Empirical Approach Using ANN for Determining Confined Compressive Strength in FRP-Strengthened Concrete Members

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SUMMARY
In the present study, a new empirical approach to obtain the confined compressive strength is developed using available experimental data by applying artificial neural networks (ANNs). With known combinations of input and output data, the neural network can be trained to extract the underlying characteristics and relationships from the data. Then, when a separate set of input data is fed to the trained network, it will produce an approximate but reasonable output. Neural networks are highly nonlinear and can capture complex interactions among input/output variables in a system without any prior knowledge about the nature of these interactions. Having parameters used as input nodes in ANN modeling such as diameter of column, concrete cover, volumetric ratio of longitudinal, lateral steel bars and also FRP and compressive strength of concrete, the target or output node was ultimate confined compressive strength. The transfer functions were assumed to be Log-sigmoid and pure-linear for hidden layer. The new approach was compared with existing empirical and experimental data and also with formulas available in concrete codes such as ACI440.2R-08. Finally the applicability of the new empirical approach to the failure prediction of strengthened members is also investigated.

Keywords: ANN, FRP, Strengthening

1. GENERAL

Bonding FRP sheets externally to strengthen RC structures has become a popular technology in the past decade. With the rapid development of this new technology, many issues related to the structural performances of FRP strengthened RC elements have been investigated. External confinement of concrete using FRPs has become a common method of column retrofitting, especially for circular columns and many recent studies have been conducted on the compressive strength of FRP-confined concrete and various models have been developed (Nanni and Bradford 1995; Karbhari and Gao 1997; Mirmiran et al. 1998; Miyauchi et al. 1999; Saafi et al. 1999; Rochette and Labossiere 2000; Xiao and Wu 2000; Matthys et al. 2005; Lam et al. 2006; Teng et al. 2007; Lee and Hegemier 2009). Using a limited test data is one of the weaknesses of existing models in which further applicability of their approaches could not be guaranteed.

In recent years, artificial neural networks have been of interest to researchers in the modelling of various civil engineering systems. The FRP-confined concrete is affected by unknown multivariable interrelationships and the existing experimental data are noisy; consequently, the models derived by regression analysis are not able to predict the behaviour well.

Artificial neural networks automatically manage the relationships between variables and adapt based on the data used for their training. So it is important to collect a large number of experimental data. In this study, a large test database built from an extensive survey of existing tests on FRP-confined circular concrete specimens is carefully examined to establish the effect of various variables. Finally, a new model is proposed based on artificial neural networks and then verified against experimental data and existing models.

2. AVAILABLE MODELS

Many researchers investigated specifically the FRP-confined concrete and consequently a considerable number of models developed. All of the proposed models were developed empirically by either doing regression analysis using existing test data or by a development based on the theory of plasticity with four or five parameters to be determined using available experimental data. The existing models can be classified into three major categories including linear, second-order and nonlinear models. Table 1 presents some important existing empirical models to predict the compressive strength of FRP-confined concrete since 1981.

Table 1. Important strength models for FRP-confined concrete

Author(s)	Year	Equation	Order
Toutanji and Matthys	2005	$f'_{cc}/f'_c = 1 + 2.3(f_l/f'_c)^{0.85}$	Nonlinear
Lam and Teng	2002	$f'_{cc}/f'_c = 1 + 2(f_l/f'_c)$	Linear
Saafi et al.	1999	$f'_{cc}/f'_c = 1 + 2.2(f_l/f'_c)^{0.84}$	Nonlinear
Miyauchi et al.	1999	$f'_{cc}/f'_c = 1 + 2.98(f_l/f'_c)$	Linear
Monti	1999	$f'_{cc}/f'_c = 0.2 + 3(f_l/f'_c)^{0.5}$	Second order
Karbhari and Ghao	1997	$f'_{cc}/f'_c = 1 + 2.1(f_l/f'_c)^{0.87}$	Nonlinear
Mander et al.	1988	$f'_{cc}/f'_c = -1.25 - 2(f_l/f'_c) + 2.25(1 + 7.94 f_l/f'_c)^{0.5}$	Second order

