# Magnitude-Recurrence Relationship and its Effect on Uniform Hazard Spectra: A Current Assessment

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**SUMMARY:**
Gutenberg and Richter showed that magnitude-recurrence relationship may be represented by a linear relationship when the log of annual rate of exceedance was plotted against magnitude. This type of recurrence model has been in use because of its simplicity and because it fits the data reasonably well over a useful range of magnitudes of engineering interest. A quadratic log frequency relationship between magnitude and the *mean annual rate* has been proposed by others and shown to fit the available data well in the high magnitude range of the magnitude-recurrence relationship. UHS (Uniform Hazard Spectra) using a quadratic log frequency and the linear relationship, has been developed and compared. It is shown that using a linear relationship would result in a more conservative design when defining UHS for ductility level earthquakes (DLE) (return periods > 5,000 years). However, UHS for strength level earthquakes (SLE) (return periods <1000 years) are not affected greatly by the use of a linear relationship.

*Keywords: U.H.S., magnitude, recurrence, quadratic, relationship*

## 1. INTRODUCTION

How often do earthquakes occur? Does some kind of pattern exist in the time scale at which earthquakes are known to occur? In a seismologically active region earthquakes occur at irregular intervals of time. It is fairly obvious that in order to extract a meaningful pattern, the length of the record must be reasonably large. The longer the record the better it is. Historical records are dated and hence it is possible for the seismologists to analyse and assess the recurrence relationship.

Gutenberg and Richter (1954) introduced the magnitude-recurrence relationship gathering data from earthquakes in the southern California region. The data (spread over a certain length of time) was organised in a manner to reflect the number of earthquakes that exceeded a certain magnitude. Also from the organised data, the *mean annual rate of being exceeded,* $\lambda_M$ , of an earthquake magnitude, $M$ , was defined as the occurrences greater than $M$ divided by the length of the time period. Or in other words the average rate at which an earthquake of some size will be exceeded. The reciprocal of the mean annual rate of being exceeded was referred to as the *mean return period* of the earthquake. It was found that the logarithm of the annual rate of exceedance of southern California earthquakes, plotted against earthquake magnitude, resulted in a linear relationship. They proposed a relationship of the form: $\log \lambda_m = a - bm$ . Where $a$ and $b$ are field constants for a particular region.

The validity of Gutenberg-Richter relationship has been questioned by various investigators (Schwartz and Coppersmith, 1984, Youngs and Coppersmith, 1985).

Because of its simplicity and as it fits the observed earthquake data reasonably well over a useful range of engineering interest, this model has been found to be convenient and is in general use today.

### 1.1 Background

Use of Quadratic Form:

One of the concerns regarding the use of a linear relationship, if un-truncated, is that it normally over-estimates the occurrence of large events and because of the scattering of data associated at the upper bound range, it does not reflect the true state.

Shlien and Toksoz (1970) have shown that, at least for magnitudes for which reliable data are available (USCGS, Duda, 1965), a quadratic log frequency relationship fits well.

Merz and Cornell (1973) introduced a more general case of a quadratic magnitude-frequency relationship with a finite upper-bound, $m_u$.

$$\log_{10} \lambda_m = \left\{ a + b_1^{'}(m - m_0) + b_2^{'}(m - m_u) \right\} \tag{1.1}$$

And the cumulative distribution function as:

$$F_M(m) = P\left[ M \le m \right] = k_{m1}^{*}[1 - \exp\{\beta_1(m - m_0) + \beta_2(m^2 - m_0^2)\}] \tag{1.2}$$

[Both $M$ and $m$ are used as symbols for magnitudes. In general, the notation adopted is that the upper case is used when variable referred to is a random variable; the lower case is used when referring to observed values of that variable].

Where

$$k_{m1}^{*} = 1 - \exp\{\beta_1(m_u - m_0) + \beta_2(m_u^2 - m_0^2)\}$$
$$\beta_1 = b_1^{'} \ln 10$$
$$\beta_2 = b_2^{'} \ln 10$$

### 1.2 Outline of the Paper

1. The quadratic form for magnitude recurrence relationship is investigated next. The quadratic form adopted is compared with the database of USCGS catalogue and those obtained by Duda and Gutenberg (1964) over a period from 1918-1968 (see Fig. 1).

