EARTHQUAKE-RESISTANT DESIGN OF SHEAR WALLS
WITH SEVERAL ROWS OF OPENINGS

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SUMMARY

A simplified method for the design of shear walls with any number of vertical rows of openings, at the damageability and ultimate limit states is proposed. The purpose of the method is a quick, rather than a particularly accurate evaluation of the parameters characterising the two above mentioned limit states.

The structure is idealised as an assemblage of one-dimensional (beam) elements with elastic-perfectly plastic moment-curvature relationship, while the seismic action is idealized as a static horizontal load.

The elastic stiffnesses of the various structural elements are reduced by means of iteratively evaluated coefficients taking into account cracks in concrete, whose effects are not negligible for the structure and the considered limit states. Particular attention is paid on the choice which maximizes the energy dissipation; a random walk optimization procedure permits to design walls under the condition of maximization of the dissipated energy. Two sets of examples are given, in order to illustrate the effectiveness of the algorithm and the behaviour of the walls with several vertical rows of openings.

INTRODUCTION

Antiseismic codes (Ref. 1, 2) quantify structural behaviour by considering the structure as always elastic and by evaluating seismic actions starting from a strong earthquake and reducing them by a coefficient, behaviour factor $K$, whose value is related to the available ductility of the structure. So doing, the reliability against serviceability and damageability limit states is directly evaluated. On the contrary, the reliability against collapse is not assessable and it is somehow guaranteed, for well defined structural types, by obeying certain general criteria of conceptual design, plus specific rules aimed at preserving integrity and conferring ductility to structural elements and to their connections.

The current position of the codes, even though justified by the difficulties of evaluating safety at collapse, can yet turn out to be too restrictive for the engineer and lead to uselessly expensive structures. Therefore it appears useful to develop non linear analysis procedures, simplified with respect to those currently available; the simplification can be attained by considering a single well-specified structural type at a time, so that its particular fea-

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tures can be utilized in order to:
- model actions in a simple way, possibly in a static form;
- model structures in a simple way and define their behaviour through few para-
meters;
- settle a single desirable mechanism of collapse in advance, via a suitable
design.
The reliability of the simplified method and the allowable values to be assi-
gned to the design parameters could be assessed via a comparison between the
results obtained with the simplified method and those provided by the more com-
plicated non-linear dynamic analysis procedures.
Following this line of operation, the writers have focused their attention
on buildings with reinforced concrete bearing walls which, for their considera-
ble overall stiffness, allow a simple static idealization of seismic actions
and, for their plan and vertical regular configuration, required by the indus-
trialized constructive technologies, allow engineers to easily reduce the ana-
lysis of an entire building to that of single walls. In particular, walls with
openings, or coupled shear walls, have been examined. These are the most com-
mon and can attain a good overall ductility, if well designed. In a previous
work a simplified method of analysis and antiseismic design of walls with only one
vertical row openings was presented (Ref. 3); the application of this method
requires constant stiffnesses of beams and walls and constant strength of beams
along the height. The validity of this method was then tested with a more com-
plicated non-linear dynamic analysis procedure (Ref. 4); finally some criteria
for the optimal choice of the design parameters were elaborated (Ref. 5). In
(Ref. 6) the proposed method has been extended to walls with more than one ver-
tical row of openings (see fig. 1). Its application again requires constant
stiffness of the single wall element and equal stiffness and strength of beams
of the same vertical row.
In the present paper, following a short description of the adopted struc-
tural model, some results obtained through a computer program based on the theo-
ry shown in Ref. 6 are given. This program can design walls with several rows
of openings, by calculating thickness, depth of connecting beams and reinfor-
cement of walls and beams under the condition of the maximum energy dissipated
through the plastic deformations of the beams; an iterative procedure permits
to take account of the reduction of stiffness of the walls due to cracking.