3. ANN MODELS

As the first step for providing sufficient information for training, verifying and testing of neural networks, a comprehensive set of test results on the axial compressive strength of FRP-confined circular concrete specimens was collected. All together, the selected database contains 213 test results including significant test programs of three recent decades (Fardis and Khalili 1981; Karbhari et al. 1993; Demers and Neale 1994; Howie and Karbhari 1994; Harmon et al. 1995; Nanni and Bradford 1995; Picher et al. 1996; Soudki and Green 1996; Karbhari and Gao 1997; Miyauchi et al. 1997; Watanabe et al. 1997; Harries et al. 1998; Mirmiran et al. 1998; Toutanji and Balaguru 1998; Matthys et al. 1999; Purba and Mufti 1999; Saafi et al. 1999; Toutanji 1999; Liu et al. 2000; Rochette and Labossiere 2000; Shahawy et al. 2000; Lam et al. 2006; Jiang and Teng 2007; and Teng et al. 2007). It is worth to mention that FRP rupture has been the failure mode for all the specimens of the test programs used in this research. The parameters used as the input nodes in the ANN modelling were the following:

- Diameter of the circular concrete specimen (*d*) in mm.
- Height of the circular concrete specimen (*L*) in mm.
- The total thickness of FRP (*t*) in mm.
- The tensile strength of the FRP in the hoop direction (f_{frp}) in MPa.
- The compressive strength of the unconfined concrete (f'_c) in MPa.
- The elastic modulus of FRP (E_{frp}) in MPa.

Having the six input nodes as described above, the target node was the compressive strength of the confined concrete (f'_{cc}). One hidden layer was used in this ANN modelling, where the transfer functions were tan-sigmoid. Before training the selected data, normalization/scaling for the whole data were made. This was done since log-sigmoid transfer function was used in the network which recognizes values between 0 and 1. In order to scale the data from 0.1 to 0.9, minimum and maximum values were taken to use linear relationship between those values. Tables 2 presents the statistical properties of collected data.

Table 2. Statistical Properties of Experimental Data

Input Nodes		d (mm)	L (mm)	f'_c (MPa)	t (mm)	f_{frp} (MPa)	E_{frp} (MPa)	f'_{cc} (MPa)
Whole Data	Mean	145.65	303.17	35.63	1.19	1537.83	114530.40	70.31
	Minimum	51.00	102.00	18.00	0.09	167.00	10500.00	30.80
	Maximum	200.00	788.00	64.20	5.31	3720.00	629600.00	241.00
	Standard Deviation	29.07	79.64	7.58	1.11	1231.30	113012.80	24.80
	Coefficient of Variation	0.20	0.26	0.21	0.93	0.80	0.99	0.35

The criterion for stopping the training of the networks was Mean Square Error (MSE) which is the average squared difference between outputs and targets. Lower values mean better performance of the network (Zero means no error). Regression values (R-values) measure the correlation between outputs and targets in the networks; An R-value of 1 means a close relationship and in contrast, 0 means a random relationship. These two criteria (MSE and R-values) were considered as the basis for selecting the idealised network. The regression values of the networks with different number of hidden nodes are presented in Fig. 1. Another filtering in the pre-elimination of networks can be seen in Fig. 2 where the maximum absolute error for each network was noted. It can be seen that all the networks were trained well, but some of them resulted large values of Mean Square Error (MSE).

