ex} = 0.00487$, $(\mu_{\Delta f})_{ex} \approx 3.0$, $(\mu_\Delta)_{ex} \approx 3.20$, and $(\mu_\Phi)_{ex} = 6.68$. For the upgraded column $\bar{M}_{up} = 1.25$, setting $(\mu_{\Delta f})_{up} = 8.0$, $I_{\Delta f} \approx 2.67$, $(\mu_\Delta)_{up} = 8.70$, $(\mu_\Phi)_{up} = 20.67$, $I_\Phi = 3.08$, $I_c = 0.815 \le 1.0$, $(\varepsilon_{cu})_{up} = 0.0122$, $(\varepsilon_{ju})_{up} = 0.00449$, $\kappa_{sh} = 0.449$, $(\mu_{jp})_{up} = 0.366$, $(K_{je})_{up} = \max(59.67,34.11) = 59.67$, $C_{sh} = 2.45$, $k_e = 0.454$, and $(t_j)_{up} = 10.88$. This thickness is approximately 6.6 % larger than the 10.20 mm jacket used in the cantilever column test by Seible et al. (1997), which performed to a displacement ductility of approximately $\mu_{\Delta f} \approx 8.0$; this is very close to the target value of $(\mu_{\Delta f})_{up} = 8.0$.

## 5. CONCLUSIONS

The design method presented in this paper is based on the strain ductility increase that results from the constant lateral kinematic restraint provided by the confining elastic FRP jacket, and is thus a strain-based approach using performance-based design principles. In the analytical design procedure, no consideration is given to the increase in axial compressive strength of FRP-confined concrete, since this increase is a secondary effect that results from axial strain induced dilation of the FRP-confined concrete core and the resultant transverse confining stresses provided by the elastic FRP jacket as transverse dilation progresses. Columns in structural systems with elastic flexibility are also considered.

The information required to determine the minimum FRP jacket thickness within the plastic hinge region of a reinforced concrete column is: (1) the target displacement ductility, (2) the geometry of the concrete section or FRP jacket; (3) the unconfined concrete core compressive strength, (4) the longitudinal reinforcement area and yield strength, (5) the column axial load, (6) the material properties of the FRP jacket; and (7) the ultimate design FRP jacket strain determined based on the FRP material and geometry of the FRP jacket.

The design procedure presented in this paper compares favorably with experimental results for columns in single curvature from the literature that were upgraded with FRP jackets and had demonstrated a substantial displacement ductility increase. In addition, the design procedure provides the minimum FRP jacket thickness required to prevent both uncontrolled transverse dilation and strain-softening in the plastic region. The displacement- based design procedure presented in this paper can also be used with existing axial stress-strain models for FRP confined concrete.

## ACKNOWLEDGMENTS

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