2. On the basis of the quadratic relationship adopted (step 1) uniform hazard spectra for return periods for SLE (500 and 1000 years) and DLE (10,000 years) are derived next.

3. UHS based on a linear relationship which is a close approximation of the dataset, except in the high magnitude range (see Fig. 1) are derived next.

4. The numerical engine for developing the UHS is the Monte Carlo simulation technique (Sen, 2006, 2009) and is briefly outlined here.

5. In the concluding part, the effects of the quadratic form on the UHS, which is the main objective of this paper, are discussed and conclusions drawn.

## 2. QUADRATIC MAGNITUDE-RECURRENCE RELATIONSHIP

Referring to equation (1.1), the more general case of a quadratic magnitude-frequency relationship with a finite lower bound $m_0$ and an upper-bound $m_u$ (proposed by Merz and Cornell, 1973) is used here and expressed as:

$$\log_{10} \lambda_m = 3.295 - 0.20(m - m_0) - 0.298(m - m_u)^2 \quad (1.3)$$

The quadratic form adopted is compared with the database of USCGS catalogue and those obtained by Duda and Gutenberg (1964) over a period from 1918-1968 and is shown in Fig.1.
[The constants have been modified to suit the USCGS and Duda (1965) data – as presented by Shlien and Toksoz (1970)].

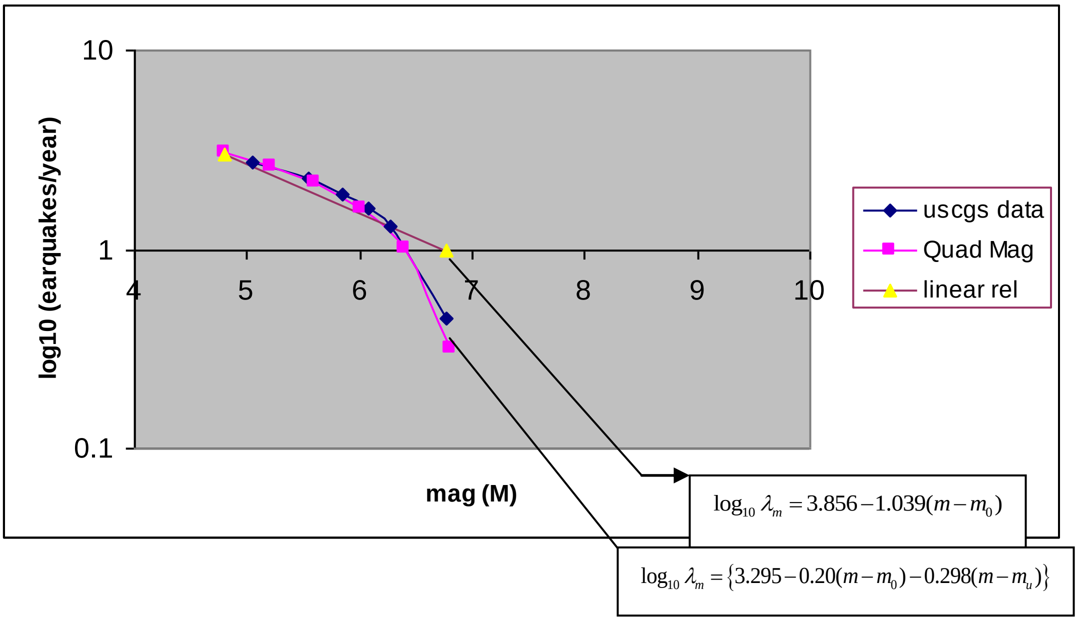


**Figure 1.** Plot of USCGS Data Points and the Quadratic Recurrence Relationship Adopted

## 3. CUMULATIVE DISTRIBUTION FUNCTION (CDF)

CDF obtained from equation (1.2) with $m_0$ =4 and $m_u$ = 8 is shown in Fig.2 (Q is the probability of M being exceeded).