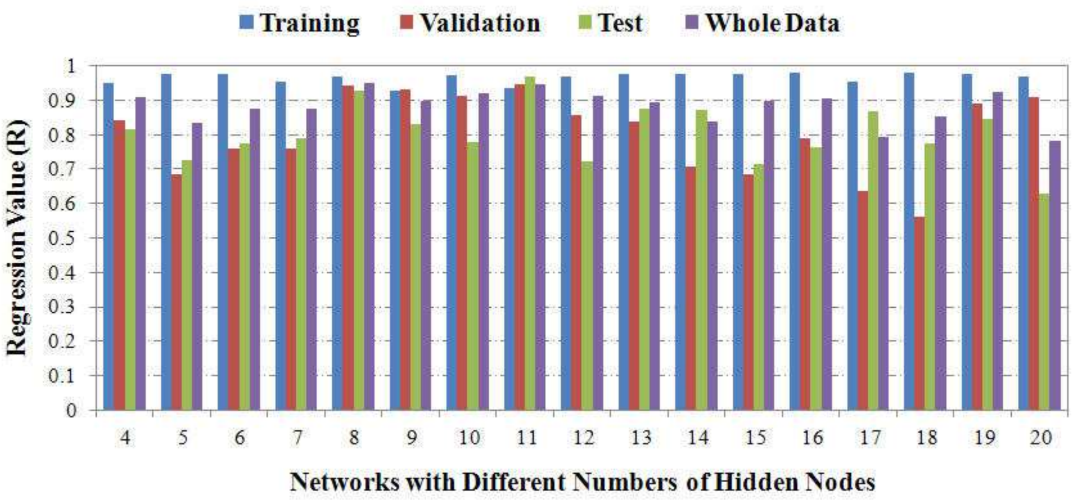

Fig. 1. Correlation Coefficient versus NN 6-n-1

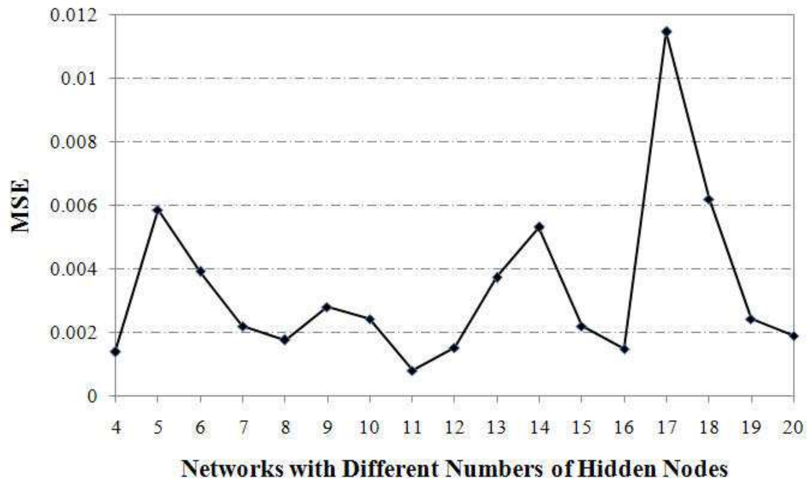

Fig. 2. Maximum Squared Error (MSE) versus Number of Hidden-Layer Neurons

After the pre-acceptance of desirable networks, the best networks are: NN 6-8-1 and NN 6-11-1. In order to arrive at a single ideal model, NN 6-11-1 was chosen since it presents good results in the case of R-values and also has the least value of MSE among all networks. The results for training the NN 6-11-1 are summarized in Figs. 3 to 5.

Figure 3 shows the mean squared error of the network starting at a large value and decreasing to a smaller value. In other words, it shows that the network is learning. The plot has three lines, because the 213 input and targets vectors are randomly divided into three sets. Training on the training vectors continues as long the training reduces the network's error on the validation vectors. After the network memorizes the training set (at the expense of generalizing more poorly), training is stopped. This technique automatically avoids the problem of over-fitting, which plagues many optimization and learning algorithms.

