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State-of-the Art Report
PROBABILISTIC SEISMIC SAFETY AND DAMAGE ASSESSMENTS OF STRUCTURES

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SUMMARY

Recent developments in the analytical tools for probabilistic structural damage and safety assessments are highlighted and discussed. Improved measures of damage have been proposed as functions of the maximum deformation and dissipated hysteretic energy. The necessary response analysis must include highly nonlinear and hysteretic behavior; accordingly, hysteretic restoring force models have been developed for this purpose. Solutions based on the Markov vector process are necessarily limited, whereas through stochastic equivalent linearization, solutions of practical significance can be more readily obtained.

INTRODUCTION

The concern for safety against structural collapse is paramount in the design of structures for seismic resistance. For economic reason, however, this concern must generally be tempered with the need to allow for some degree of structural damage under a strong-motion earthquake. Traditionally, these dual requirements have been accomplished by providing sufficient ductility in the design of a structure, such that it can sustain some inelastic response and thus absorb or dissipate energy through hysteresis.

It is, therefore, universally recognized that some tolerable structural damage is unavoidable when subjected to strong seismic forces, even though collapse or severe damage must be avoided. This presents a special challenge for the analysis or assessment of structural safety. The level of safety or damage will, of course, also depend on the maximum seismic hazard expected during the life of a structure.

Presented herein is an attempt to summarize and/or highlight some of the representative or typical works (familiar to the author) that have been developed in recent years addressing or relating to the above problems of seismic structural safety and damage assessments. Many of the papers in this special theme session of this conference (Session STH) will elaborate on the subject, including the nonlinear-inelastic response analysis necessary for the assessment of safety and damage.

MODELS AND ANALYSIS OF SEISMIC STRUCTURAL DAMAGE

The subject of seismic damage assessment is a topic of fairly recent research interest (Refs. 1-6). First of all, measures of damage are necessary in order to define the degree or severity of structural damage. Such measures must necessarily be functions of one or more response quantities of a structure. These include the maximum response (or ductility ratio) and the cumulative inelastic deformation (or hysteretic energy), among others. The analysis of damage, therefore, must necessarily include the inelastic dynamic response and the hysteretic structural behavior, including the range approaching collapse. Furthermore, earthquake loadings and corresponding responses need to be modeled as nonstationary stochastic processes.

Definition and Measures of Seismic Damage The ductility ratio (or ductility factor)

$$\mu = \frac{\delta_{max}}{\delta_y} \tag{1}$$

where: δ_{max} = the maximum deformation response; and
δ_y = the yield deformation,

is often used as the measure of structural damage (e.g., Ref. 7). Clearly, the maximum deformation of a structure is a major contributor to its damage; however, damage can be caused also by the repeated oscillatory response from an earthquake loading. This recognition has led to the development of damage measures that included the effect of repeated loadings.

Banon and Veneziano (Ref. 1) defined damage as a function of two response ratios,

$$D = f(K, N) \tag{2}$$

where:

$K = k_i/k_s$, a stiffness ratio; and

$$N = \frac{\sum_{i=1}^{n} |\theta_{pi}|}{\theta_y}$$, a measure of cumulative rotational deformations;

in which, k_i = the initial flexural stiffness;
k_s = the reduced secant stiffness;
θ_{pi} = plastic rotation at the ith cycle; and
θ_y = yield rotation.

Park and Ang (Ref. 8) defined damage in terms of an index,

$$D = \frac{\delta_{max}}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \tag{3}$$

where: δ_{max} = the maximum response deformation;
$\int dE$ = the dissipated hysteretic energy;
δ_u = the ultimate static deformation;
Q_y = the static yield strength; and
β = a constant, dependent on the type of material.

