UPPER BOUND FLOOR RESPONSE SPECTRA

A.J. Philippacopoulos (I)
P.C. Wang (II)
C.A. Miller (III)
C.J. Costantino (IV)
Presenting Author: A.J. Philippacopoulos

SUMMARY

The upper bound floor spectra presented in this paper are generated by a "worst-case" analysis that takes into account past recorded earthquakes as well as structural characteristics. The objective is to maximize the floor spectral accelerations at all frequencies for a linear combination of seismic accelerograms that can be used for a particular site. The spectral curves thus generated are higher than any of those produced by the accelerograms applied individually to the structure. The conservativeness introduced is within reasonable design limits.

INTRODUCTION

Floor response spectra are extensively used for the seismic evaluation of light subsystems such as piping and electrical or mechanical equipment. These spectra are obtained from mathematical models of the buildings that are supporting such items. Provided that decoupling is justified, the motion of the building at the locations of the supports can be used for the evaluation of the subsystem response. This motion is conventionally filtered through a single-degree-of-system oscillator to produce the floor spectrum.

To obtain floor response spectra, various methods are available. The most common in the present state of the art is the time history method. Other methods have been proposed by Biggs and Roesset (Ref. 1), Biggs (Ref. 2), Kapur and Shao (Ref. 3) and recently by Singh (Ref. 4). All these approaches deal with the question of how for a given building and input ground motion one can obtain floor spectra. In the present paper an upper bound approach is presented which considers both the site dependency as well as the system dependency of the floor spectra. Furthermore, the values of the floor spectral accelerations are obtained through a maximization procedure. The approach is based on the concept of the critical

---

(I) Assoc. Scientist, Brookhaven National Laboratory, Upton, NY 11973

(II) Brookhaven National Laboratory, Prof. of Civil Engineering, Polytechnic Institute of New York

(III) Prof. of Civil Engineering, City College of New York

(IV) Prof. of Civil Engineering, City College of New York

excitation. A similar approach has been reported previously for the seismic assessment of life-line structures (Ref. 5).

Although the method can be easily applied to individual structures, a standarization of it is also demonstrated. This is accomplished here by considering variations of the spectral frequency at which the dominant spike occurs. This frequency depends, in general, on the soil-structure system characteristics. Typical variations of soil-structure system parameters that reflect conditions met in practice were considered to establish a data basis containing individual upper bound floor spectra. From these data, generic floor spectra are obtained.

APPROACH

Assume that a collection of N earthquake recorded accelerograms i.e., $\ddot{x}_j(t)$; j = 1, ..., N are available for a given site. Such records may have been recorded either at this particular site or at other sites with similar geological conditions. The floor response spectra of a building located at the given site due to the above individual accelerograms can be generated using available methods.

Consider now a class C of records constructed by linear combinations of $\ddot{x}_j(t)$ i.e.,

$$\ddot{z}(t) = \sum_{j=1}^{N} w_j \ddot{x}_j(t) \tag{1}$$

where w_j are the weighing coefficients of the superposition. Furthermore, the class C is characterized by the condition that the intensities of its members do not exceed a value E appropriate for the given site. The intensity measure used here is the root-square (RS) of the record. Thus the above condition can be mathematically expressed as:

$$\left(\int_o^t \ddot{z}(t)dt\right)^{1/2} \leq E \tag{2-a}$$

or:

$$\{w\}^T[G]\{w\} \leq E^2 \tag{2-b}$$

where:

$$g_{ij} = \int_o^t \ddot{x}_i(t)\ddot{x}_j(t)dt \tag{2-c}$$

Any record $\ddot{z}(t)$ which belongs to the class C can be used to generate floor response spectra for the building located at the site. The upper bound floor spectra presented here are associated with the particular member of the class C whose the weighing coefficients w_j will maximize the floor spectral acceleration. The mathematical formulation of the method is given next.