STRUCTURAL MODEL

The model is obtained by replacing the connecting beams of each vertical
row with an equivalent comb-like continuum made up of infinitesimal axially ri-
gid laminae and by idealising wall elements and laminae as one-dimensional ele-
ments with elastic-perfectly plastic behaviour. In particular the elastic cha-
acteristic are evaluated on cracked sections since, as a rule, first yieldings
happen when previous seismic waves have already caused cracking in most of the
structure. Wall elements are considered as fixed at the base, in view of the
considerable stiffness of usual foundation structures.
The desired mechanism of collapse occurs when all the connecting beams

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yield before the first of the wall elements yields. In addition the collapse is conventionally defined as happening since the first wall element yields, instead of requiring, according to statics, that all the wall elements yield. Such assumption has the purpose of avoiding the high stresses (normal stresses in lower beams and shear stresses at the foot of the walls) which occur when the wall elements yield. Such stresses considerably reduce the ductility of the concerned structural elements while the plastic rotations of the yielded wall elements, occurring when even only one wall element is still elastic, are almost negligible and give rise to no appreciable dissipation of energy (see Ref. 6). Further reasons for such a conventional definition of collapse are:
- the desire to localize residual deformations in beams only, in order to make repair operations easier and less expensive,
- the larger energy dissipation capacity and lesser deterioration of beams with respect to wall elements,
- the opportunity of maintaining an overall stiffness in order to keep a control on the damage to non-structural elements even near collapse.
Furthermore, there should not necessarily be an increase in initial costs. Wall elements, in fact, remain elastic and therefore do not require the large transverse reinforcement needed to guarantee a ductile behaviour. The reduction of transverse reinforcement ought to offset, if not completely at least to a large
extent, the increase of longitudinal reinforcement due to the larger stresses.

The proposed method is substantially oriented to verify safety against collapse
limit state and also allow to check damage to non-structural elements.

From what was previously said, a wall must be designed so that, under the ac-
tion of a horizontal static load whose resultant is \( W_h \), the following condi-
tions are satisfied:

(a) All the wall elements remain elastic;
(b) All the connecting beams yield;
(c) The ductility demand in the connecting beams is less than its allowable
maximum value \( \eta_{\text{max}} \);
(d) The interstorey drift is less than its allowable maximum value \( \lambda_{\text{max}} \).

These conditions are equivalent to the following bounds:

\[
1 < \omega_i; \quad \tilde{\eta}_i \leq \eta_{\text{max}}; \quad \tilde{\lambda} \leq \lambda_{\text{max}};
\]

where:

\[
\omega_i = \frac{W_i}{W_{y_i}} = \text{Load ratio for the } i-\text{th row of beams};
\]

\[
\tilde{\eta}_i = \text{Maximum required ductility for the } i-\text{th row of beams};
\]

\[
\tilde{\lambda} = \text{Maximum interstorey drift};
\]

\[
W_{y_i} = \text{Horizontal load corresponding to the yielding of the beams of the}
\]

\( i-\text{th row.} \)

The design procedure which guarantees the fulfillment of these conditions
has been developed and is described in detail in (Ref. 6).

The choice of the \( \omega_i \) and \( \tilde{\eta}_i \) values, instead of being carried out a priori, can
become a part of the design procedure if the condition of the maximum energy
dissipation through the plasticization of the beams is added to the conditions
already mentioned. The problem of assigning the values which maximize the dissi-
ipated energy \( E_d \) to the design variables \( \omega_i, \tilde{\eta}_i \) can be solved through a nume-
rical procedure by assuming \( E_d \) as the objective function being maximized in the
2m-dimensional space of the \( \omega_i \)'s, \( \tilde{\eta}_i \)'s (\( i = 1, \ldots, m \)).

In Ref. 6 an analytical expression for the dissipated energy of wall with one
row of openings was found; for this case the differences among the values of
\( E_d \), for different values of \( \omega \) and \( \tilde{\eta} \), are rather large and, in general, \( E_d \) at-
tains its maximum for \( \omega = 1.6 \), while the dependence on \( \omega \) diminishes when incre-
asing \( \tilde{\eta} \). The extent of such differences and the existence of quite a distinct
maximum in the range of the acceptable values of \( \tilde{\eta} \), emphasizes the importance
of a good choice of the \( \omega_i \)'s and \( \tilde{\eta}_i \)'s.