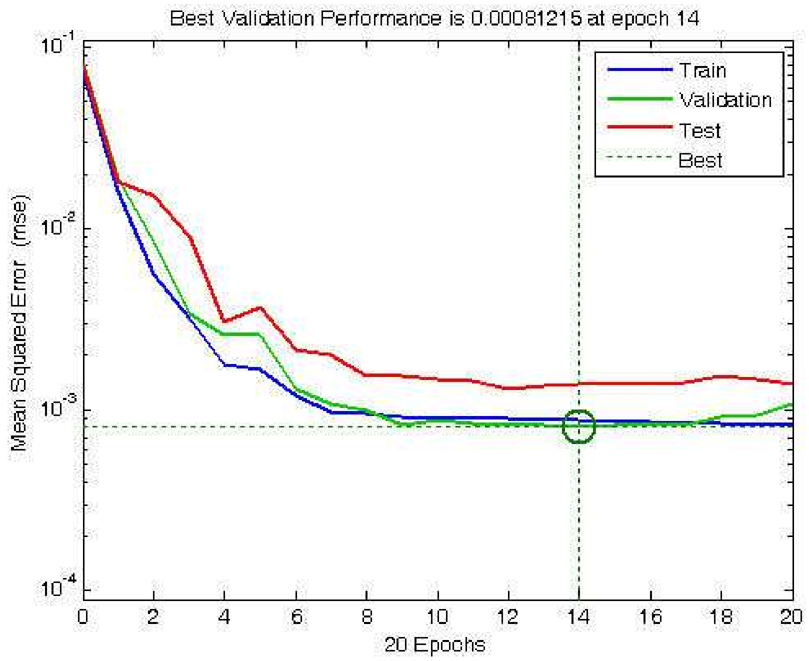

Fig. 3. Performance of NN 6-11-1

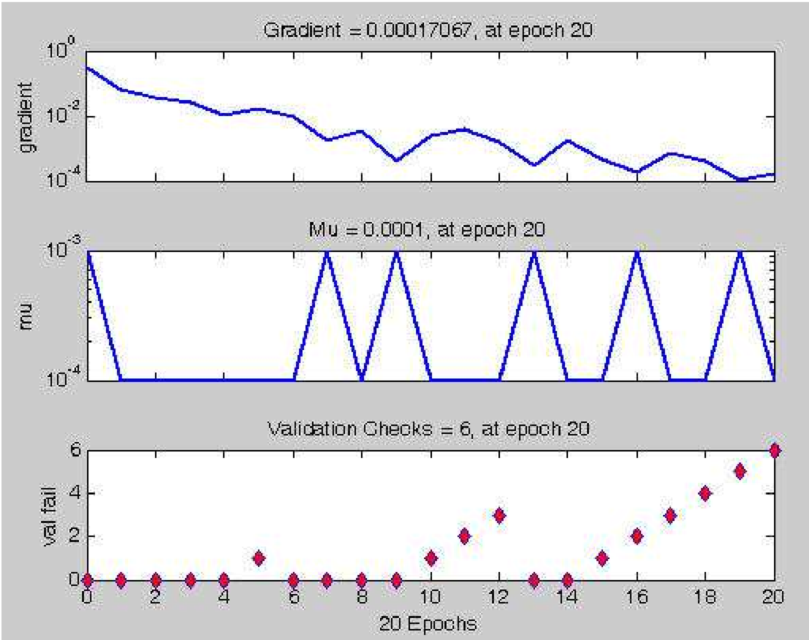

Fig. 4. Training state of NN 6-11-1

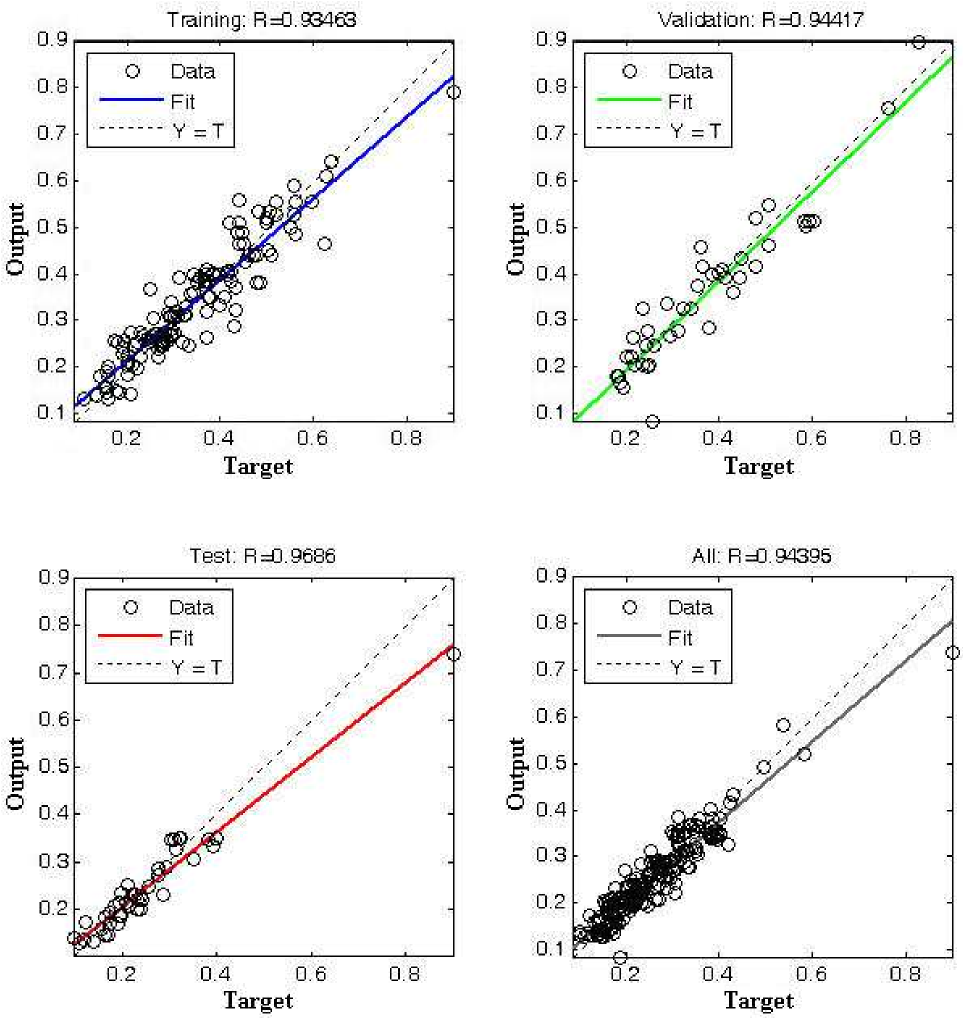

Fig. 5. Regressions of training, validation and test data simulated by NN 6-11-1

4. ANN MODEL VERSUS EXISTING MODELS

The three important models for verification of the ANN model selected including the strength models proposed by Toutanji and Matthys in 2005 (as the representative for nonlinear models), Lam and Teng in 2002 (as the representative for linear models) and finally Mander et al. in 1988 (second-order). The simulated compressive strengths of the FRP-confined concrete from idealised neural network compared to the three existing strength models are plotted against the experimental values in Fig. 6. The error distribution of the models, in terms of the percentage difference between simulated and experimental results is summarized in Table 4. From both Fig. 6 and Table 3, it can be observed that there is reasonably good performance of the neural network in predicting the experimental results. The average error for the ANN model for predicting the experimental results is equal to 8.9% while the average error for the other three models including Toutanji and Matthys', Lam and Teng's and Mander's are 13.2%, 16.1% and 20.6% respectively. Actually, more than 90% of the simulated results are within $\pm 20\%$ of the experimental values for ANN model but the accuracy of other models is lower than 80% in the same range.