FORMULATION

The motion of the building of interest located at the given site is usually described by a descrete mathematical model which employs certain number of degrees-of-freedom (DOF). The floor spectra of the building are associated with its DOF's. The acceleration response associated with the i-th DOF (i.e., floor acceleration time history) due to the j-th accelerogram $\ddot{x}_j(t)$ is:

$$\ddot{y}_{ij}(t) = h_i(t) * \ddot{x}_j(t) \tag{3}$$

where * denotes convolution operation and $h_i(t)$ is the unit impulse response of the building that corresponds to the transfer function between the free-field and the floor acceleration. Such functions are easily obtained by standard methods of structural dynamics. Furthermore, for linear structural behavior the acceleration response of the i-th DOF due to the input $\ddot{z}(t)$ is:

$$\ddot{u}_i(t) = \sum_{j=1}^{N} w_j \ddot{y}_{ij}(t) \tag{4}$$

Provided that decoupling is justified, the floor response spectrum associated with the i-th DOF of the building is:

$$S_a = \max_t |\ddot{q}(t)| \tag{5}$$

where $\ddot{q}(t)$ represents the acceleration response function of a single degree-of-freedom oscillator due to $\ddot{u}_i(t)$. This response can be obtained by a convolution operation as:

$$\ddot{q}(t) = h_a(t) * \ddot{u}_i(t) \tag{6}$$

where $h_a(t)$ is the acceleration unit impulse response of the oscillator for given frequency and damping.

From Eqs. 3, 4 and 6 it is obtained:

$$\ddot{q}(t) = \sum_{j=1}^{N} w_j \left[h_a(t) * \ddot{y}_{ij}(t)\right] = \sum_{j=1}^{N} w_j \{h_a(t) * [h_i(t) * \ddot{x}_j(t)]\}$$

$$= \sum_{j=1}^{N} w_j \{[h_a(t) * h_i(t)] * \ddot{x}_j(t)\} = \sum_{j=1}^{N} w_j \ddot{v}_{ij}(t) \tag{7}$$

using known properties of the convolution. Equation 7 can be written in matrix form as:

$$\ddot{q}(t) = \{w\}^T \{\ddot{v}_i\} \tag{8}$$

where the N-vector $[\ddot{v}_i]$ has typical elements $\ddot{v}_{ij}(t)$; $j = 1, \ldots, N$. The function $\ddot{v}_{ij}(t)$ is the spectral floor acceleration time history of the i-th DOF of the building due to the j-th earthquake. Thus the floor spectral value S_a for a given set of frequency and damping values due to the j-th earthquake is:

$$S_a = \max_t |\ddot{v}_{ij}(t)| \tag{9}$$

The upper bound floor spectral value S_a^U for the same set of spectral frequency and damping is obtained here as:

$$S_a^U = \max_t |\ddot{r}(t)| \tag{10}$$

where $\ddot{r}(t)$ is obtained by the following maximization under a constraint:

$$\left.\begin{aligned} \ddot{r}(t) &= \max_{w_j} |\ddot{q}(t)| \\ \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j g_{ij} &\leq E^2 \end{aligned}\right\} \quad j = 1,\ldots,N \tag{11}$$

The maximization under the constraint expressed by Eqs. 11 can be carried out using the method of Lagrange's multipliers. Based on this, the maximizing vector $\ddot{r}(t)$ is:

$$\ddot{r}(t) = E\left(\{\ddot{v}_i\}^T [G]^{-1} \{\ddot{v}_i\}\right)^{1/2} \tag{12}$$

From Eqs. 10 and 12 it is obtained:

$$S_a^U = \max_t |Es(t)| \tag{13-a}$$

where:

$$s(t) = \left(\{\ddot{v}_i\}^T [G]^{-1} \{\ddot{v}_i\}\right)^{1/2} \tag{13-b}$$

Equation 13-a gives the upper bound floor spectral value for given spectral frequency and damping.

In order to acertain the conservatism introduced by this approach, the upper bound floor spectral value S_a can be compared with an envelope spectral value which may be defined as:

$$S_a^E = \max_j (S_a) = \max_j \max_t |\ddot{v}_{ij}| \tag{14}$$

that is, the maximum floor spectral value among those produced individually by the selected earthquakes for the given spectral frequency and damping. Spectral curves shown latter indicate:

$$S_a^U > S_a^E \tag{15}$$

by a reasonable margin which may be justified for particular design cases.

APPLICATION

In order to demonstrate the concept of the upper bound floor spectrum a typical soil-structure interaction system was employed. This system consists of a single-mode superstructure resting on a flexible foundation.

It combines a flexural structural mode with two rigid body modes i.e., translation and rocking. Although it may be argued that this system is simple as compared with more complex models used for structures such as nuclear, it nevertheless, incorporates the most important features of the interaction. Furthermore, such simple models have been used to study the behavior of nuclear structures (Ref. 6).