A statistical examination of over 400 tests of reinforced concrete members (beams and beam-columns) revealed that at collapse the above damage index has a lognormal distribution with a median of 1.0 and a c.o.v. of around 50% (Ref. 9). Observe that the first term is damage caused by maximum deformation (i.e., ductility ratio) whereas the second is damage caused by the dissipation of hysteretic energy. Values of $D = 0$ means the absence of damage; $D \geq 1.0$ means total collapse; whereas intermediate values define increasing degrees of damage.

Based on the concept of low-cycle fatigue, Stephens and Yao (Ref. 4) proposed the following measure of damage:

$$D = \left(\frac{\Delta\delta_p^+}{\Delta\delta_{pu}^+}\right)^{\alpha} \tag{4}$$

where: $\Delta\delta_p^+$ = positive change in plastic deformation;
$\Delta\delta_{pu}^+$ = positive change in plastic deformation at failure; and
$\alpha = 1 - br_p$,

in which b is a constant and r_p is the ratio between negative and positive plastic deformation changes in one cycle.

Suzuki and Minai (Ref. 5) discussed several single-parameter damage measures described in terms of respective differential equations. Specifically, for example, for the maximum ductility ratio, μ, the differential equation is

$$\dot{\mu} = |\dot{x}|U(x\dot{x})U(|x|-\mu) \tag{5}$$

where: x = displacement response; and
$U(\cdot)$ = the unit step function.

Also, if the unloading and loading stiffnesses are the same, the dissipated hysteretic energy, $\varepsilon(t)$, can be represented by the differential equation

$$\dot{\varepsilon}(t) = (1-r)z(\dot{x}-g_z) \tag{6}$$

where: r = ratio of post-yield stiffness to pre-yield stiffness;
z = normalized hysteretic component; and
g_z = a nonlinear function.

Shinozuka and Tan (Ref. 2) defined damage in terms of damage probability matrices. An initial damage probability vector, M_o, represents the probabilities that an undamaged structure will sustain various degrees of damage (e.g., minor, moderate, major) when subjected to an earthquake of given intensity. Subsequently, the probabilities that a structure in a given state of damage will reach other damage states when subjected to an earthquake of intensity I is then described by a conditional damage probability matrix, $M_i(I)$, which is a lower triangular Markovian transition matrix. The components of the matrix $M_i(I)$ are transition probabilities,

$$P_{D_iD_j}(I) = P\{\text{damage state} = D_i \,|\, \text{damage state} = D_j \text{ under earthquake } I\}$$

If the initial damage is D_o, the conditional probability that the subsequent damage will be D_n after n earthquakes of intensities $I_1, I_2, \ldots I_n$ is a component in the matrix

$$M_{n,1} = M_n(I_n)\cdot M_{n-1}(I_{n-1}) \ldots M_1(I_1) \tag{7}$$

whereas if the structure is originally undamaged, the damage probability vector after the n earthquakes will be,

$$M_{n,o} = M_{n,1} \cdot M_o \tag{8}$$

The validity and utility of the above formulation have been examined (Ref. 2) through Monte Carlo simulations.

Hysteretic Restoring Force Models In order to include the oscillatory nature of a seismic motion, the response quantities in any damage measure must include the hysteretic behavior of the restoring force of a structure.

The restoring force may be described as,

$$q = rx + (1-r)z \tag{9}$$

where: x = the nonhysteretic component of response; and
z = the hysteretic component.

In particular, for the bilinear hysteretic model, the differential equation for the hysteretic component is (Ref. 5)

$$\dot{z} = \dot{x}[1-U(\dot{x})U(z-1) - U(-\dot{x})U(-z-1)] \tag{10}$$

For the slip type hysteresis, the hysteretic component is given by,

$$\dot{z} = \dot{x}\{U(x-u^+)U(\dot{x})[1-U(z-1)] + U(x)U(-\dot{x})[1-U(-z)] + U(-x-u^-)U(-\dot{x})[1-U(-z-1)] + U(-x)U(\dot{x})[1-U(z)]\} \tag{11}$$

where u^+ and u^- are control variables. If the additional state variables u^+ and u^- are neglected, the above Eq. 11 reduces to the slip model proposed by Iwan (Ref. 10).