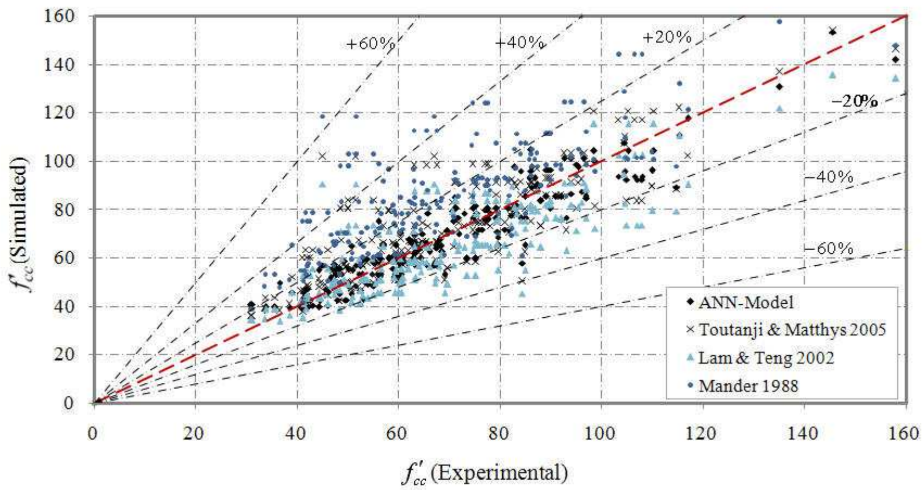

Fig. 6. Comparison of various predicted values of f'_{cc} versus experimental data for different strength models

Table 3. Distribution of errors for different strength models relative to experimental values

RANGE OF ERROR	NUMBER OF DATA IN THE RANGE FOR MODELS				PERCENTAGE TO WHOLE DATA (213 TESTS) FOR MODELS			
	MANDER (1988)	LAM & TENG (2002)	TOUTANJI & MATTHYS (2005)	ANN-MODEL	MANDER (1988)	LAM & TENG (2002)	TOUTANJI & MATTHYS (2005)	ANN-MODEL
±5%	26	45	55	72	12.2%	21.1%	25.8%	33.8%
±10%	51	82	107	132	23.9%	38.5%	50.2%	62.0%
±15%	79	118	149	180	37.1%	55.4%	70.0%	84.5%
±20%	112	147	165	196	52.6%	69.0%	77.5%	92.0%
±25%	141	168	179	206	66.2%	78.9%	84.0%	96.7%
±30%	171	183	193	211	80.3%	85.9%	90.6%	99.1%
±35%	182	192	202	212	85.4%	90.1%	94.8%	99.5%
±40%	190	210	209	212	89.2%	98.6%	98.1%	99.5%
±50%	209	213	211	213	98.1%	100.0%	99.1%	100.0%
±60%	212	213	213	213	99.5%	100.0%	100.0%	100.0%
±65%	213	213	213	213	100.0%	100.0%	100.0%	100.0%

5. NEW ANN FORMULA

As it was indicated in the previous section, the simulated results from the neural network are in reasonably good agreement with the experimental data. But it is not convenient to use the network in engineering design since the network contains many weights and biases together with transfer functions and consequently the final equations will become very complicated. In order to come up with this problem, the neural network should be employed to generate empirical design charts and equations for use in design. The range and reference value for each of the six input parameters are first chosen to be close to their mean values and are presented in Table 4.

Table 4. Range of Input Parameters and Their Corresponding Reference Values Used in Derivation of Empirical Design Approach

Input parameters	Minimum	Maximum	Reference
d (mm)	51.00	200.00	140
L (mm)	102.00	788.00	300
f'_c (MPa)	18.00	64.20	35
t (mm)	0.09	5.31	1.2
f_{frp} (MPa)	167.00	3720.00	1500
E_{frp} (MPa)	10500.00	629600.00	115000

The pattern formula used here for predicting the compressive strength of FRP-confined concrete was introduced by Leung et al. (2006) for determining ultimate FRP strain of FRP-strengthened concrete beams. As the first step, f'_{cc} is first plotted against E_{FRP} assuming the other five input parameters are kept constant at their respective reference values. To account for the effect of these parameters on f'_{cc}, a correction function has to be derived. The correction function can be written in the following form:

$$F(d, L, f'_c, t, f_{frp}) = C(d) * C(L) * C(f'_c) * C(t) * C(f_{FRP}) \tag{1}$$