The above said optimization represents a constrained problem; the con-
straints are, besides the conditions on \( \omega_i, \tilde{\eta}_i, \lambda \), the conditions on the wall
thickness and on the depth of the beams. The former condition depends on the
shear strenght of the wall, the latter on the architectural design.

NUMERICAL RESULTS

The above described design method has been codified in FORTRAN ASCII, to
obtain the program WALLOP. Starting from the input data, WALLOP calculates the
design seismic forces in accordance with (Ref. 7) and determines the design pa-
rameters and quantities which maximize the energy dissipated through the plas-
tic deformations of the connecting beams. The optimization is performed in the
2n-dimensional space of the parameters $\omega_i$, $\tilde{\eta}_i$ ($i = 1, \ldots, n$), via a random
walk procedure; upper and lower bounds are imposed on $\omega_i$ and $\tilde{\eta}_i$ and on the
depth of the beams $h_c$ through suitable penalty functions. The objective func-
tion is given by the sum of the two amounts of the dissipated energy, for sei-
smic forces coming from right and from left respectively; a further constraint
is imposed in order the difference between the two amount to be less than 10% of
their sum.

Two iterative procedures, external to the optimization algorithm, permit
to account for cracking in evaluating mechanical characteristics (areas and
flexural inertias) of walls and to control the maximum interstory drift.

For each designed (and eventually optimized) wall, the first procedure
calculates minimum reinforcements and mechanical characteristics of the cracked
sections at each story of every single wall, determines, using these charac-
teristics, displacements and rotations at each story and calculates the values
of the reduction coefficients which, if assumed constant along the height of
the single wall, minimize the mean square differences between the above said
rotations and vertical displacements and the corresponding ones obtained with
constant coefficients. If such coefficients are not sufficiently similar to the
previously assumed ones, a new design is performed, assuming the new coeffi-
cients.

The second procedure compare the interstory drift $\tilde{\lambda}$ of the designed wall
with the maximum allowable value $\lambda_{\max}$; in case $\tilde{\lambda} > \lambda_{\max}$, the thickness $s$ is
changed and a new design is performed.

Two sets of numerical experiments have been made, in order to test the ef-
fectiveness of the algorithm and the behaviour of the walls, as regards to both
the dissipated energy (first set) and the stiffness reduction (second set).

The first set of experiments is performed on symmetrical walls with ($n_g$
10 and 15 stories, ($h = 3.00$ m) and with ($n_p$) 1 ($B_1 = B_2 = 7.00$ m, $b_1 = 1$ m),
2 ($B_1 = B_2 = B_3 = 4.33$ m, $b_1 = b_2 = 1.00$ m) and 3 ($B_1 = B_2 = B_3 = B_4 = 3.00$ m,
$b_1 = b_2 = b_3 = 1.00$ m) vertical rows of openings. These walls are optimized
with respect to the dissipated energy, but no iterations are made on the reduc-
tion coefficients. The following bounds are imposed:

$$1 \leq \omega \leq 2; \quad 1 \leq \tilde{\eta} \leq 13; \quad 0 \leq h_c \leq 0.60 \text{ m}; \quad 0 \leq \lambda \leq 8\%; \quad s \geq 0.15 \text{ m}.$$  

The values of the design parameters and of the main quantities are shown in
Tab. 1, along with the values of the dissipated energy. As can be seen unsymme-
trical values of $\omega_i$ and $\tilde{\eta}_i$ are obtained for symmetrical walls, because of the
slow convergence in the neighbourhood of the maximum point. This is due to the
flatness of the objective function in that region, as is clearly shown in the
last two rows of Tab. 1, where parameters and quantities relevant to the calcu-
lated and to the actual maximum of the dissipated energy are reported. The de-
sign parameters, however, tends to assume the same values $\omega_i = 1.72; \tilde{\eta} = 13,$
for all the rows, independently of their number in the wall. Correspondingly depth and ultimate shear of beams decrease, while the dissipated energy increases, with the number of rows increasing. It is interesting to notice the marked increment of energy dissipation, when passing from 10 to 15 stories. This is due to a more uniformly distributed energy dissipation consequent to a nearly constant rotation of the beams along the height of the wall.