Other differential equation forms for z were also discussed by Suzuki and Minai (Ref. 5), including the Clough and Johnston model (Ref. 11) for reinforced concrete and the Kato-Akiyama model (Ref. 12) for steel structures.

For a large class of smooth hysteresis, Wen (Ref. 13) extended the Bouc (Ref. 14) model as follows,

$$\dot{z} = \frac{1}{\eta}[A\dot{x} - \nu(\beta|\dot{x}||z|^{n-1}z - \gamma\dot{x}|z|^n)] \tag{12}$$

where the parameters A, β, γ, η, ν, and n define the amplitude, hysteretic loop shape, and inelastic properties. Equation 12 can describe both softening and hardening inelasticity.

Degradation in both strength and stiffness can be introduced through the parameter A as a function of the hysteretic energy dissipated $\varepsilon(t)$; e.g.,

$$A(t) = A_o - \delta_A \varepsilon(t)$$

where: δ_A = rate of degradation, and

$$\varepsilon(t) = (1-\alpha)k \int_0^t z(\tau)\dot{x}(\tau)d\tau.$$

Degradation may also be a function of the response amplitude (Ref. 15).

The above smooth hysteretic restoring force model has been generalized for two directions. To include the interaction in the two directions, the hysteretic components, z_x and z_y, are described by the following coupled differential equations (Ref. 16):

$$\dot{z}_x = A\dot{u}_x - \beta|\dot{u}_x z_x|z_x - \gamma\dot{u}_x z_x^2 - \beta|\dot{u}_y z_y|z_x - \gamma\dot{u}_y z_x z_y \tag{13a}$$

$$\dot{z}_y = A\dot{u}_y - \beta|\dot{u}_y z_y|z_y - \gamma\dot{u}_y z_y^2 - \beta|\dot{u}_x z_x|z_y - \gamma\dot{u}_x z_y z_x \tag{13b}$$

where u_x and u_y are the displacements in the x and y directions, respectively. In this case, the hysteretic energy is defined by,

$$\varepsilon(t) = \int_0^t [z_x(\tau)\dot{u}_x(\tau) + z_y(\tau)\dot{u}_y(\tau)]d\tau$$

The versatility and capability of the above restoring force models have been demonstrated, including for complex force-displacement relationships of reinforced concrete structures. For example, in Fig. 1 are shown the experimental results for a reinforced concrete frame and corresponding analytical representation with Eq. 12; whereas similar comparison of results for biaxial loading of a reinforced concrete column are shown in Fig. 2.

STRUCTURAL RESPONSE AND RELIABILITY ANALYSIS

By modeling the excitation as a filtered shot noise, and using the differential equation form of the restoring force, the nonlinear stochastic response analysis necessary for damage assessment may be formulated on the basis of the Markov vector process, with the transition probability governed by the Fokker-Planck equation.

Solution to the resulting partial differential equation for the transition probability, however, is difficult to obtain. Successful solutions including the method of stochastic averaging, thus far, have been limited to single-degree-of-freedom systems (Refs. 13, 17, and 18).

The Markov vector process approach for nonlinear-inelastic response analysis has been under intensive study by Kobori, et al. (Ref. 19), and Minai and Suzuki (Ref. 5) using the differential equation forms for the hysteretic restoring force as well as for the damage measure. A unified formulation has been developed for directly evaluating the required structural reliability against a specified level of ductility ratio without the need for level-crossings or the use of the Fokker-Planck equations. Instead of solving for the transition probability, solutions for the response moments are obtained through the well-known Ito equation. With an appropriate series expansion of the joint probability density function, the moments are evaluated as products of normal and gamma probability density functions and use of orthogonal polynomials; ordinary differential equations for the moments are obtained which may be solved numerically. Excellent results have been indicated, as shown in Fig. 3. The extension to multi-degree-of-freedom systems, however, would require greatly increased effort -- both analytical and numerical.