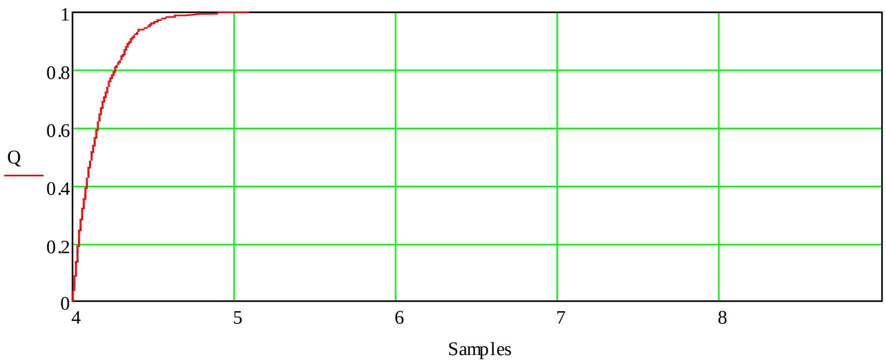


**Figure 2.** Cumulative Distribution Function for Quad-Mag Recurrence Relation

### 3.1. General Scheme – Monte Carlo Simulation

Monte Carlo simulation is an established technique to solve probabilistic models. The probabilistic model outlined by Cornell (1968) may be solved by the Monte Carlo simulation process. The numerical approach is different. It mirrors real life events as may be evinced from the computational scheme shown in the flow chart (see Fig.3).

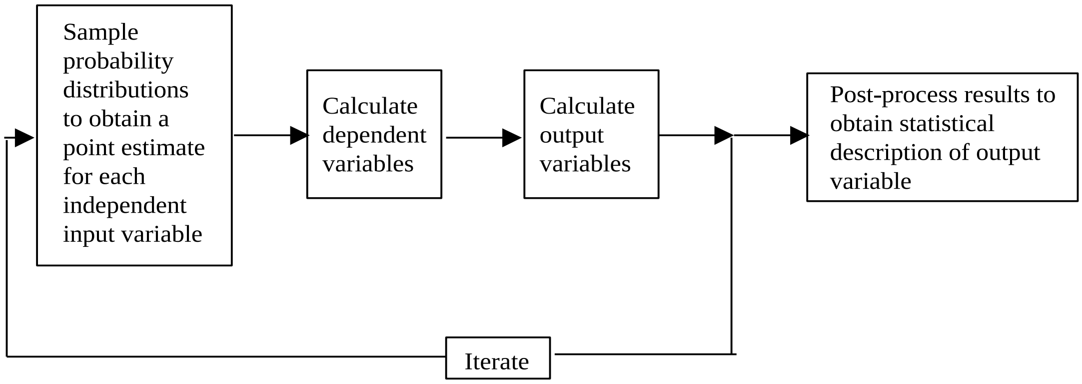


**Figure 3.** General Scheme of Monte Carlo Simulation

All computations for generating plots of exceedance of acceleration against return periods, were carried out with the mathematical software *Mathcad 14* . The software has a random number generator incorporated within which is used to obtain a point estimate for each independent input variable.

### 3.2 Preliminary Steps

The preliminary steps are as follows:

1. Fault Line

As an example the following were considered:

The fault line is 650 km long and earthquakes can occur anywhere along the fault line. Thus, uniform random distribution may be assumed. The following numerical results are obtained for a site located a minimum surface distance, $\Delta$, of 40 km from a line source of earthquakes at a depth of 20 km. In the above intensity relationship $R$ is related to the position where the earthquake originates which happens to be randomly distributed.

2. Relationship Used

Before we consider constructing the uniform hazard spectra for a site, we must know the probability of peak ground motion (peak acceleration, velocity etc) being exceeded. Cornell's probabilistic methodology outlined above can be applied on any functional relationship between site ground motion variable $Y$ (peak acceleration, velocity and displacement), and $M$ and $R$.

Though not quite appropriate this would nevertheless be adequate for illustrating the effects of the quadratic form on the UHS, which is the main objective of this paper. The functional relationship provided by McGuire (1974) working on data from sites in Western USA, is shown below:

$$S_a = b_1^{'} 10^{b_2 M} (R+25)^{-b_3} \quad \text{cm/sec}^2 \tag{1.4}$$

or

$$\log S_a = b_1 + b_2 M - b_3 \log(R+25) \tag{1.5}$$

The constants for the above equation are shown in Table 3.

$$S_a = b_1' 10^{b_2 M} (R + 25)^{-b_3}$$
$$\log S_a = b_1 + b_2 M - b_3 \log(R + 25)$$

(with permission, reproduced from Dowrick, 1987)