The variation of f'_{cc} with each parameter is assumed to be independent of the other parameters. The correction factor $C(d)$ will be derived first. To derive $C(d)$, master curves are first obtained with different L values, but with d fixed at the reference value of 140. For each combination of d and L, f'_{cc} is obtained from the neural network. The number of vectors to draw the correction factors was chosen to be about 25% of whole data which is approximately equal to 50 data series. By dividing the network simulated value by the value read off from the master curves, the correction factor *C(d)* can be obtained. By considering all curves for *C(d)*, a line that fits the curve with the minimum least square error was found. The same procedure has been applied to obtain the correction factors for the other input parameters.

After finishing the process, the following equations for correction factors are summarized as:

$$C(d) = -0.490(\frac{d}{140}) + 1.494 \tag{2}$$

$$C(L) = 0.159 \ln(\frac{L}{300}) + 1.009 \tag{3}$$

$$C(f_c') = 1.082\left(\frac{f_c'}{35}\right)^4 - 5.071\left(\frac{f_c'}{35}\right)^3 + 8.209\left(\frac{f_c'}{35}\right)^2 - 5.025\left(\frac{f_c'}{35}\right) + 1.798 \quad (4)$$

$$C(t) = -0.064\left(\frac{t}{1.2}\right)^2 + 0.669\left(\frac{t}{1.2}\right) + 0.387 \quad (5)$$

$$C(f_{FRP}) = -0.213\left(\frac{f_{FRP}}{1500}\right)^4 + 0.901\left(\frac{f_{FRP}}{1500}\right)^3 - 1.008\left(\frac{f_{FRP}}{1500}\right)^2 + 0.723\left(\frac{f_{FRP}}{1500}\right) + 0.604 \quad (6)$$

Consequently, the compressive strength of FRP-confined concrete will be obtained from equation (7):

$$f_{cc}' = \left(f_{cc}'\right)_{chart} * C(d) * C(L) * C(f_c') * C(t) * C(f_{FRP}) \quad (7)$$

With random selection of 113 data from experimental database, the proposed ANN model is compared with the three existing models used in Section 4.1 which is shown in Fig. 23. Similarly, the error distribution of the models is calculated and presented in Table 6. The average error for the ANN model for predicting the experimental results is equal to 10% while the average error for the other three models including Toutanji and Matthys', Lam and Teng's and Mander's are 10.9%, 16.3% and 16.5% respectively. Actually, more than 90% of the simulated results are within $\pm 20\%$ of the experimental values for ANN model but the accuracy of other models is lower than 80% in the same range. Again this is an indication that the network has learned to generalize the information well and reflects good precision in simulating. Moreover, concentrating on the Fig. 23, it can be seen that the values simulated by the ANN model sets spread around the 45° line which implies neither over-estimation nor under-estimation.