Stochastic Linearization Method The method of equivalent linearization was first used for stochastic problems by Caughey (Ref. 20), and its theoretical basis has been discussed by Iwan (Ref. 21). With the use of smooth hysteretic restoring force models, the linearization can be accomplished in closed form without resorting to the Krylov-Bogoliubov approximation (Ref. 22), thus greatly

facilitating its application to multi-degree-of-freedom systems, including those with degradation.

For example, the restoring force described by Eq. 12 can be linearized as,

$$\dot{z} + c_1\dot{x} + c_2 z = 0 \tag{14}$$

where the coefficients c_1 and c_2 are evaluated on the basis of minimum mean squared error; thus, assuming Gaussian excitation and response,

$$c_1 = \sqrt{\frac{2}{\pi}}\,[\gamma\,\frac{E(\dot{x}z)}{\sigma_{\dot{x}}} + \beta\sigma_z] - A$$

$$c_2 = \sqrt{\frac{2}{\pi}}\,[\gamma\sigma_{\dot{x}} + \beta\,\frac{E(x\dot{z})}{\sigma_z}]$$

The equation of motion for a SDF system together with Eq. 14 then describe the equivalent linearized system of governing equations. Under a filtered Gaussian shot noise excitation, the response covariance matrix, S, of the response variables (x, $\dot{x}$, and z) is governed by the following matrix ordinary differential equation:

$$\frac{dS}{dt} + GS + SG^t = B \tag{15}$$

where: t stands for the transpose;
G = matrix representing the structural system, including the linearization coefficients and filter parameters; and
B = matrix of the response vector and the excitation.

Because the linearization coefficients are functions of the response statistics, iterative numerical solution is generally required. The effectiveness of the equivalent linearization technique has been demonstrated extensively for practical purposes (e.g., Refs. 6, 15, and 16).

<u>Some Applications</u> The above method has been applied with the damage index of Eq. 3 for the damage analysis of reinforced concrete buildings (Ref. 3). For buildings, the total damage of a structure may be defined as the weighted story damages,

$$D_T = \sum_i w_i D_i \tag{16}$$

where: D_i = the damage index of the ith story; and
$w_i = E_i/\sum_i E_i$, in which E_i = the hysteretic energy dissipated at the ith story.

Reliability against a tolerable seismic damage D_a is then defined as,

$$R_a = P(D_T < D_a) \tag{17}$$

whereas the corresponding reliability against structural collapse is,

$$R_u = P(D_T < D_u) \tag{18}$$

in which D_u is the ultimate capacity defined in terms of the damage index. An application to reinforced concrete buildings is discussed in Ref. 9.

Applying the equivalent linearization technique for multiple degree-of-freedom soil-structure systems with multi-linear hysteretic behavior, Asano and Suzuki (Ref. 23) formulated the differential equations for the response covariance under a nonstationary earthquake excitation. The reliability against specified ductility ratios are then evaluated by assuming Poisson-distributed level crossings and joint Gaussian displacement and velocity processes. The method was applied by Asano (Ref. 6) to examine the safety of two buildings that survived the Miyagi earthquake of 12 June 1978, in terms of the respective nonexceedance probabilities of specified story ductility ratios (≤ 2.0) during the earthquake.

Kobori, et al. (Ref. 24) investigated the nonlinear stochastic response of multi-story space frame buildings with bilinear elasto-plastic components subjected to nonstationary seismic excitations. The ductility ratios of the story displacements and of the joint rotations were evaluated numerically, and associated first-excursion probabilities were calculated on the basis of Poisson assumption and pseudo-Gaussian approximations.

ON SEISMIC HAZARD ANALYSIS

The reliability or nonexceedance probabilities evaluated above are for a given or assumed level of earthquake intensity; i.e., the calculated probabilities are conditional probabilities. The seismic hazard, however, may be specified only in terms of probability; therefore, the seismic safety of a structure must be evaluated as the combination (i.e., convolution) of the calculated conditional structural reliability with the probabilities associated with all the significant levels of seismic intensity. The latter information requires a seismic hazard analysis.