**Table 3.** McGuire's Attenuation Expressions for Spectral Acceleration with 5% Damping (1974)

| Period (s) | $b_1'$ | $b_1$ | $b_2$ | $b_3$ | Coeff. of Var. of $S_a$ |
|---|---|---|---|---|---|
| 0.1 | 1610 | 3.173 | 0.233 | 1.341 | 0.651 |
| 0.2 | 2510 | 3.373 | 0.226 | 1.323 | 0.577 |
| 0.3 | 1478 | 3.144 | 0.290 | 1.416 | 0.560 |
| 0.5 | 183.2 | 2.234 | 0.356 | 1.197 | 0.591 |
| 1.0 | 6.894 | 0.801 | 0.399 | 0.704 | 0.703 |
| 2.0 | 0.974 | -0.071 | 0.466 | 0.675 | 0.941 |
| 3.0 | 0.497 | -0.370 | 0.485 | 0.709 | 1.007 |
| 4.0 | 0.291 | -0.620 | 0.520 | 0.788 | 1.191 |

### 3.3 Monte Carlo Simulation Plots for Peak Ground Acceleration

We can apply the Monte Carlo process to derive the plot for peak ground acceleration vs. return period, $R$, (or probability of being exceeded). The ground acceleration exceedance curve for period = 0.2 sec is shown in Fig. 4.

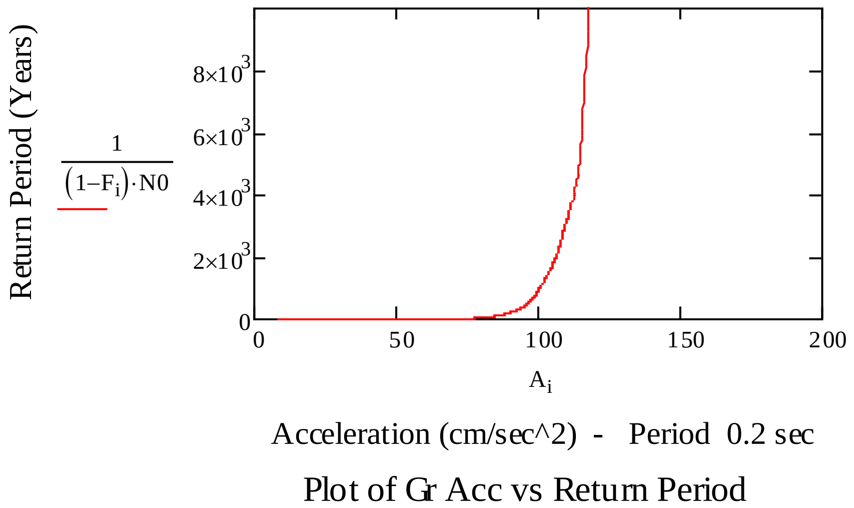


**Figure 4.** Ground Acceleration Exceedance Curve for Period = 0.2 sec

## 3.4 Uniform Hazard Response Spectrum (UHRS)

### *3.4.1 Quadratic-Magnitude Recurrence Relationship*

Step 1
Calculation of peak ground accelerations for periods 0.1 – 4.0 sec (as shown in Table 3) is carried out first (plots not shown here) by modifying the *Mathcad* , V14 sheets.

Step 2
The concept and methodology is shown in Fig. 5.

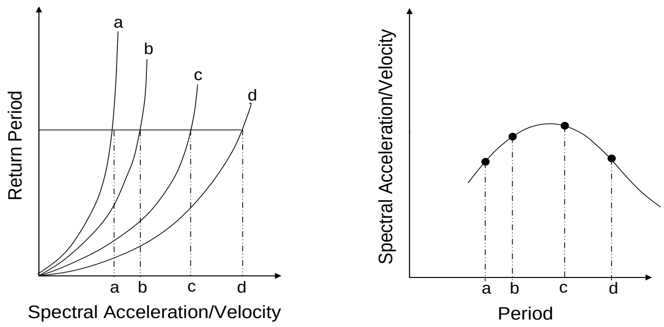


**Figure 5.** Construction of UHRS

Step 3
Finally, we follow the plan outlined in Fig. 5 for constructing the UHRS. The plot for UHRS for a return period of 500, 1000 and 10,000 years is shown in Fig. 6.