A closed form solution for transfer functions was carried out. The transfer function between the free-field and the floor acceleration was found to be:

$$H(\omega) = -(\omega_o^2 + 2\xi_o\omega_o\omega i)H_o(\omega) \tag{16-a}$$

where:

$$H_o(\omega) = \frac{T_1 + i\omega T_2}{P_1 + 2\omega P_2} \tag{16-b}$$

Expressions for T_1, T_2, P_1 and P_2 are given in the appendix. ω_o and ξ_o are the fixed-base structural circular frequency and the damping respectively. The unit impluse response $h_i(t)$ in Eq. 7 can be obtained from $H(\omega)$ by Fourier transformation.

NUMERICAL RESULTS

Floor spectral curves were generated according to the approach described previously using a CDC 7600 computer system. A set of twelve horizontal recorded earthquake accelerograms were selected from available data (Ref. 7) to represent $\ddot{x}_j(t)$ $j = 1, .., N$. The intensity of the El Centro 1940 earthquake was used as the reference intensity E. In the evaluation of the spectral curves, the structural and the soil damping were treated in a direct manner according to Eq. 16. Therefore, composite type of damping was not used in deriving the floor spectra. Finally, a typical range of structural and interaction parameters was used according to Ref. 8. A comparison between the upper bound floor spectrum and the envelope one is shown in Fig. 1 for a given value of the fixed-base structural frequency and for different interaction frequencies. A set of upper bound curves similar to the one depicted in Fig. 1 were obtained by varying the structural frequency from 2 to 15 Hz. From these latter curves, a generic one was obtained and it is depicted in Fig. 2.

CONCLUSION

An approach for generating upper bound floor response spectra is presented. The spectral curves are produced from recorded ground motions and are higher than any of those produced by these motions acting individually to the structure. The conservatism introduced by this approach is within reasonable engineering limits and may be justified in the seismic analysis of certain important subsystems.

REFERENCES

1. Biggs, J.M. and Roesset, J.M., "Seismic Design of Nuclear Power Plants", MIT Press, Cambridge, MA, 1970.

2. Biggs, J.M., "Seismic Response Spectra for Equipment Design in Nuclear Power Plants", First International Conference on Structural Mechanics in Reactor Technology, Berlin, September 1971, Paper K 4/7.

3. Kapur, K.K., Shao, L.C., "Generation of Seismic Floor Response Spectra for Equipment Design", Specialty Conference on Structural Design of Nuclear Plant Facilities, ASCE, Chicago, IL, 1973.

4. Singh, M.P., "Generation of Seismic Floor Spectra", Journal of the Engineering Mechanics Division, ASCE, October 1975.

5. Wang, P.C. and Philippacopoulos, A.J., "Critical Assessment of Life-line Structures", Proceedings of the Seventh World Conference on Earthquake Engineering, Instanbul, Turkey 1980.

6. Roesset, J.M., "The Use of Simple Models in Soil-Structure Interaction", Second ASCE Conference on Civil Engineering and Nuclear Power, Vol. II, Geotechnical Topics, Tennessee, September 1980.

7. Strong Motion Earthquakes Accelerograms, California Institute of Technology, Pasadena, California.

8. Philippacopoulos, A.J., Miller, C.A., "An Assessment of Soil-Structure Interaction Effects Based on Simple Models", 7th International Conference on Structural Mechanics in Reactor Technology, Chicago, IL, August 1983, Paper K 10/3.

APPENDIX

Parameters T_1, T_2, P_1 and P_2 in Eq. 16-b are

$$T_1(\omega) = r_1\omega_t \quad (r_2-1)\omega_t + 4r_2\xi_t\xi_r\omega_r \quad \omega^2 - r_2\omega_t\omega_r^2$$

$$T_2(\omega) = 2r_1\omega_t \quad (r_2-1)\xi_t\omega^2 - r_2(\xi_t\omega_r + \xi_r\omega_t)\omega_r$$

$$P_1(\omega) = q_6\omega^6 + q_4\omega^4 + q_2\omega^2 + q_o$$

$$\omega P_2(\omega) = q_5\omega^5 + q_3\omega^3 + q_1\omega$$

where:

ω_t, ω_r = interaction circular frequencies in translation and rocking.

ξ_t, ξ_r = interaction damping in translation and rocking.

r_1, r_2 = mass ratios between superstructure and foundation.

$q_o, \ldots, q_6$ = polynomials of structural and interaction parameters.

ω = frequency variable.