Several models have been developed for such hazard analysis. All the models require assumptions on (i) the occurrence of significant events in the region of influence; (ii) the relative frequencies of earthquake magnitudes; and (iii) the attenuation relationship. The occurrence of significant earthquakes is usually assumed to constitute a Poisson process; however, models based on non-Poisson assumptions have also been proposed, including models that are time-dependent as well as magnitude-dependent. The application of any non-Poisson model, of course, would require more information and data than that of the corresponding Poisson model.

The energy released from an earthquake is generally assumed to originate from a point (the focus); however, the energy release over the rupture length of an earthquake has also been developed. In spite of continuing improvements in the modeling for earthquake hazard analysis, there remain high uncertainties in the calculated results (probabilities), particularly those associated with the attenuation relation. Moreover, in regions of low historical seismicity (rare occurrences), the parameters of a hazard model (such as the occurrence rate, the maximum possible magnitude) will also contain significant uncertainty. Methods or approaches to reduce these uncertainties remain unresolved.

CONCLUDING REMARKS

The subject of seismic structural damage and safety assessments remains important. Even though there have been significant recent developments, further studies and improvements are required, including the formulation of reliability-based design procedures that can admit some tolerable degree of damage but will minimize the risk of collapse.

Seismic structural safety is a function also of the expected level of seismic hazard; the assessment of such hazards remains clouded with high degree of uncertainty.

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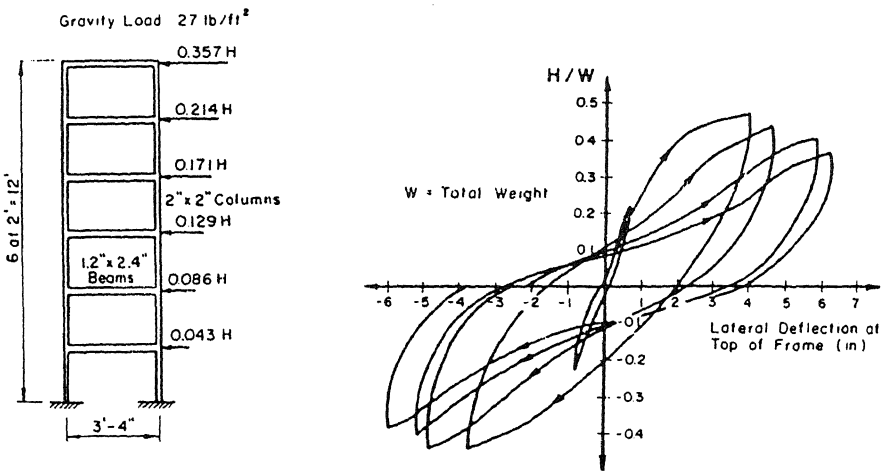
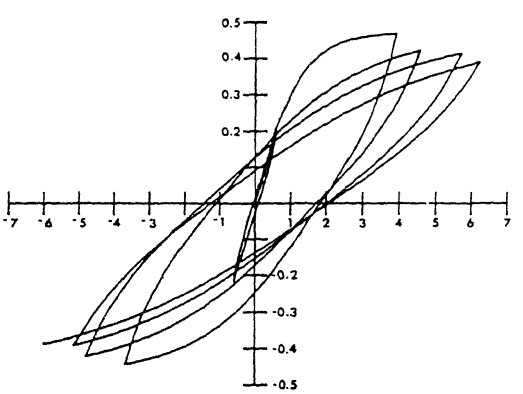
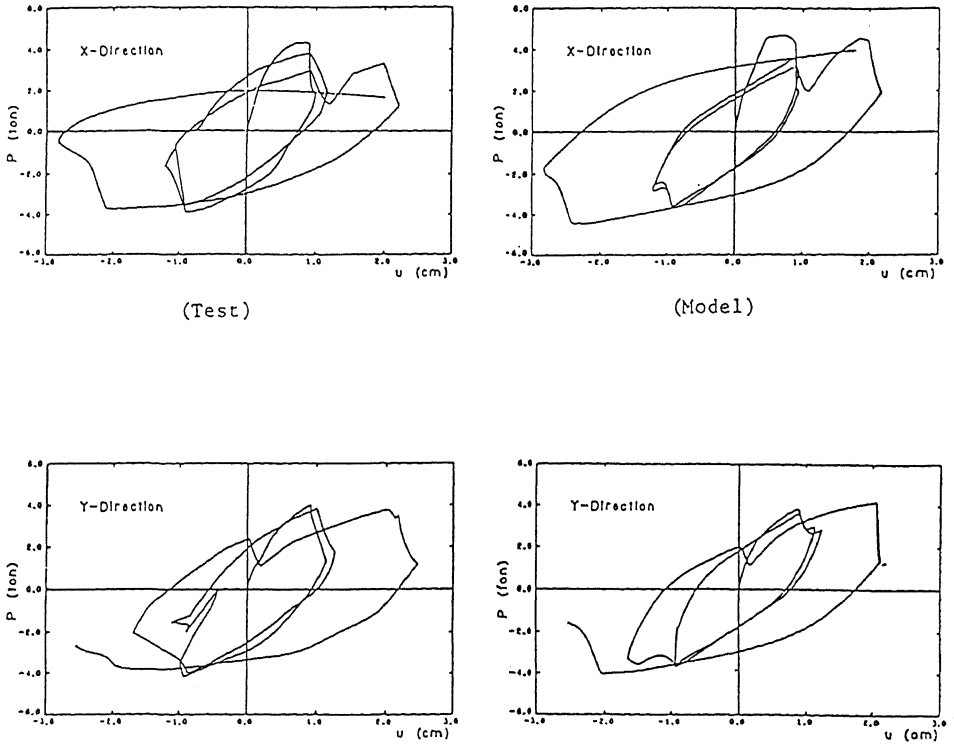