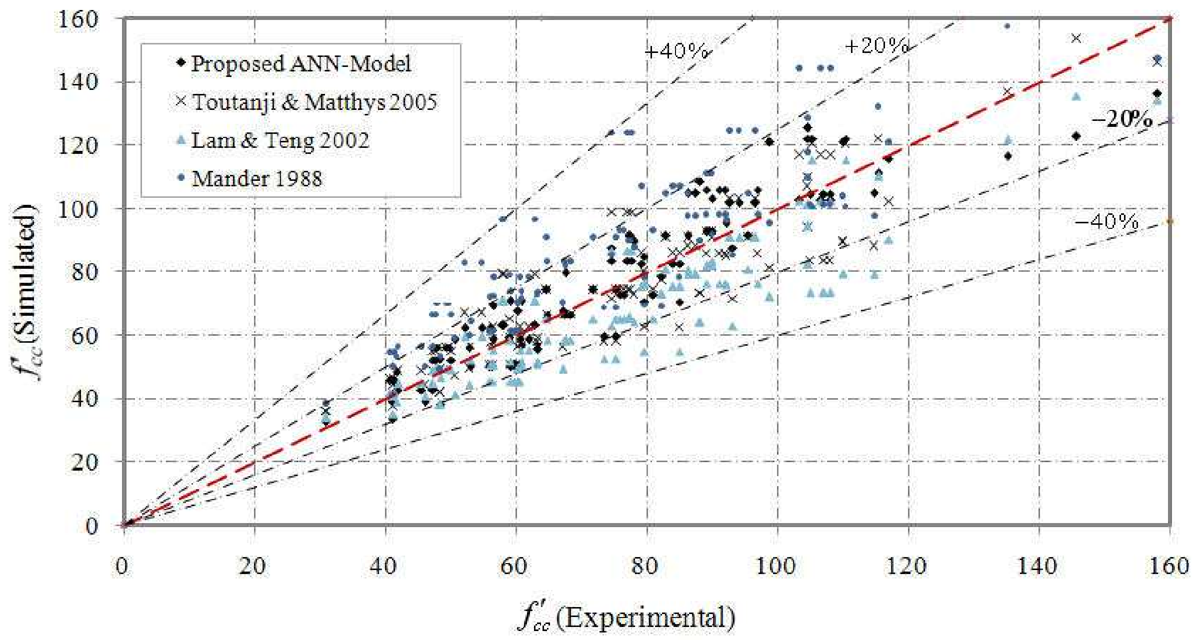

Fig. 7. Comparison of predicted values of f_{cc}' versus experimental data for proposed ANN equation against three existing models

Table 5. Distribution of errors for different strength models relative to experimental values

RANGE OF ERROR	NUMBER OF DATA IN THE RANGE FOR MODELS				PERCENTAGE TO WHOLE DATA (213 TESTS) FOR MODELS			
	MANDER (1988)	LAM & TENG (2002)	TOUTANJI & MATTHYS (2005)	ANN-MODEL	MANDER (1988)	LAM & TENG (2002)	TOUTANJI & MATTHYS (2005)	ANN-MODEL
±5%	19	24	34	30	16.8%	21.2%	30.1%	26.5%
±10%	37	41	65	62	32.7%	36.3%	57.5%	54.9%
±15%	53	60	86	86	46.9%	53.1%	76.1%	76.1%
±20%	72	78	94	107	63.7%	69.0%	83.2%	94.7%
±25%	89	89	102	111	78.8%	78.8%	90.3%	98.2%
±30%	104	98	111	113	92.0%	86.7%	98.2%	100.0%
±35%	107	105	112	113	94.7%	92.9%	99.1%	100.0%
±40%	112	113	113	113	99.1%	100.0%	100.0%	100.0%
±50%	113	113	113	113	100.0%	100.0%	100.0%	100.0%

6. CONCLUSIONS

Developing neural networks, the compressive strength of FRP-confined concrete was related to six input parameters including diameter and height of concrete specimen, total thickness of FRP, tensile strength of the FRP in the hoop direction, elastic modulus of FRP and compressive strength of the unconfined concrete. After training the 17 neural networks with different number of hidden neurons, by considering the performance of the networks (MSE and R), one of the networks was selected for simulation which showed effective performance through training, testing, and validation. In order to verify the performance of the network, the results from ANN simulations was compared to the results of three important existing strength models (linear, nonlinear and second-order models). The average error for the ANN model for predicting the experimental results was lower than 9% while the average errors for the other three models were more than 13%. On the other hand, more than 90% of the simulated results were within $\pm 20\%$ of the experimental values for ANN model but the accuracy of other models was lower than 80% in the same range which indicated that the network was learned to generalize the information well. Also values simulated by the ANN model set spread around the 45^{o} line which implied neither over-estimation nor under-estimation. In order to use the simulated results obtained from ANN model in prediction of compressive strength of FRP-confined concrete in the absence of the idealised network, an equation was derived which predicts the compressive strength independently from the network.

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