### *3.4.2 UHRS - Linear Relationship*

The linear relationship adopted for comparison with the quadratic relationship is shown in Fig.1. The relationship is of the form: $\log_{10} \lambda_m = 3.856 - 1.039(m - m_0)$ .

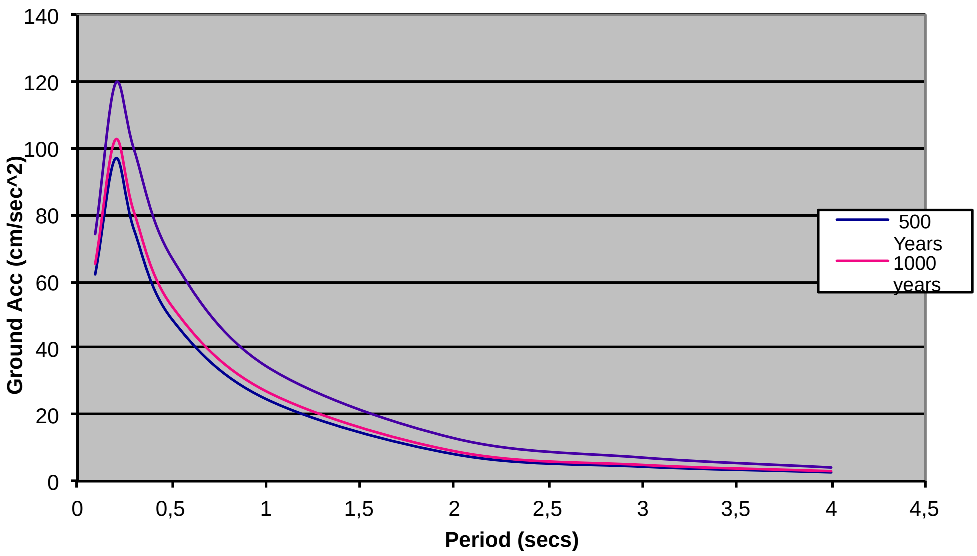


**Figure 6.** Plot of UHRS (Return Period: 500, 1000 and 10,000 years)

## 4. CONCLUDING REMARKS

Comparison of the UHS developed with the quadratic and the linear relationship is shown in Figs. 7, 8 and 9.