(a) Experimental Load-Deflection Curves for Reinforced Concrete Building Model (after Park and Paulay, Ref. 25)



(b) Hysteretic Curves from Displacement-Based Stiffness Degrading Model

Fig. 1 Comparison of Test and Model Hysteretic Curves

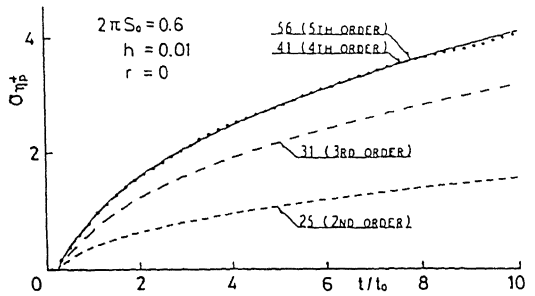


(Test) (Model)

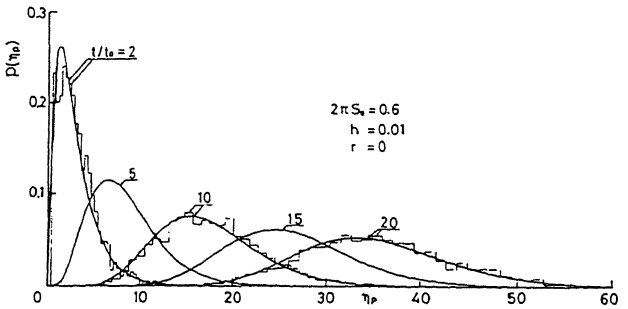
(Test) (Model)

(a) (b)

Fig. 2 Comparison of Test (after Takizawa and Aoyoma, Ref. 26) and Model Results



(a) rms values



(b) Probability Density Functions

Fig. 3 Comparison of Series Expansion Solutions with Simulations