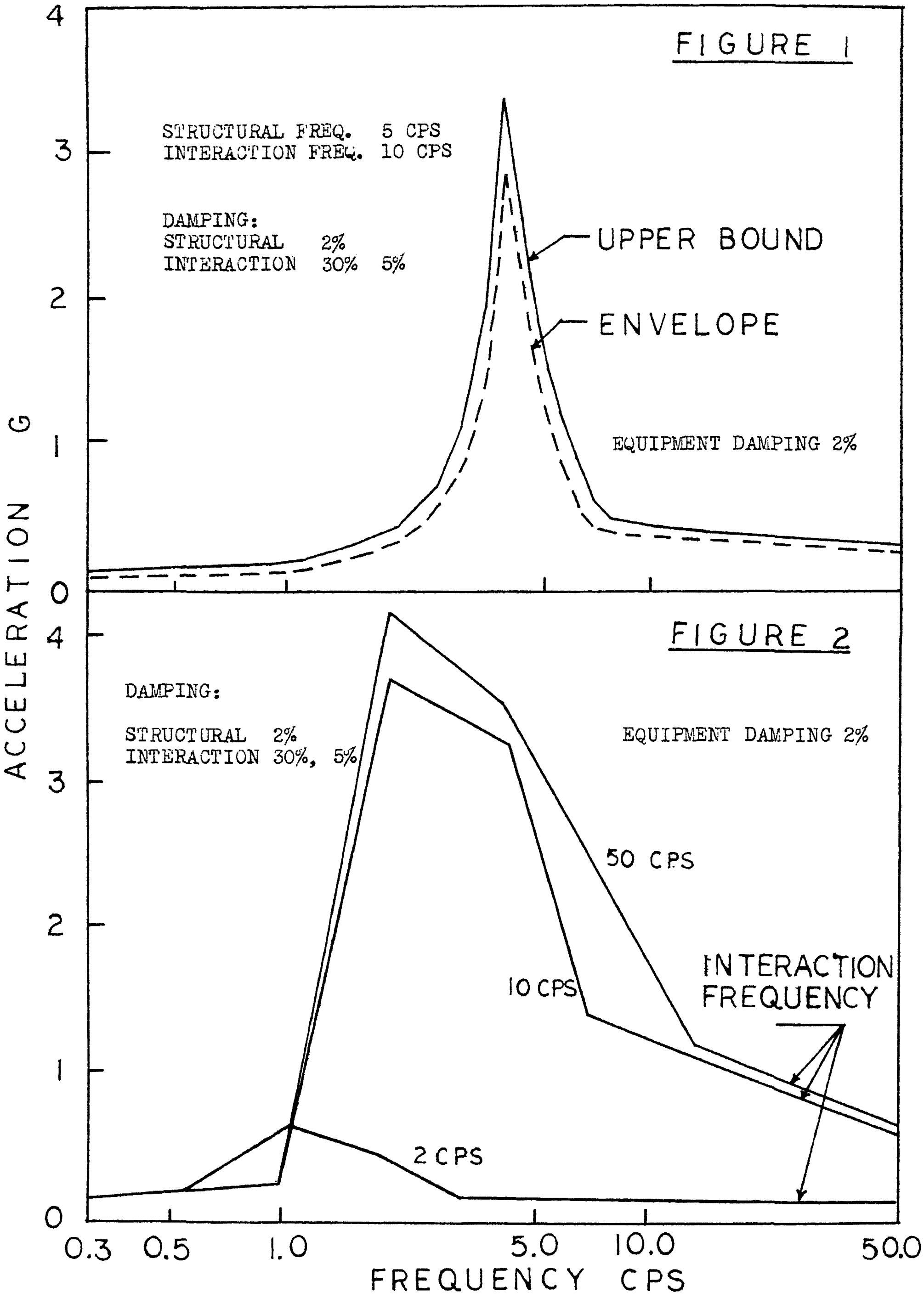
FIGURE 1
STRUCTURAL FREQ. 5 CPS
INTERACTION FREQ. 10 CPS
DAMPING:
STRUCTURAL 2%
INTERACTION 30% 5%
UPPER BOUND
ENVELOPE
EQUIPMENT DAMPING 2%
FIGURE 2
DAMPING:
STRUCTURAL 2%
INTERACTION 30%, 5%
EQUIPMENT DAMPING 2%
50 CPS
10 CPS
2 CPS
INTERACTION FREQUENCY
ACCELERATION G
0
1
2
3
4
0.3
0.5
1.0
5.0
10.0
50.0
FREQUENCY CPS