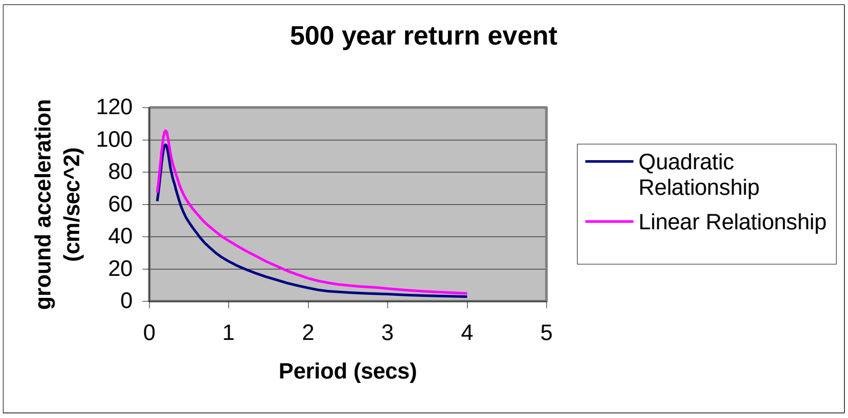


**Figure 7.** 500 year Return Event

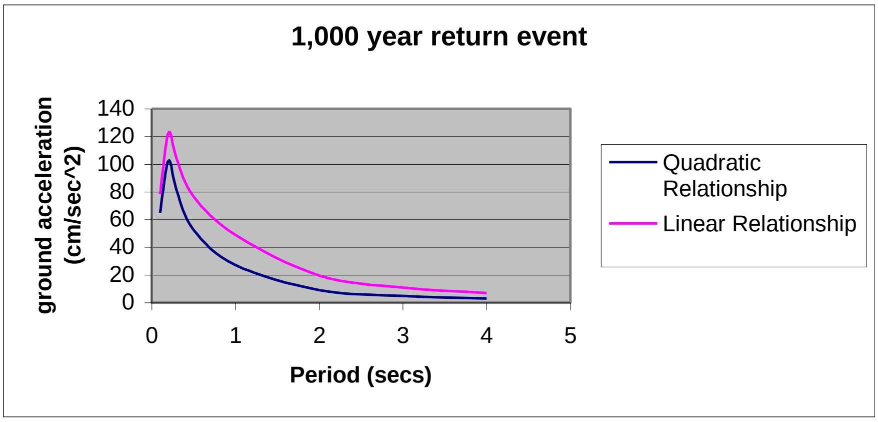


**Figure 8.** 1,000 year Return Event

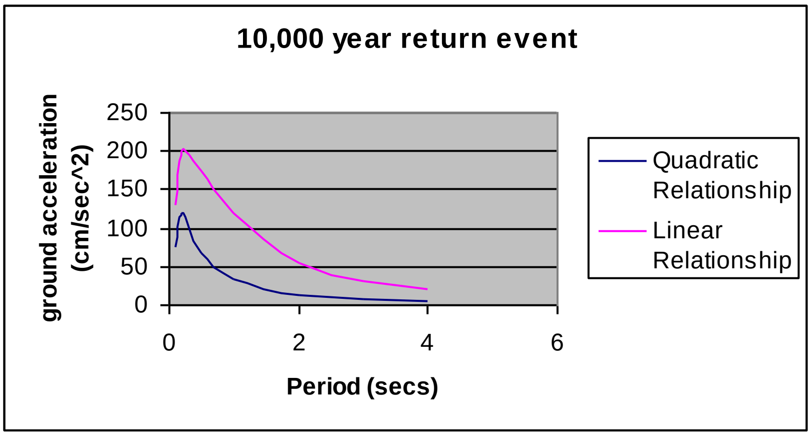


**Figure 9.** 10,000 year Return Event

The quadratic relationship predicts fewer events of high magnitudes and as expected, Fig. 9 shows the difference in using a quadratic instead of a linear magnitude-frequency relationship and is significant for the 10,000 year return event, as ground accelerations for this low-risk event are higher.

In the UHRS for the 500 and 1,000 year return period, where ground accelerations expected are much lower, and there is no appreciable difference in UHS from an engineering perspective (see Figs. 7 and 8).

Merz and Cornell (1973) had arrived at a similar conclusion when risks associated with high ground accelerations were compared with the two magnitude-frequency recurrence relationship.

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## REFERENCES

Cornell, C.A. (1968), 'Engineering Seismic Risk Analysis', *Bull. Seismol. Soc. Amer., 58, 1583-1606.*

Dowrick, D.J., (1987), *'Earthquake Resistant Design'*, John Wiley & Sons, Chichester.

Duda, S.J. (1965), 'Secular Seismic energy Release in the Circum-Pacific belt', *Tectonophysics*, 2(5), 409-452.

Gutenberg, B. and Richter, C.F. (1954), '*Seismicity of the Earth'*, Princeton University Press.

McGuire, R.K. (1974), 'Seismic structural response risk analysis incorporating peak response regressions on earthquake magnitude and distance', Res. Report R74-51, Dept of Civil Engineering, Massachusetts Institute of Technology.

Merz, H.A. and Cornell, C.A. (1973), 'Seismic Risk Analysis Based on a Quadratic Magnitude-Frequency Law', *Bull. Seismol. Soc. Amer.*, Vol 63, No. 6, 1999-2006.

Schwartz, D. P. and Coppersmith, K. J. (1984), 'Fault behaviour and characteristic earthquakes:examples from the Wasutch and San Andreas fault zones' *J. Geophysical Research*, Vol 89, No B7, 5681-5698. .

Sen, T.K., (2006), 'Construction of Uniform Hazard Response Spectra Using Monte Carlo Simulation', Proc. First European Conf. Earthquake Engineering and Seismology, Geneva, Switzerland, 3-8 Sept. 2006.

Sen, T.K., (2009), *'Fundamentals of Seismic Loading on Structures'*, John Wiley & Sons, Chichester, UK.

Shlien, S. and Toksoz, M.N. (1970), 'Frequency-Magnitude Statistics of Earthquake Occurrences', *Earthquake Notes*, Vol. XLI, No. 1.

Youngs, R. R. and Coppersmith, K. J. (1985), 'Implications of fault slip rates and earthquake recurrence models to probabilistic seismic hazard estimates'. *Bull. Seismol. Soc. Amer.,* Vol. 75, 939-964.