The second set is relevant to walls with 10 and 15 stories (h = 3.00 m) with only one row of openings; the width of the openings is maintained constant for all the walls (b_1 = 1), while their position is changed; they are centered (eccentricity e_τ = 0, B_1 = B_2 = 7) or eccentric (e_τ = 3, B_1 = 9, B_2 = 5; e_τ = 4, B_1 = 11, B_2 = 3). These walls have been designed without maximizing the dissipated energy; the values assumed for the design parameters are ω = 1.2, 1.5 and 5, 10. The initial values of the reduction coefficients are 0.5 for flexural inerties and 0.7 for areas, for all the walls. In Tab. 2 the reduction coefficients for areas (A) and flexural inerties (J) of wall 1 and 2, for seismic forces from right (R) and from left (L) are shown. For ω = 1.5, 5 there are no possible solutions with h_c > 0 and the results for h_c = 0 are not reported. As will be seen, their values are quite different, when the single wall is in tension (1L, 2R) or in compression (1R, 2L) by the shear forces of the beams. They are also little sensitive to the variations of the design parameters and to the number of stories. The eccentricity plays an important role only for the smaller of the two single walls. In fig. 2 the variations of the coefficients during the iterative procedure are shown for two of the designed walls. It is worthwhile to notice that the procedure is rapidly convergent except for wall 2 (the smaller one) when loaded from left (compressed); in this case the coefficient tends to assume the midvalue between two successively calculated values.

![Graphs](image)

a) ω = 1.2, 5 = 10  
b) ω = 1.5, 5 = 10

*Fig. 2 - Convergence of the iterative procedure for evaluating the stiffness reduction coefficients.*

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CONCLUSION

The results obtained with the design method presented in this paper, suggest some interesting observations on the effectiveness of the method and on the behaviour of the walls with one or more vertical rows of openings.

In particular it has been seen that the variations of axial and flexural stiffness, due to cracking in the walls, are very marked; therefore they must be correctly calculated and taken into account when designing coupled walls.

The optimal values of the design parameters are, at least for symmetrical walls, univocally defined and are equal for all the vertical rows, independently of the total number of rows. In proximity of its maximum, however, the objective function (i.e. the energy dissipation) is little sensitive to variations of the parameters, so that little differences with respect to the optimal values cause negligible decrements of the dissipated energy. As seen in a previous paper (Ref. 6) for a single row, the maximum value of the objective function is located on the frontier \( \tilde{\eta} = \tilde{\eta}_{\text{max}} \) and corresponds to a relative maximum with respect to \( \tilde{\eta} \).

These considerations suggest further developments of the research, both for improving the algorithm and for better understanding the behaviour of the walls. In particular it appears important to calculate (iteratively) the stiffness reduction of the coupling beams, since it can considerably affect their ductility ratio. Extensive parametric investigations are still needed, in order to assess the behaviour of walls with several unsymmetrically arranged rows of openings.

REFERENCES


ACKNOWLEDGMENTS

The authors are indebted to Dr. Fabio Lozupone for the program WALLOP.
### Tab. 1 - Design parameters, energy dissipated and design quantities for the first set of walls

<table>
<thead>
<tr>
<th>WALL MOD.</th>
<th>DESIGN PARAMETERS</th>
<th>DISS. ENER.</th>
<th>DESIGN QUANTITIES (KN, m)</th>
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<tr>
<td></td>
<td>( n_s ) ( n_r ) ( \omega_1 ) ( \omega_2 ) ( \eta_1 ) ( \eta_2 ) ( \eta_3 ) ( E_d ) ( s ) ( W_L ) ( h_{C1} ) ( h_{C2} ) ( h_{C3} ) ( \nu_{U1} ) ( \nu_{U2} ) ( \nu_{U3} ) ( d_{%} )</td>
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### Tab. 2 - Stiffness reduction coefficients and design quantities for the second set of walls

<table>
<thead>
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<th>WALL MODEL</th>
<th>DESIGN PARAMET.</th>
<th>STIFFNESS REDUCTION COEFFICIENTS</th>
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<td></td>
<td>( e_f ) ( n_s ) ( \omega ) ( \lambda )</td>
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